A MANUAL

OF

LAND SURVEYING

COMPRISING

AN ELEMENTARY COURSE

OF

PRACTICE WITH INSTRUMENTS

AND A TREATISE UPON THE

SURVEY OF PUBLIC AND PRIVATE LANDS

PREPARED

FOR USE OF SCHOOLS AND OF SURVEYORS

BY

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" Let things that have to be done be learned by doing them "

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PREFACE.

In making subdivisions of Government Surveys, or in resurveying old boundary lines, every surveyor has felt the need of definite instructions relating to a multitude of questions found to arise in the work.

The function of a surveyor in most of these cases is a judicial one, and the answers to those questions are to be found only in the decisions of courts which are practically inaccessible to him.

When for the want of this knowledge, which was beyond his reach, the surveyor has gone astray in his conclusions, the courts have sharply criticised his actions, and he has been looked upon and treated as a disturber of the public peace. This state of affairs early claimed the attention of the Michigan Association of Surveyors and Engineers, and was emphasized to them by Judge T. M. Cooley, who told them that "Courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his actions by the same lights and the same rules that govern theirs."

To meet this desideratum, this flourishing society early appointed a Committee to prepare a manual which should give the instruction so much needed. This Committee began the work in 1881, and ever since have prosecuted it as diligently as the professional duties of its members would permit. *The decisions of the Supreme Courts of the United States and of the several States have been ransacked almost from beginning to end,* and out of the great mass of material thus acquired, *the substance of about 160 Supreme Court decisions has been selected,*

giving authoritative rules for, as nearly as possible. every case which has arisen or is likely to arise in the surveyor's practice. *Laws were collected, instructions from the General Land Office gathered,* and the *reports of Surveyors' Associations* examined and culled from, until everything having an important bearing upon the subject has been brought together in a compact and usable shape.

It soon, however, became apparent to the Committee that the matter they were preparing ought of propriety to be incorporated in the text-book of students in land surveying; and thus it was determined to make the Manual a book for use of schools, as well as one of reference for the surveyor in the field. In carrying out this purpose, a mathematical treatise, in somewhat the usual form of text-books upon surveying, has been prepared, as leading up to the portion of the work above referred to.

This part of the work will be found to embrace the necessary definitions, descriptions of instruments and their adjustments, and an elementary course of practice with them in the field.

The attempt has been made to introduce a new department in books designed for use of students in learning the art of surveying, as based upon the principle that the only way to learn an art, successfully, is by actually performing the operations involved in its pursuit. Accordingly, it is thought that *the true way to learn to survey is by surveying.*

In developing this plan, we have begun with the chain or tape, as the simplest and safest instrument to be placed in the hands of the beginner, and have set the student to measuring lines and, with the platting instruments, drawing them to scale. From this he proceeds to the solution with the chain of a few simple problems in setting out perpendiculars and parallels, measuring inaccessible lines and finding the contents of fields of various forms and under different plans of measurement.

A similar course is pursued with the compass, the transit, and the level.

The design of the work thus proposed is, however, not so much to instruct or familiarize the student concerning any manner of solving the problems of surveying, as it is to provide him practice with the instrument employed, to the end of learning something of its powers and adaptations, and of acquiring some degree of handiness in its use. Different methods of solution of the same problem are in many cases suggested, with a view to extending the use of the problem to purposes of additional practice.

The form generally employed, of giving suggestions as *to how* the problem may be solved rather than so-called solutions, has been dictated by two considerations: (1) The real solution of a problem in surveying is the operation of obtaining the required result, as performed on the ground; (2) The simple and off-hand way in which a surveyor would naturally indicate to a class of boys the manner of performing a given operation with an instrument, is quite in contrast with the stiffness and formality of diction which usually obtain in a written solution.

Again, as to particular procedures indicated by the suggestions, it is hoped that the field has been left entirely open for any who may occupy it to offer any different or additional advice to the student with the same freedom we have assumed for ourselves. We trust we have left ample opportunity of this kind. Doubtless things have been omitted which some would regard as important to have introduced. Such omissions will readily be supplied by teachers at their pleasure and convenience.

In order to keep the book within what was regarded as proper limits of size, and to present a treatment of the subject on a level with the average opportunities for its pursuit in schools at large, it has been as necessary to determine what might safely be left out, as to decide what ought properly to be put in.

Again, account is to be taken of the fact that no book,

however full, can make a surveyor. At best, any treatise upon the subject can be but an introduction to surveying —can serve only as a door leading into the actual experiences of the "field" which alone can give proficiency in the art.

Our purpose has been simply to point out what we conceived to be the rational way of approach to those experiences, and thus to make some slight contribution to what may be termed the methodology of surveying.

The task, undertaken with no little hesitation in view of the considerable burden of duties regularly devolving upon us as a teacher, is now done, and we commit our work alike either to kindly criticism or to generous approval.

The following acknowledgments are proper to be made in the present connection:

(1) We desire to confess our indebtedness to the many excellent works on Surveying which have been consulted in the preparation of this volume, and especially to those of Davies and Gillespie, for valuable suggestions incorporated in the course of the work.

(2) We wish to express in this place our most sincere thanks to Messrs. W. and L. E. Gurley, manufacturers of engineers' and surveyors' instruments, at Troy, N. Y., for granting us, most cordially, the use of their plates for our cuts of instruments, and for the privilege of using the descriptions of instruments and adjustments contained in their Manual.

(3) Our thanks are due also to the officers and members of the Michigan Engineering Society, and with them to many other surveyors and mathematical teachers throughout the country, for the hearty interest and helpful spirit they have manifested in the publication of this treatise.

C. F. R. BELLOWS.

YPSILANTI, May 1, 1886.

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ERRATA.

P. 16.—Seventh line of Art. 19, for 2 sin 20°, read 2 sin 30°.

P. 77.—Second line of Ex. 3, for east of *A*, read north of *A*; and for from *A* run S., read from *A* run W. Also, fourth line, for east from *C*, read west from *C*.

Seventh line of Art. 72, for ⅓ of a second, read ⅓ of a minute.

P. 165.—Second line from bottom, for Fig. 109, read Fig. 111.

P. 170.—Fifteenth line, for Fig. 109, read Fig. 111.

P. 172.—For Section VII, read Section VI.

P. 196.—For Section VIII, read Section VII.

P. 215.—Second line of Remarks, for Secs. 13 and 14, read Secs. 11 and 14. Also, last line of Remarks, for 5 27° W., read S. 27° W.

P. 216.—Tenth line from bottom, for sections, read stations.

A MANUAL

OF

LAND SURVEYING.

SECTION I.

INTRODUCTION.

A brief treatment of the subjects of Logarithms and Trigonometrical Formulas is here introduced for the purpose of review and reference by such as may desire it.

I. LOGARITHMS.

1. **The Logarithm** of any given number is the exponent with which a fixed number must be affected to produce the given number.

The fixed number is called the **Base**.

The base of the common system of logarithms is 10.

Since $10^0 = 1$, the logarithm of 1 is 0.
" $10^1 = 10$, " " " 10 is 1.
" $10^2 = 100$, " " " 100 is 2.
etc., etc., etc.

Thus, the logarithms of all powers of the base are *integral numbers*.

2. The logarithms of numbers intervening between exact powers of the base are composed of an integral and a fractional part. The integral part of a logarithm is called the **Index** or **Characteristic**; and the fractional part, usually expressed decimally, is called the **Mantissa**, or simply the **decimal part**.

3. Properties of Common Logarithms.—From the nature of exponents considered in connection with the decimal notation result the following principles:

I. *The logarithm of any exact power of 10 is a positive integer one less than the number of places of figures in the expression of the number.*

For, by definition, $\log 10^p = p$. But 10^p is expressed decimally by 1 with p ciphers annexed, and hence contains $p + 1$ places of figures.

II. *In general, the characteristic of the common logarithm of any integral number is one less than the number of figures in the expression of the number.*

For, denoting the characteristic of the logarithm by c, and the decimal part by d, we may write $\log 10^{c+d} = c+d$. But, as $d < 1$, 10^{c+d}, expressed decimally, contains $c+1$ figures.

III. *The characteristic of the common logarithm of any fractional number less than* 1, *expressed decimally, is negative and numerically one more than the number of zeros immediately following the decimal point.*

For, let $\frac{10^{c+d}}{10^p}$ denote the fraction.

The numerator 10^{c+d}, under the decimal notation, has $c+1$ figures, as already observed, while the expression of the fraction by the same notation requires p places of figures. Since the decimal under consideration is, by hypothesis, not a mixed one, the number $c+1$ of figures in the numerator cannot *exceed* the number p of figures at the right of the decimal point, but may be less, the deficiency being supplied by the zeros prefixed to the $c+1$ figures of the numerator.

In either case $p-(c+1)$ is the number of zeros immediately following the decimal point. Let $p-(c+1)=n$, whence $c-p = -(n+1)$. Now $\frac{10^{c+d}}{10^p} = 10^{c-p+d}$, and $\log 10^{c-p+d} = c-p+d$, of which $c-p$ is the index.

Substituting the value of $c-p$ found above, we have

$$\log \frac{10^{c+d}}{10^{p}} = -(n+1)+d.$$

SCHOLIUM.—In the notation of the negative characteristic, the minus sign is usually written above the characteristic.

Thus, $\log 0.00045 = \bar{4}.653213$.

IV. *The characteristic of the common logarithm of a mixed decimal is the same as the characteristic of the logarithm of the integral part.*

For, using the same notation as above, $c-p+1$ is the number of places of figures in the integral part; and $c-p$ is the characteristic of the logarithm of the given decimal.

V. *The decimal part of the common logarithm of a number is not changed by multiplying or dividing the number by any power of 10.*

For the logarithm of 10^{c+d} is $c+d$, and the logarithm of $10^{c+d\pm p}$ is $c\pm p+d$ in which, since c and p are integers, $c\pm p$ is the characteristic and d is the decimal part, the same after the multiplication or division of 10^{c+d} by 10^{p} as it was before.

4. Table of Logarithms.—In Table I at the end of the book, are given the logarithms of numbers from 1 to 10000, true to six decimal places.

The characteristics, however, are usually not entered in the Table, but are supplied by inspection (Prin. II).

The manner of using the Table is illustrated in the solution of the following problems:

Prob. 1.—*Given a number, to find its logarithm.*

CASE 1.—*When the number is between 1 and 1000.*

Solution.—We find the given number in the left hand column of the Table, and opposite to it in the next column, the decimal part of the logarithm sought, the two left hand figures of the next preceding full set of six figures being understood where only four figures occur. To the decimal part of the logarithm as thus obtained,

we prefix the proper characteristic as found by inspection.

Thus, log 87 = 1.939519; log 237 = 2.374748.

CASE 2.—*When the number is between 1000 and 10000.*

Solution.—We find the number expressed by the left hand three figures of the given number in the left hand column, and then pass horizontally across the page to the column headed by the right hand figure of the given number. Here we find either the six figures of the mantissa or the right hand four figures of it. In the latter case, the left hand two figures of the next preceding number of six figures are to be prefixed, together with the proper characteristic.

Thus, log 4712 = $3.673\overset{205}{\cancel{942}}$; log 9354 = 3.970997.

CASE 3.—*When the number exceeds 10000.*

Solution.—We find as in Case 2 the mantissa of the logarithm of the number expressed by the left hand four figures of the given number. We then take the number opposite in the column headed "Diff" and multiply it by the remaining figures of the given number, rejecting from the right of the product as many figures as there are in the multiplier. We then add the part of the product retained, to the mantissa as before found, and prefix the proper characteristic.

SCH.—When the left hand figure of those rejected is 5 or more, the part retained is to be increased by 1.

Thus, log 34256 = $4.534661+127\times.6=4.534737$;

and log 283745 = $5.452859+153\times.45=5.452928$.

CASE 4.—*When the number is a decimal.*

Solution.—We find the mantissa, regarding the number as integral, and prefix the proper characteristic, (Prin. III or IV).

Thus, log 0.472 $=\bar{1}.673942$; log 37.25=1.571126.

Prob 2.—*Given any logarithm, to find the corresponding number.*

CASE 1.—*When the mantissa is found in the Table.*

Solution.—We take out the corresponding number and

place a decimal point agreeably with the given characteristic.

Thus, of the logarithm 2.359076, we find 228.6 as the corresponding number; of the logarithm 4.542825, we find 34900 as the number sought.

CASE 2.—*When the mantissa is not found in the Table.*

Solution.—We take from the Table the next less mantissa and the corresponding number. We then subtract that mantissa from the given one, and divide the remainder by the number opposite in column "Diff." We annex the figures of the quotient to the number taken from the Table and place a decimal point agreeably with the given characteristic. Thus, given the logarithm 2.456789, we take from the Table the mantissa .456670 and the corresponding number 2862. Then (.456789—.456670)÷152=0.78, we annex to 2862 and move the decimal point one place toward the left, forming 286.278 as the expression of the number sought.

Uses of Logarithms.—From the nature of a logarithm as an exponent of a fixed number, it follows that the addition of logarithms answers to the operation of multiplying together the corresponding numbers; that the subtraction of logarithms answers to the division of the corresponding numbers, one by the other; and that the multiplication or division of a logarithm by an integer answers, respectively, to raising the corresponding number to a power or extracting a root of it.

Whence the following rules:

5. To multiply one number by another.

RULE.—*Add the logarithm of the numbers, and find the number corresponding to their sum.*

Thus, to multiply together 34.23, 7.581 and 0.039 we have

$$\begin{aligned}
\log 34.23 &= 1.534407 \\
\log 7.581 &= 0.879726 \\
\log 0.039 &= \bar{2}.591065 \\
\hline
\log \text{prod.} &= 1.005198
\end{aligned}$$

Corresponding number 10.1204, which is the product sought.

6. To divide one number by another.

RULE.—*Subtract the logarithm of the divisor from the logarithm of the dividend, and find the number corresponding to the remainder.*

Thus, to divide 32.75 by 4.087 we have

$$\begin{aligned} \log 32.75 &= 1.515211 \\ \log 4.087 &= 0.611405 \\ \hline \log \text{quot.} &= 0.903806 \end{aligned}$$

Corresponding number 8.0132, which is the quotient sought.

SCH.—From the rule we have

$$\log (m \div n) = \log m - \log n = \log m + 10 - \log n - 10,$$

by adding and subtracting the base. The remainder, as $10 - \log n$, obtained by subtracting a logarithm from the base is called the **Arithmetical Complement** of the logarithm, and written *ar. co. log*. We may therefore write

$$\log (m \div n) = \log m + \text{ar. co. } \log n - 10.$$

Whence the following

RULE.—*To the logarithm of the dividend add the arithmetical complement of the logarithm of the divisor, and subtract 10. Find the number corresponding to the resulting logarithm, as the quotient sought.*

The arithmetical complement of a logarithm is conveniently found by subtracting each figure of the logarithm from left to right, from 9 until the last which is subtracted from 10.

Thus, ar. co. log 3.107486 is 6.892514.

In this way the ar. co. log is conveniently written directly from the Table.

7. To raise a number to a power.

RULE.—*Multiply the logarithm of the number by the exponent of the power, and find the number corresponding to the product.*

Thus, to raise 2.143 to the 10th power, we have

$$\log 2.143 = 0.331022.$$
$$\log \text{power} = 0.331022 \times 10 = 3.310220.$$

Corresponding number 2127.73, which is the power sought.

8. To extract a root of a number.

RULE.—*Divide the logarithm of the number by the index of the root, and find the number corresponding to the quotient.*

Thus, to extract the 5th root of 153.4 we have

$$\log 153.4 = 2.185825.$$

$$\log \text{root} = 2.185825 \div 5 = 0.437165.$$

Corresponding number 2.7363, which is the root sought.

SCH.—A general demonstration of the above rules will be readily given by students familiar with "Theory of Exponents."

EXERCISES.

9. 1. Find the product of 35.68 by 117, by use of logarithms. Result, 4174.56.

2. Find the product of 58.327 by 3.56; of 23.8612 by 0.0345.

3. Find the continued product of 1378, 278.63 and 0.275.

4. Find the quotient of 2578 divided by 123.

Result, 20.9593.

5. Find quotient of 4.26 divided by 0.345; of 0.000376 divided by 12.34; of 58.6185 divided by 321.78.

6. Divide the product 3827×63.5 by 3.58.

Operation.—

$$\begin{aligned} \log 3827 &= 3.582858 \\ \log 63.5 &= 1.802774 \\ \text{ar. co. } \log 3.58 &= \underline{9.446117} \\ \log \text{quotient} &= 4.831749 \end{aligned}$$

Corresponding number, 67881.1.

7. Find the fourth term of the proportion 47.863 : 0.58 :: 0.0375 : ——

8. Find 5th power of 15 by logarithms.

Operation.—

$$\begin{aligned} \log 15 &= 1.176091 \\ 5 \log 15 &= 5.880455 \end{aligned}$$

Corresponding number, 759373.7.

This result differs a little from the exact 5th power of 15, since the logarithm of 15 is only approximately given.

9. **Find 19^4, 136^3 and 4785^2 by logarithms.**

10. **Find the cube root of 37854.**

Operation.—$\log 37854 = 4.578112$

$\frac{1}{3} \log 37854 = 1.526037$

Corresponding number, 33.5766.

11. Find $\sqrt[5]{123456}$; $\sqrt[3]{45.61}$; and $\sqrt[7]{0.00037}$.

12. Given $x = \dfrac{135^2 \times \sqrt[3]{15}}{\sqrt[4]{11} \times 17^3}$.

Operation.—

$$\begin{aligned} \log\ 135^2 &= 4.260668 \\ \log \sqrt[3]{15} &= 0.392030 \\ \text{ar. co.} \log \sqrt[4]{11} &= 9.739902 \\ \text{ar. co.} \log\ 17^3 &= 6.308653 \\ \hline \log x &= 0.701253 \\ x &= 5.02636 \end{aligned}$$

It will be noticed that 10 is subtracted for each ar. co. log. which is used. The above example affords a good illustration of the utility of logarithms in abridging numerical computations.

II. Trigonometrical Formulas.

10. Functions of Angles.—Let $BAC = A$ be any angle. Let $BD = p$ be a perpendicular upon AC.

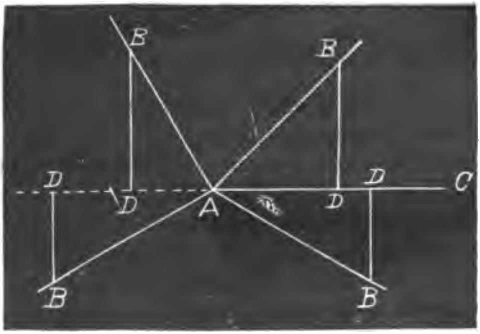

Fig. 1.

Let $AB = h$ and $AD = b$. Then

the ratio $\frac{p}{h} =$ Sine A, written $\sin A$.

" " $\frac{b}{h} =$ Cosine A, " $\cos A$.

" " $\frac{p}{b} =$ Tangent A, " $\tan A$.

" " $\frac{b}{p} =$ Cotangent A, " $\cot A$.

" " $\frac{h}{b} =$ Secant A, " $\sec A$.

" " $\frac{h}{p} =$ Cosecant A, " cosec A.

These ratios are called **Functions** of the angle.

SCH.—If $h = 1$. the functions are lines, and are called NATURAL FUNCTIONS.

11. Signs of Functions.—The signs of the functions depend upon the signs of p and b in accordance with the following conventionalities:

I. *Perpendiculars above the initial line are regarded as positive, and hence those below are negative.*

II. *Bases at the right of the vertex of the angle are regarded as positive, and hence those at the left are negative.*

Accordingly, for positive angles,* from 0° to 180°, the sine and cosecant are positive; and from 180° to 360°, the sine and cosecant are negative.

From 0° to 90°, or from 270° to 360°, the cosine and secant are positive; and from 90° to 270°, the cosine and secant are negative.

From 0° to 90° or from 180° to 270°, the tangent and cotangent are positive, and from 90° to 180° or from 270° to 360° the tangent and cotangent are negative.

SCH.—For negative angles, that is those regarded as generated by right-handed rotation, each of the functions, except the cosine and secant, changes sign.

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* Angles generated by the revolution of AC toward the left.

Thus, $\sin(-A) = -\sin A$, $\cos(-A) = \cos A$.
$\tan(-A) = -\tan A$, $\cot(-A) = -\cot A$.
$\sec(-A) = \sec A$, $\operatorname{cosec}(-A) = -\operatorname{cosec} A$.

12.—Values of Functions.—For the particular angles considered, the functions have the following numerical values:

$\sin 0°$ or $180° = 0$; $\sin 90°$ or $270° = 1$.
$\cos 0°$ or $180° = 1$; $\cos 90°$ or $270° = 0$.
$\tan 0°$ or $180° = 0$; $\tan 90°$ or $270° = \infty$.
$\cot 0°$ or $180° = \infty$; $\cot 90°$ or $270° = 0$.
$\sec 0°$ or $180° = 1$; $\sec 90°$ or $270° = \infty$.
$\operatorname{cosec} 0°$ or $180° = \infty$; $\operatorname{cosec} 90°$ or $270° = 1$.

SCH.—In using the above values, due regard must be had to their algebraic signs, (Art. 11).

13.—Relations of Functions of the Same Angle.—From the formulas of definitions (Art. 10), are readily deduced the following relations:

(*a*) *Of sine and cosine.* Let A be any angle and p, b and h, above described, the terms of the ratios constituting the functions of that angle, we have by definition,

$\operatorname{Sin} A = \frac{p}{h}$ and $\cos A = \frac{b}{h}$. Squaring and adding,

$\sin^2 A + \cos^2 A = \frac{p^2+b^2}{h^2} = \frac{h^2}{h^2} = 1$, whence are obtained

$$\operatorname{Sin} A = (1-\cos^2 A)^{1/2} \qquad (1)$$
$$\text{and } \cos A = (1-\sin^2 A)^{1/2} \qquad (2).$$

(*b*) *Of tangent and cotangent.*

By definition $\tan A = \frac{p}{b}$, whence $\tan A = \frac{1}{\frac{b}{p}} = \frac{1}{\cot A}$ (3).

whence also $\cot A = \frac{1}{\tan A}$ (4).

(*c*) *Of tangent and cotangent to sine and cosine.*

$$\operatorname{Tan} A = \frac{p}{b} = \frac{\frac{p}{h}}{\frac{b}{h}} = \frac{\sin A}{\cos A} \qquad (5).$$

$$\text{Cot } A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} \qquad (6).$$

(*d*) *Of secant and cosine.*

$$\text{Sec } A = \frac{h}{b} = \frac{1}{\dfrac{b}{h}} = \frac{1}{\cos A} \qquad (7).$$

$$\text{Whence also } \cos A = \frac{1}{\sec A} \qquad (8).$$

(*e*) *Of cosecant and sine.*

$$\text{Cosec } A = \frac{h}{p} = \frac{1}{\dfrac{p}{h}} = \frac{1}{\sin A} \qquad (9).$$

$$\text{Whence } \sin A = \frac{1}{\text{cosec } A} \qquad (10).$$

(*f*) *Of tangent and secant.*

$\text{Tan}^2 A = \dfrac{p^2}{b^2}$. $\text{Tan}^2 A+1 = \dfrac{p^2+b^2}{b^2} = \dfrac{h^2}{b^2} = \sec^2 A$, from which are obtained

$$\tan A = (\sec^2 A-1)^{\frac{1}{2}}, \qquad (11)$$

$$\text{and } \sec A = (\tan^2 A+1)^{\frac{1}{2}} \qquad (12).$$

(*g*) *Of the cotangent and cosecant.*

$\text{Cot}^2 A = \dfrac{b^2}{p^2}$. $\text{Cot}^2 A+1 = \dfrac{p^2+b^2}{p^2} = \dfrac{h^2}{p^2} = \text{cosec}^2 A$, from which are obtained

$$\cot A = (\text{cosec}^2 A - 1)^{\frac{1}{2}}, \qquad (13),$$

$$\text{and cosec } A = (\cot^2 A + 1)^{\frac{1}{2}}, \qquad (14).$$

14. Relations of Functions of Complemental and Supplemental Angles—The remainder from subtracting an angle from 90° is called the **Complement** of the angle, and from subtracting it from 180° is called the **Supplement** of the angle. The terms cosine, cotangent and cosecant as applied to an angle mean respectively, the sine, tangent and secant of the complement of the

angle. Thus, $\cos A = \sin(90° - A)$, $\cot A = \tan(90° - A)$ and $\operatorname{cosec} A = \sec(90° - A)$. Since in these expressions A is any angle, we may substitute for it $90° - A$, obtaining $\cos(90° - A) = \sin A$, $\cot(90° - A) = \tan A$ and $\operatorname{cosec}(90° - A) = \sec A$. Collecting and reversing members, we have

$$\left.\begin{array}{ll} \sin & (90° - A) = \cos A \\ \cos & (90° - A) = \sin A \\ \tan & (90° - A) = \cot A \\ \cot & (90° - A) = \tan A \\ \sec & (90° - A) = \operatorname{cosec} A \\ \operatorname{cosec} & (90° - A) = \sec A \end{array}\right\} \qquad (15)$$

If in (15), we suppose A to be $-A$, we have, (Art. 11, sch.),

$$\left.\begin{array}{ll} \sin & (90° + A) = \cos A \\ \cos & (90° + A) = -\sin A \\ \tan & (90° + A) = -\cot A \\ \cot & (90° + A) = -\tan A \\ \sec & (90° + A) = -\operatorname{cosec} A \\ \operatorname{cosec} & (90° + A) = \sec A \end{array}\right\} \qquad (16)$$

If in (16), we suppose A to be $90° - A$, we have on reducing by (15),

$$\left.\begin{array}{ll} \sin & (180° - A) = \sin A \\ \cos & (180° - A) = -\cos A \\ \tan & (180° - A) = -\tan A \\ \cot & (180° - A) = -\cot A \\ \sec & (180° - A) = -\sec A \\ \operatorname{cosec} & (180° - A) = \operatorname{cosec} A \end{array}\right\} \qquad (17)$$

Supposing A in (17) to be $-A$ and reducing, we obtain

$$\left.\begin{array}{ll} \sin & (180° + A) = -\sin A \\ \cos & (180° + A) = -\cos A \\ \tan & (180° + A) = \tan A \\ \cot & (180° + A) = \cot A \\ \sec & (180° + A) = -\sec A \\ \operatorname{cosec} & (180° + A) = -\operatorname{cosec} A \end{array}\right\} \qquad (18)$$

In a similar manner are obtained

$$\left.\begin{array}{ll} \sin & (270° \mp A) = -\cos A \\ \cos & (270° \mp A) = \mp \sin A \\ \tan & (270° \mp A) = \pm \cot A \\ \cot & (270° \mp A) = \pm \tan A \\ \sec & (270° \mp A) = \mp \operatorname{cosec} A \\ \operatorname{cosec} & (270° \mp A) = -\sec A \end{array}\right\} \qquad (19)$$

And

$$\left.\begin{aligned}\sin\ (360^\circ \mp A) &= \mp \sin A\\ \cos\ (360^\circ \mp A) &= \cos A\\ \tan\ (360^\circ \mp A) &= \mp \tan A\\ \cot\ (360^\circ \mp A) &= \pm \cot A\\ \sec\ (360^\circ \mp A) &= \sec A\\ \operatorname{cosec}\ (360^\circ \mp A) &= \mp \operatorname{cosec} A\end{aligned}\right\} \qquad (20)$$

15. Functions of the Sum or Difference of two Angles.—Let BAD and EAB be any two angles represented by A and B, respectively. From any point E, draw EF and EB perpendicular to AD and AB, and from B draw BG parallel to AD.

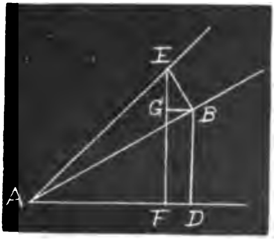


FIG. 2.

Then FB is a rectangle. Angle BEG = angle $BAD = A$.

Now, $\sin(A+B) = \dfrac{EF}{AE} = \dfrac{GF+EG}{AE} = \dfrac{BD}{AE} + \dfrac{EG}{AE}$

$$= \frac{BD}{AB} \times \frac{AB}{AE} + \frac{EG}{EB} \times \frac{EB}{AE}$$

$$= \sin A \cos B + \cos A \sin B \qquad (21)$$

Also, $\cos(A+B) = \dfrac{AF}{AE} = \dfrac{AD - FD}{AE} = \dfrac{AD}{AE} - \dfrac{GB}{AE}$

$$= \frac{AD}{AB} \times \frac{AB}{AE} - \frac{GB}{EB} \times \frac{EB}{AE}$$

$$= \cos A \cos B - \sin A \sin B \qquad (22)$$

If in (21) and (22), we suppose B to be $-B$, we have

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \qquad (23)$$

and $\cos(A-B) = \cos A \cos B + \sin A \sin B \qquad (24)$

The other functions of $A \pm B$ may be obtained from the above by means of formulas, Art. 13.

Thus,

$$\tan(A \pm B) = \frac{\sin(A \pm B)}{\cos(A \pm B)} = \frac{\sin A \cos B \pm \cos A \sin B}{\cos A \cos B \mp \sin A \sin B}$$

Dividing numerator and denominator by $\cos A \cos B$,

$$\tan(A \pm B) = \frac{\frac{\sin A}{\cos A} \pm \frac{\sin B}{\cos B}}{1 \mp \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}} = \frac{\tan A \pm \tan B}{1 \mp \tan A \times \tan B} \quad (25)$$

Similarly, $$\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A} \quad (26)$$

16. Functions of 2A, 3A, and ½A.—If in (21), we suppose $B = A$, we have

$$\sin 2A = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A \quad (27)$$

If in (22) we suppose $B = A$,

$$\cos 2A = \cos^2 A - \sin^2 A \quad (28)$$

$$\text{or } \cos 2A = 2 \cos^2 A - 1 \quad (29)$$

Eliminating $\cos^2 A$ instead of $\sin^2 A$,

$$\cos 2A = 1 - 2 \sin^2 A \quad (30)$$

$$\text{Tan } 2A = \frac{\sin 2A}{\cos 2A} = \frac{2 \sin A \cos A}{\cos^2 A - \sin^2 A} = \frac{2 \tan A}{1 - \tan^2 A} \quad (31)$$

$$\text{Cot } 2A = \frac{\cos 2A}{\sin 2A} = \frac{\cot^2 A - 1}{2 \cot A} \quad (32)$$

If in (21), we suppose $B = 2A$ and reduce, we obtain

$$\sin 3A = \sin A\,(3 - 4 \sin^2 A) \quad (33)$$

Similarly, from (22)

$$\cos 3A = \cos A\,(4 \cos^2 A - 3) \quad (34)$$

$$\text{Tan } 3A = \frac{\sin 3A}{\cos 3A} = \tan A \left\{ \frac{3 - \tan^2 A}{1 - 3 \tan^2 A} \right\} \quad (35)$$

If in (30), we suppose A to be $\frac{1}{2}A$,

$$\cos A = 1 - 2 \sin^2 \tfrac{1}{2}A, \text{ whence,}$$

$$\sin \tfrac{1}{2}A = \left\{ \tfrac{1}{2}(1 - \cos A) \right\}^{\frac{1}{2}} \quad (36)$$

If in (29), we suppose A to be $\frac{1}{2}A$,

$$\cos A = 2 \cos^2 \tfrac{1}{2}A - 1, \text{ whence}$$

$$\cos \tfrac{1}{2}A = \left\{ \tfrac{1}{2}(1 + \cos A) \right\}^{\frac{1}{2}} \quad (37)$$

$$\text{Tan } \tfrac{1}{2}A = \frac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} = \left\{ \frac{1 - \cos A}{1 + \cos A} \right\}^{\frac{1}{2}} = \frac{\sin A}{1 + \cos A} \quad (38)$$

$$\text{Cot } \tfrac{1}{2}A = \frac{1}{\tan \frac{1}{2} A} = \left\{\frac{1+\cos A}{1-\cos A}\right\}^{\frac{1}{2}} = \frac{\sin A}{1-\cos A} \quad (39)$$

Or since tangent and cotangent are each the reciprocal of the other

$$\tan \tfrac{1}{2}A = \frac{1-\cos A}{\sin A} \quad (40)$$

and

$$\cot \tfrac{1}{2}A = \frac{1+\cos A}{\sin A} \quad (41)$$

17. Sum or Difference of the Sines or the Cosines of two Angles.—Let A and B be any two angles and assume the identical equations:

$$A = \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B) \quad (a)$$

and

$$B = \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A-B) \quad (b)$$

From (a) and (b) by (21) and (23),

$$\sin A = \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) + \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (c)$$

and

$$\sin B = \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) - \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (d)$$

From (c) and (d) by addition and subtraction,

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) \quad (42)$$

and

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (43)$$

Again from (a) and (b) by (22) and (24),

$$\cos A = \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) - \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (e)$$

and

$$\cos B = \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) + \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (f)$$

From (e) and (f)

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B) \quad (44)$$

and

$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B) \quad (45)$$

18. Ratios of Sum and Difference of Sines and Cosines of two Angles.—From the above formulas by division and reduction:

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)} \quad (46)$$

$$\frac{\sin A + \sin B}{\cos A - \cos B} = \tan \tfrac{1}{2}(A + B) \quad (47)$$

$$\frac{\sin A + \sin B}{\cos A - \cos B} = -\cot \tfrac{1}{2}(A - B) \quad (48)$$

$$\frac{\sin A - \sin B}{\cos A + \cos B} = \tan \tfrac{1}{2}(A - B) \quad (49)$$

$$\frac{\sin A - \sin B}{\cos A - \cos B} = -\cot \tfrac{1}{2}(A + B) \quad (50)$$

19. Numerical Values of Functions.—Among the foregoing formulas are certain ones which are applicable to the computation of the numerical values of the functions of angles.

Thus, to find the values of the functions of an angle of 60° or of 30°, we have (27)

$$\sin 60° = 2 \sin 30° \cos 30°.$$

But $\sin 60° = \cos 30°$, (Art. 14).

Whence $2 \sin 30° = 1$

$$\left.\begin{aligned} \text{or } \sin 30° &= \cos 60° = \tfrac{1}{2} \\ \cos 30° &= \sin 60° = (1 - \tfrac{1}{4})^{\frac{1}{2}} = \tfrac{1}{2}\sqrt{3} \\ \tan 30° &= \cot 60° = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \tfrac{1}{3}\sqrt{3} \\ \cot 30° &= \tan 60° = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3} \end{aligned}\right\} \quad (51)$$

etc., etc., etc.

Again, to find the values of the functions of 45° we have $\sin 45° = \cos 45°$, and $\sin 90° = 1 = 2 \sin 45° \cos 45°$.

Whence $2 \sin^2 45° = 1$

$$\left.\begin{aligned} \text{or } \sin 45° &= \cos 45° = \tfrac{1}{2}\sqrt{2} \\ \tan 45° &= \cot 45° = 1 \\ \sec 45° &= \operatorname{cosec} 45° = \sqrt{2} \end{aligned}\right\} \quad (52)$$

20. The values of the sine and cosine for each minute of angle from 0° to 90° have been computed and are given in Table II. The values of the tangent and co-tangent are shown in Table III. In Table IV are given the logarithms, plus 10, of each of the above functions. The purpose of adding ten to the logarithm is to avoid negative characteristics. Logarithms thus augmented are called **Tabular Logarithms.** The necessary account of the augmentation of true logarithms in the use of tabular logarithms is easily taken in any given case. In the above tables, numbers of degrees to 45° appear at the top of the page and numbers of minutes at the left. From 45° to 90° the numbers of degrees appear at the bottom of the page and of minutes at the right. The manner of using the Tables will be seen in the following:

21. *Examples.*—1. Required the natural sine and cosine of 12° 18′; of 53° 35′.

Solution.—Turning to Table II, on page showing 12° at the top, and opposite 18′ on the left, we find

$$\sin 12^\circ\ 18' = .21303$$
$$\text{and } \cos 12^\circ\ 18' = .97705.$$

Again, on page showing 53° at the bottom, and opposite 35′ on the right, we find

$$\sin 53^\circ\ 35' = .80472$$
$$\text{and } \cos 53^\circ\ 35' = .59365.$$

2. Required the natural sine and cosine of 27° 35′ 43″.

Solution.—As in Ex. 1 we find

$$\sin 27^\circ\ 36' = .46330$$
$$\text{and } \sin 27^\circ\ 35' = .46304$$

$$\begin{aligned}
\text{Diff. for } 1' &= .00026\\
\text{" " } 1'' &= .00026 \div 60 = .0000043\\
\text{" " } 43'' &= .0000043 \times 43 = .00018
\end{aligned}$$

$$\text{Whence, } \sin 27^\circ\ 35'\ 43'' = .46304 + .00018 = .46322.$$

$$\begin{aligned}
\text{Again, } \cos 27^\circ\ 36' &= .88620\\
\text{and } \cos 27^\circ\ 35' &= .88634
\end{aligned}$$

$$\begin{aligned}
\text{Diff. for } 1' &= -.00014\\
\text{" " } 1'' &= -.00014 \div 60 = -.0000023\\
\text{" " } 43'' &= -.0000023 \times 43 = -.00010
\end{aligned}$$

Whence, cos 27° 35′ 43′′ = .88634 — .00010 = .88624.

3. Given the sine of an angle, to find the angle.

Solution.—Suppose given sine = .48357.

We find sine next less in Table = .48354 = sin 28° 55′

Diff. = .00003

sin 28° 56′ = .48379

sin 28° 55′ = .48354

Diff. for 1′ = .00025

" " 1′′ = .00025 ÷ 60 = .000004.

Dividing .00003 by .000004, we obtain 7 as the number of seconds to add to 28° 55′. Whence, 28° 55′ 7′′ as the angle sought.

4. Given the cosine of an angle, to find the angle.

Solution.—Let given cosine = .52479

Cosine next less in Table = .52473 = cos 58° 21′

Diff = .00006.

sin 58° 22′ = .52448

cos 58° 21′ = .52473

Diff. for 1′ = — .00025

" " 1′′ = — .00025 ÷ 60 = — .000004.

Dividing .00006 by — .000004, we obtain — 15 as the number of seconds to add to 58° 21′. Whence 58° 20′ 45′′ as the angle sought.

SCH.—In the manner of the foregoing examples may be found from Table III. the tangent or cotangent of a given angle, or the angle corresponding to a given tangent or cotangent.

5. Required the tabular logarithmic sine and cosine of 34° 23′; of 72° 40′.

Solution.—Turning to Table IV, on page showing 34° at the top and opposite 23′ on the left, we find

log sin 34° 23′ = 9.751839

and log cos 34° 23′ = 9.916600.

On page showing 72° at the bottom and opposite 40′ on the right, we find

log sin 72° 40′ = 9.979816

and log cos 72° 40′ = 9.474115.

6. Required the logarithmic sine and cosine of 43° 25′ 17′′.

Solution.—We find as in the last example

$$\log \sin\ 43°\ 25' = 9.837146.$$

We then take 2.23* standing opposite in Col. D. 1′′ and multiply it by 17, the given number of seconds, obtaining $2.23 \times 17 = 36.91$ (millionths). Rejecting the decimal part of the product, but adding 1 to the integral part since the decimal exceeds 5 tenths, we add the product to the sine of 43° 25′, obtaining

$$\log \sin 43°\ 25'\ 17'' = 9.837146 + .000038 = 9.837184.$$

Again, log cos 43° 25′ = 9.861161.

Multiplying 1.99 (millionths) in Column D. 1′′ by 17 we obtain 33.83 (millionths) or .000034 nearly. This we subtract from the log cosine of 43° 25′ obtaining

$$\log \cos 43°\ 25'\ 17'' = 9.861161—.000034 = 9\ 861127.$$

The reason for subtracting the correction for the seconds is that the cosine decreases by an increase of the angle. The student is liable to make mistakes at this point.

7. Given the log sine, or log cosine of an angle to find the angle.

Solution.—Suppose the given log sin = 9.348526
We find in table for 12° 53′ log sin = 9.348240

Diff. = 0.000286

We divide this difference by 9.20 (millionths) found opposite in column D. 1′′ and add the quotient $286 \div 9.20 = 31 +$ as a number of seconds to 12° 53′, obtaining 12° 53′ 31′′ as the angle sought.

Again, suppose the given log cos = 9.254319
We find in table for 79° 40′ log cos = 9.253761

Diff. = 0.000558

We divide the difference 558 by 11.54 found opposite in column D. 1′′ obtaining 48, which we *subtract* as a num-

*Really 2.23 millionths.

ber of seconds from 79° 40′, obtaining 79° 39′ 12′′ as the angle sought.

SCH.—1. Really, we should take from the table the logarithmic function *nearest* the given one, whether it be greater or less, and the corresponding angle. Then we should add or subtract the correction as the case may require.

SCH.—2. The manner of finding the tabular logarithmic tangent or cotangent of a given angle, or the angle answering to a given tabular logarithmic tangent or cotangent is the same as above shown for the sine and cosine.

EXERCISES.

22. 1. Find natural sine, cosine, tangent and cotangent of 56° 30′ 40′′.

2. Find natural sine, cosine, tangent and cotangent of 120° 15′ 24′′.

SUGGESTIONS.—Regard 120° 15′ 24″ as 90° + A formula 16, whence A = 30° 15′ 24″.

Or regard 120° 15′ 24″ as 180° — A formula 17, whence A = 59° 44′ 36″.

3. Given the following natural functions, to find from each the corresponding angle:

sin = .66966; cos = .51160; tan = .63758; cot = 1.98625.

Results, 42° 2′ 27′′; 59° 13′ 46′′; 32° 31′ 15′′; 26° 43′ 25′′.

4. Find tabular logarithmic sine, cosine, tangent and cotangent of 40° 37′ 23′′.

5. Find tabular logarithmic sine, cosine, tangent and cotangent of 100° 20′ 30′′.

SUGGESTION.—As under formulas 16 and 17, A = 10° 20′ 30″ or A = 79° 39′ 30″.

6. Find the angle whose tab log sin = 9.745164.
Result, 33° 47′ 15′′ or 146° 12′ 45′′.

7. Find the angle whose tab log cos = 9.959954.
Result, 24° 13′ 43′′ or 335° 46′ 17′′.

8. Find the angle whose tab log tan = 10.475606.
Result, 71° 30′ 18′′ or 251° 30′ 18′′.

9. Find the angle whose tab log cot = 9.927318.
Result, 49° 46′ 20′′ or 229° 46′ 20′′.

10. Find tab log sine, cosine, tangent and cotangent of an angle whose nat. sin = .62579.

23. Relations of Sides and Functions of Angles of Triangles.—Let ABC be any plane triangle. From either angle, as C, draw CD perpendicular to the opposite side.

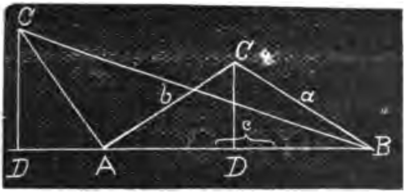


FIG. 3.

Representing the sides of the triangle by small letters corresponding to the letters denoting the angles opposite, we have, from familiar geometrical theorems, accordingly as the angle A is acute or obtuse,

$$a^2 = b^2 + c^2 \mp 2\,c \times AD. \quad (a).$$

By definition, $\cos A = \pm \dfrac{AD}{b}$, the sign depending upon whether A is acute or obtuse.

Whence $AD = \pm\, b \cos A$. Substituting in (a), we have

$$\left.\begin{array}{ll} & a^2 = b^2 + c^2 - 2\,bc \cos A. \\ \text{Similarly,} & b^2 = a^2 + c^2 - 2\,ac \cos B, \\ \text{and} & c^2 = a^2 + b^2 - 2\,ab \cos C. \end{array}\right\} \quad (53)$$

That is, *The square on either side of a plane triangle equals the sum of the squares on the other sides minus twice the rectangle on these sides multiplied by the cosine of their included angle.*

Cor. From (53) are derived

$$\left.\begin{array}{ll} & \cos A = \dfrac{b^2 + c^2 - a^2}{2\,bc}, \\[2ex] & \cos B = \dfrac{a^2 + c^2 - b^2}{2\,ac} \\[2ex] \text{and} & \cos C = \dfrac{a^2 + b^2 - c^2}{2\,ab} \end{array}\right\} \quad (54)$$

That is, *The cosine of either angle of a plane triangle equals the sum of the squares on the sides including the angle, minus the square on the side opposite, divided by twice the product of the including sides.*

Adding the members of the first and second of (53) and reducing, we have

$$c = a \cos B + b \cos A \quad (b).$$

Subtracting the members of the second of (53) from those of the first, we obtain

$$a^2 - b^2 = c\,(a \cos B - b \cos A) \quad (c).$$

Substituting in (c) the value of c from (b), we have

$$a^2 - b^2 = a^2 \cos^2 B - b^2 \cos^2 A,$$

which, by transposition and factoring, becomes

$$a^2\,(1 - \cos^2 B) = b^2\,(1 - \cos^2 A).$$

Reducing by (1), we have

$$a^2 \sin^2 B = b^2 \sin^2 A$$

$$\text{whence } a \sin B = b \sin A$$

$$\left.\begin{aligned} &\text{or } \frac{a}{b} = \frac{\sin A}{\sin B}. \\ &\text{Similarly, } \frac{a}{c} = \frac{\sin A}{\sin C} \\ &\text{and } \frac{b}{c} = \frac{\sin B}{\sin C}. \end{aligned}\right\} \quad (55)$$

That is, *The sides of a plane triangle are proportional to the sines of the angles opposite.*

Substituting in the first and second of (53) the value of c^2 in the third, we obtain

$$b = a \cos C + c \cos A$$

$$\text{and } a = b \cos C + c \cos B.$$

From these results by addition and subtraction, we have

$$(a + b)\,(1 - \cos C) = c\,(\cos A + \cos B)$$

$$\text{and } (a - b)\,(1 + \cos C) = c\,(\cos B - \cos A).$$

Whence by division

$$\frac{a+b}{a-b} \times \frac{1-\cos C}{1+\cos C} = \frac{\cos A + \cos B}{\cos B - \cos A} \quad (d).$$

From (38) $\dfrac{1-\cos C}{1+\cos C} = \tan^2 \tfrac{1}{2}C$, and from (44) and (45) by division

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \frac{\cot \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}.$$

Also, since $C = 180° - (A+B)$, we have $\frac{1}{2}C = 90° - \frac{1}{2}(A+B)$.

Whence $\tan \frac{1}{2}C = \tan \left\{ 90° - \frac{1}{2}(A+B) \right\} = \cot \frac{1}{2}(A+B)$.

Substituting these results in (d) we have

$$\frac{a+b}{a-b} \times \cot^2 \tfrac{1}{2}(A+B) = \frac{\cot \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}.$$

Dividing members by $\cot^2 \frac{1}{2}(A-B)$, remembering that $\dfrac{1}{\cot} = \tan$, we have

$$\left.\begin{aligned} \frac{a+b}{a-b} &= \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}. \\ \text{Similarly,} \quad \frac{a+c}{a-c} &= \frac{\tan \frac{1}{2}(A+C)}{\tan \frac{1}{2}(A-C)} \\ \text{and} \quad \frac{b+c}{b-c} &= \frac{\tan \frac{1}{2}(B+C)}{\tan \frac{1}{2}(B-C)}. \end{aligned}\right\} \quad (56)$$

That is, *The sum of two sides of a plane triangle is to their difference as the tangent of one-half the sum of the opposite angles is to the tangent of one-half the difference of those angles.*

SCH.—1. Formulas (54), (55) and (56) serve in finding the remaining parts of a plane triangle in the following cases:

CASE 1. *Given two sides and an opposite angle.*
CASE 2. *Given two angles and an opposite side.*
CASE 3, *Given two angles and included side.*

In either of these, we use [55].

CASE 4. *Given two sides and included angle.*

In this case, we employ [56].

CASE 5. *Given the three sides.*

In this case, we may employ [54].

SCH.—2. Since (54) are not adapted to application of logarithms, it is customary to transform them as follows;

Subtracting each member of the first from unity,

$$1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{2bc - b^2 - c^2 + a^2}{2\,bc}$$

$$= \frac{a^2 - (b-c)^2}{2\,bc} = \frac{(a+b-c)\,(a+c-b)}{2\,bc} \quad (e)$$

Assuming $a + b + c = 2s$, we have

$a + b - c = 2s - 2\,c$ and $a + c - b = 2s - 2b.$

We have also (36), $1 - \cos A = 2 \sin^2 \frac{1}{2} A.$

Making these substitutions in (e) and reducing,

$$\left.\begin{aligned} \sin \tfrac{1}{2} A &= \left\{\frac{(s-b)\ (s-c)}{bc}\right\}^{\frac{1}{2}} \\ \text{Similarly, } \sin \tfrac{1}{2} B &= \left\{\frac{(s-a)\ (s-c)}{ac}\right\}^{\frac{1}{2}} \\ \text{and} \qquad \sin \tfrac{1}{2} C &= \left\{\frac{(s-a)\ (s-b)}{ab}\right\}^{\frac{1}{2}} \end{aligned}\right\} \quad (57)$$

Or, in another way, by adding the members of [54] to unity and reducing in a manner similar to the above, using, however, formula [37], we obtain

$$\left.\begin{aligned} \cos \tfrac{1}{2} A &= \left\{\frac{s\,(s-a)}{bc}\right\}^{\frac{1}{2}} \\ \text{Similarly} \qquad \cos \tfrac{1}{2} B &= \left\{\frac{s\,(s-b)}{ac}\right\}^{\frac{1}{2}} \\ \text{and} \qquad \cos \tfrac{1}{2} C &= \left\{\frac{s\,(s-c)}{ab}\right\}^{\frac{1}{2}} \end{aligned}\right\} \quad (58)$$

Or, again, dividing (57) by (58), we find

$$\tan \tfrac{1}{2} A = \left\{ \frac{(s-b)(s-c)}{s(s-a)} \right\}^{\frac{1}{2}}$$

$$\tan \tfrac{1}{2} B = \left\{ \frac{(s-a)(s-c)}{s(s-b)} \right\}^{\frac{1}{2}}$$

and $$\tan \tfrac{1}{2} C = \left\{ \frac{(s-a)(s-b)}{s(s-c)} \right\}^{\frac{1}{2}}$$

From the above, are obtained

$$\left.\begin{aligned} \tan \tfrac{1}{2} A &= \frac{1}{s-a}\left\{ \frac{(s-a)(s-b)(s-c)}{s} \right\}^{\frac{1}{2}} = \frac{r}{s-a} \\ \tan \tfrac{1}{2} B &= \frac{1}{s-b}\left\{ \frac{(s-a)(s-b)(s-c)}{s} \right\}^{\frac{1}{2}} = \frac{r}{s-b} \\ \text{and } \tan \tfrac{1}{2} C &= \frac{1}{s-c}\left\{ \frac{(s-a)(s-b)(s-c)}{s} \right\}^{\frac{1}{2}} = \frac{r}{s-c} \end{aligned}\right\} \quad (59)$$

in which r denotes the radical factor, or the radius of the inscribed circle of the triangle.

In the solution of right triangles, the formulas given directly by the definitions of the functions, (Art. 10), are employed. Thus, in the triangle BAC right-angled at A we have

$$\frac{b}{a} = \sin B$$

and $$\frac{b}{c} = \tan B$$

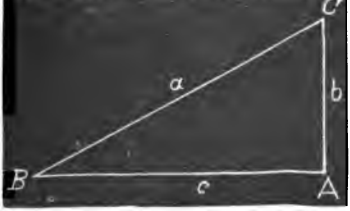


FIG. 4.

$$\left.\begin{aligned} &\text{Whence } b = a \sin B = c \tan B \\ &\text{Similarly, } c = a \sin C = b \tan C. \end{aligned}\right\} \quad (60)$$

That is, *Either perpendicular side of a right triangle equals the hypotenuse multiplied by the sine, or the other*

perpendicular side multiplied by the tangent of the angle opposite the first mentioned side.

Cor. 1 Since the angles B and C are the complements of each other, formulas (60) give

$$\left.\begin{aligned} b &= a \cos C = c \cot C \\ \text{and} \quad c &= a \cos B = b \cot B \end{aligned}\right\} \qquad (61)$$

That is, *Either perpendicular side of a right triangle equals the hypotenuse multiplied by the cosine, or the other perpendicular side multiplied by the cotangent of the angle adjacent the first mentioned side.*

Cor. 2. From (61), we obtain

$$\left.\begin{aligned} a &= \frac{b}{\cos C} = b \sec C \\ \text{and} \quad a &= \frac{c}{\cos B} = c \sec B \end{aligned}\right\} \qquad (62)$$

That is, *The hypotenuse of a right triangle equals either perpendicular side multiplied by the secant of the adjacent angle.*

III. Solution of Triangles.

I. Of Right Triangles.

24. 1. In the right triangle ABC, given the hypotenuse $a = 255$ ft., and the angle $B = 57° 14'$, to find the angle C and the sides b and c.

Solution.—1. The angle $C = 90° - 57° 14' = 32° 46'$.

2. By (60), $b = a \sin B$, whence, by logarithms, (Art. 5),

$a = 255$ ft.	log	2.406540
$B = 57° 14'$	log sin	9.924735
$b = 214.42$ ft.	log	2.331275

REMARK.—We drop 10 from the sum of the logarithms since the tabular log sin B is ten too large, (Art. 20).

3. By (61), $c = a \cos B$, whence

$a = 255$	log	2.406540
$B = 57° 14'$	log cos	9.733373
$c = 138.01$	log	2.139913

2. In the right triangle ABC, given the side $b = 150$ and the angle $B = 40° 30'$, to find the angle C, the hypotenuse a, and the side c.

Solution.—1. The angle $C = 90° - 40° 30' = 49° 30'$.

2. By (60), $a = \dfrac{b}{\sin B}$, whence, by logarithms. (Art. 6),

$b = 150$	log	2.176091
$B = 40° 30'$	log sin	9.812544
$a = 230.96$	log	2.363547

3. By (60), $c = \dfrac{b}{\tan B}$, whence

$b = 150$	log	2.176091
$B = 40° 30'$	log tan	9.931499
$c = 175.63$	log	2.244592

3. In the right triangle ABC, given the hypotenuse $b = 243$ and the side $a = 120$, to find the other parts.

Solution.—1. By definition, $\sin A = \dfrac{a}{b}$, whence

$a = 120$	log	2.079181
$b = 243$	log	2.385606
$A = 29° 35' 33''$	log sin	*9.693575

2. The angle $C = 90° - 29° 35' 33'' = 60° 24' 27''$.

3. The side $c = (b^2 - a^2)^{\frac{1}{2}} = [(b + a)(b - a)]^{\frac{1}{2}}$, whence

$b + a = 363$	log	2.559907
$b - a = 123$	log	2.089905
c	log	4.649812
$c = 211.30$	log	2.324906

4. In a right triangle ABC, given the side $b = 385$, and the side $c = 256$, to find the other parts.

Solution.—1. By definition, $\tan C = \dfrac{c}{b}$, whence

* Since tabular logarithms are true logarithms plus 10.

$c = 256$	log	2.408240
$b = 385$	log	2.585461
$C = 33° \ 37' \ 17''$	log tan	9.822779

2. The angle $B = 90° - 33° \ 37' \ 17'' = 56° \ 22' \ 43''$.

3. By definition, $\sin C = \dfrac{c}{a}$ or $a = \dfrac{c}{\sin C}$, whence

$c = 256$	log	2.408240
$C = 33° \ 37' \ 17''$	log sin	9.743277
$a = 462.34$	log	2.664963

II. Of Oblique Triangles.

25. Case 1.—*Given two sides and an opposite angle.*

In the triangle ABC, given $a = 60$, $b = 50$, and $A = 60° \ 40' \ 30''$, to find the angles B and C and the side c

Solution.—1. By (55), $\sin B = \dfrac{b \sin A}{a}$, whence

$b = 50$	log	1.698970
$A = 60° \ 40' \ 30''$	log sin	9.940444
$a = 60$ ar. co.	log	8.221849
$B = 46° \ 35' \ 51''$	log sin	9.861263, (Art. 6, Sch.).

2. $C = 180° - (A+B) = 72° \ 43' \ 39''$.

3. By (55), $c = \dfrac{a \sin C}{\sin A}$, whence

$a = 60$	log	1.778151
$C = 72° \ 43' \ 39''$	log sin	9.979960
$A = 60° \ 40' \ 30''$	ar. co. log sin	0.059554
$c = 65.71$	log	1.817667

Sch.—Under the above case, in the solution of triangles there are often two solutions—two values of each of the unknown parts, which are consistent with the given ones. This arises from the fact that the angle required, opposite one of the given sides, is found from its sine, which is the same precisely, (17), for the possible value of the required angle and for its supplement. A full discussion of the case cannot conveniently be taken up here. We give simply its conclusion as enunciated in the following

Principle —*If the side opposite the given angle is equal to or greater than the other given side, the required angle is acute and the triangle has but one solution; but if the side opposite the given angle is less than the other given side, the required angle may be acute or obtuse, right or imaginary, and the triangle has two, one, or no solution.*

The fact of a right or an imaginary angle is readily detected in the course of the solution.

26. CASE 2. *Given two angles and an opposite side.*

In the triangle ABC, given the angle $A = 40° \ 15'$, the angle $B = 35° \ 12' \ 8''$ and the side $a = 50$ ft., to find the angle C and the sides b and c.

Solution.—1. The angle $C = 180° - (A + B) = 104° \ 32' \ 52''$.

2. From (55), $b = \dfrac{a \sin B}{\sin A}$, whence

$a = 50$ ft.	log	1.698970
$B = 35° \ 12' \ 8''$	log sin	9.760772
$A = 40° \ 15'$	ar. co. log sin	0.189684
$b = 44.61$ ft.	log	1.649426

3. Again from (55), $c = \dfrac{a \sin C}{\sin A}$, whence

$a = 50$ ft.	log	1.698970
$C = 104° \ 32' \ 52''$	log sin	9.985847
$A = 40° \ 15'$	ar. co. log sin	0.189684
$c = 74.90$ ft.	log	1.874501

27. CASE 3. *Given two angles and included side.*

In the triangle ABC, $A = 54° \ 28'$, $B = 70° \ 40' \ 20''$ and $c = 125$, to find the other parts.

Solution.—Left to the student.

$$\text{Results} \begin{cases} C = 54° \ 51' \ 40'' \\ a = 124.4 \\ b = 144.24 \end{cases}$$

28. CASE 4. *Given two sides and included angle.*

In the triangle ABC, given $a = 89$, $b = 57$ and $C = 75° \ 4' \ 15''$ to find the other parts.

Solution.—1. $\frac{1}{2}(A+B) = 90° - \frac{1}{2}C = 52° \ 27' \ 52.5''$.

2. From (56), $\tan \frac{1}{2}(A-B) = \dfrac{(a-b) \tan \frac{1}{2}(A+B)}{a+b}$, whence

$a - b$	$= 32$	log	1.505150
$\frac{1}{2}(A+B)$	$= 52° \ 27' \ 52.5''$	log tan	10.114464
$a+b$	$= 146$	ar. co. log	7.835647
$\frac{1}{2}(A-B)$	$= 15° \ 55' \ 19''$	log tan	9.455261

Having now $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, we find

$$A = \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B) = 68° \ 23' \ 11.5''$$

and $$B = \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A-B) = 36° \ 32' \ 33.5''$$

3. From (55), $c = \dfrac{a \sin C}{\sin A}$, whence $c = 92.5$.

29. CASE 5. *Given the three sides.*

In the triangle ABC, given $a = 120$, $b = 102$ and $c = 75$, to find the angles.

Solution.—From (59), $r = \left\{\dfrac{(s-a)(s-b)(s-c)}{s}\right\}^{\frac{1}{2}}$ whence

$s - a =$	28.5		log 1.454845
$s - b =$	46.5		log 1.667453
$s - c =$	73.5		log 1.866287
$s =$	148.5	ar. co.	log 7.828274
r^2			log 2.816859
r			log 1.408429

Also, from (59), $\tan \frac{1}{2}A = \dfrac{r}{s-a}$,

$$\tan \tfrac{1}{2}B = \frac{r}{s-b},$$

$$\tan \tfrac{1}{2}C = \frac{r}{s-c}, \text{ whence}$$

r		log	1.408429
$s - a$		log	1.454845
$\frac{1}{2}A$	$= 41° \ 56' \ 38.5''$	log tan	9.953584

$A = 83° \ 53' \ 17''$		
r	log	1.408429
$s - b$	log	1.667453
$\frac{1}{2}B = 28° \ 50' \ 42''$	log tan	9.740976
$B = 57° \ 41' \ 24''$		

Similarly, is found $C = 38° \ 25' \ 19''$

EXERCISES.

30. In a triangle ABC,

1. Given $A=30°$, $B=40° \ 50'$, and $a=83$, to find C, b and c.

2. Given $a=50$, $b=40$, and $B=32°$, to find A, C and c.

3. Given $A=25° \ 30'$, $B=50° \ 20'$, and $c=100$, to find C, a and b.

4. Given $A=32° \ 22' \ 12''$, $b=125$, and $c=212$, to find B, C and a.

5. Given $a=134$, $b=143$, and $c=314$, to find A, B and C.

6. Given $A=90°$, $a=234$, and $B=40° \ 15'$, to find C, b and c.

7. Given $B=90°$, $b=84$, and $a=62.5$, to find A, C and c.

8. Given $C=90°$, $A=34° \ 45' \ 10''$, and $b=50$, to find B, a and c.

9. Given $A=90°$, $b=75$, and $c=45$, to find B, C and a.

10. Given $A=90°$, $a+b=207$, and $c=69$, to find B, C, a and b.

11. Given $C=29° \ 46'$, $a-b=30$, and $\sin A : \sin B :: 7 : 4$, to find A, B and c.

12. Given $a+b+c=54$, $a-b=6$, and $b-c=6$, to find A, B and C.

SECTION II.

DEFINITIONS AND INSTRUMENTS.

31. **Surveying** is the art of making the necessary measurements for determining the relative location of points on the surface of the earth.

It is of two kinds or branches—*Plane* and *Geodesic.*

32 **Plane Surveying** is that branch in which the curvature of the earth is disregarded.

It is limited to surveys of moderate extent, as of farms, town sites, etc.

33 **Geodesic Surveying** is that branch in which, for reason of the extent of surface comprised by the work, account is required to be taken of the curvature of the earth.

The terms Land Surveying, Railroad Surveying, Topographical Surveying, and some others, are used with reference to certain special purposes or practical objects to which the work is applied.

34. The operations comprised in Land Surveying may be classed under *Field Work, Platting,* and *Computation of Area.*

35. **Field Work** includes all operations required to be performed on the ground, as finding points which have been previously established, measuring to establish the location of other points, keeping memoranda, etc.

The memoranda constitute what are called the **Field Notes.**

The measurements usually required in Surveying are of two kinds—the measurement of distances or lines and the measurement of direction or angles. Accordingly, two classes of instruments are employed.

36. The principal instrument of linear measure, in Surveying, is the **Chain.**

The Steel Tape is, however, rapidly usurping the place of the chain. It is lighter and much more convenient to use. It is also a much more accurate measure than the chain.

The chain is 66 feet in length and consists of 100 links. Each tenth link is marked by a piece of brass, notched in a manner to indicate the number of tens from the end of the chain. Swivels are also inserted to prevent twisting.

Steel tapes are made of various lengths. One of 66 ft., divided in links, is adapted to chain measurement.

37. Ten iron or steel pins, called **Marking Pins**, accompany the chain. They are usually about a foot in length and an eighth of an inch in thickness. They are pointed at one end for sticking in the ground, and bent into a ring at the other so as to be strung upon a ring for convenience of carrying.

Instead of ten pins, eleven are sometimes employed, and in place of a ring for carrying, a quiver is sometimes used.

38 Stakes with flags attached, and slender rods variously painted so as to be readily distinguished at a distance, are employed in ranging out long lines and marking points toward which measurements are required to be made.

39. Chain Measure Table:

7.92	inches	make	1 link (l.)
100	links	"	1 chain (ch.)
80	chains	"	1 mile (mi.)

Links are conveniently expressed as hundredths of a chain. Thus 5 l., 17 l., 8 ch. 12 l. may be written 0.05 ch., 0.17 ch., 8.12 ch. Any number of chains is reduced to feet by multiplying by 66, and, conversely, by dividing by 66. Thus, 5.13 ch. $= 5.13 \times 66$ ft. $= 338.58$ ft., and 215 ft. 7 in. $= 215.583$ ft. $= 215.583 \div 66 = 3.27$ ch., nearly.

40. Chaining.—Preliminary to any operation in Surveying, it is important to consider whether the instru-

ments required in the work are in proper condition and adjustment for use. So simple an instrument as the chain forms no exception to this rule. It is liable to become lengthened by the wearing or opening of the links. or shortened by the links becoming bent.

It should, therefore, be frequently tested by comparing it with a carefully measured length of 66 feet, set off in a convenient place for use.

An error of length not readily remedied should be noted and the proper correction made of apparent measurements.

In measuring lines, marks should be set up at the extremities, and also at one or more intermediate points when the extremities are not visible from each other.

The operation of chaining will be better learned by practice than by precept. We, accordingly, omit any formal description of the process in this connection.

The following suggestions may, however, be serviceable:

In starting to measure a line, the forward chainman should see that he has the right number of marking pins, and that the chain or tape is free from kinks, and straight.

He should keep close to the line when chaining—never wandering off to either side. He must guide himself within narrow bounds by ranging forward or backward.

He should draw the chain taut and always at the same tension. It is sometimes more important that the measuring be uniform than that it be absolutely exact. The chain should be pulled steadily, never with a jerk, especially in going up hill.

The marking pins must be set plumb. The work should proceed briskly—no dawdling or gaping about is allowable. The forward chainman should not stop and look around until the rear chainman halts him.

In the counting of odd links, care must be taken not to count from the wrong end of the chain, giving 60 instead of 40, etc.; nor to count the wrong way from the tag, giving 38 instead of 42, etc. Too much care cannot be taken to avoid making mistakes.

The rear chainman should observe the condition of the chain as the head chainman pulls out.

It is for him also to see that the forward chainman keeps in line. He should call "Halt" in time to avoid the chain being brought up with a jerk. He should not pull up the marking pin before the leader has set his in the right place and called out "Stuck." He should then step forward promptly, not requiring the leader to drag him in addition to the chain.

He should count the pins before handing them to the head chainman, or changing positions with him.

He ought to carry in his eye the place where the leader sticks a pin, so as not to pass it or to have to hunt about for it.

41. Distances are required to be measured *horizontally*. Accordingly, in chaining up and down slopes, one end of the chain must be raised to a level with the other. A rod held vertically or a plumb-line will often be found necessary, in such cases, in locating the proper points on the ground.

Upon steep slopes it may be impossible to use the entire chain at a time. The part used may be any convenient portion of a chain, as one half, one-fourth, one-fifth, etc., or such parts may be used as taken in succession make a full chain.

The forward chainman, however, must be careful to leave a pin *only at the end of each entire chain.*

Many surveyors adopt the practice of always chaining down slopes rather than up them, since more accurate measurements are thus obtained.

Let the student now practice some with the chain, as suggested in the following

EXERCISES.

42 1. Mark two points several chains apart, and measure the distance between them.

Measure back and compare results.

2. Mark two points on sloping ground, and measure between them.

Measure back and compare results.

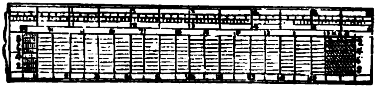
43. Platting.—The operation of delineating a survey on paper is called **Platting**.

The delineation is called a **Plat**.

In platting a survey, lines are drawn to some convenient scale, that is, a certain number of miles, chains or rods is represented by an inch, for example, on the plat. Angles are drawn equal to those on the ground.

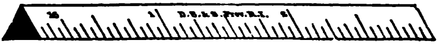
Thus, as the lines are made proportional and the angles equal, the plat and the "field" are *similar* figures.

The principal platting instruments are



(1) The *Scale* of equal parts used in laying down lines to scale.

FIG. 5.



(2) The *Dividers* or *Compasses* for describing arcs of circles, and for taking off the lengths of lines from the scale, etc.

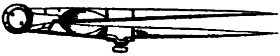


FIG. 6.

(3) The *Ruler* and *Right-line Pen* for drawing straight lines.

FIG. 7.

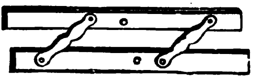


FIG. 8.

(4) The *Parallel Ruler* for drawing parallels. The *Ruler* and *Triangle* are used for the same purpose.

(5) The *Semi-circular Protractor* for laying down angles.

A *Scale of Chords* is often used for the same purpose.

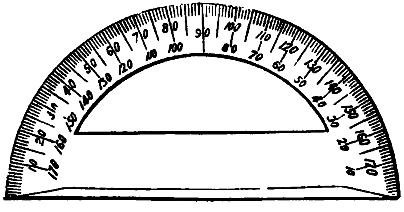


FIG. 9.

A detailed description of the above instruments is deemed unnecessary, the student being doubtless already quite familiar with them.

For the same reason, instruction in the manner of using them is also omitted, any suggestion which the student may need in this respect being left to the teacher to make.

EXERCISES.

44. 1. To draw a straight line equal to a given straight line.*

2. To make an angle equal to a given angle.

3. To draw through a given point a line parallel to a given line.

4. To draw through a given point a line perpendicular to a given line. Two cases.

5. To bisect a given line; a given angle.

6. To construct lines proportional to given lines.

7. To construct a polygon similar to a given polygon.

8. Plat the following lines:

(1) 8 ch., to scale of 2 chains to the inch.

(2) 10 ch., to scale of 5 chains to the inch.

(3) 10 ch., to scale of 4 chains to the inch.

(4) 17.25 ch., to scale of 3 chains to the inch.

(5) 25.40 ch., to scale of 4 chains to the inch.

9. Plat a triangle whose sides are 13.50 ch., 14.25 ch. and 16.20 ch., on a scale of 5 chains to an inch; on a scale of 3 chains to an inch.

10. Plat a rectangle whose adjacent sides are 9.24 ch. and 13.78 ch., on a scale of 4 chains to the inch.

45. Area.—The area of land is expressed in the denominations, *square links, square feet, square rods* or *perches, square chains, roods, acres, square miles,* or *sections* and *townships.*

46. Land Measure Table:

625 square links (sq. l.)	make	1 square rod (sq. rd.)
16 sq. rd. or perches (P.)	"	1 square chain (sq. ch.)
$2\frac{1}{2}$ sq. ch. or 40 P.	"	1 rood (R.)
10 sq. ch. or 4 R.	"	1 acre (A.)
640 A.	"	1 square mile (sq. mi.)
36 sq. mi. or sections	"	1 township (Tp.)

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*The first seven exercises are the elementary problems of Geometry, and are designed to be solved on paper by use of the dividers and ruler.

SECTION III.

CHAIN SURVEYING.

The following geometrical problems are designed to be solved by the student in the field. Convenient ground may be chosen somewhere on the commons for the field work, to be followed by exercises in platting.

47 Prob. 1 *To let fall a perpendicular from a given point upon a given line.*

CASE 1. *When the distance of the point from the line is less than one chain.*

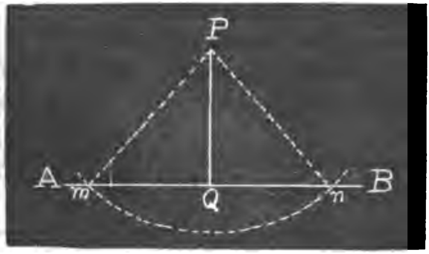

FIG. 10.

SUGGESTION.—(1) Let P be the given point and AB the given line. Fasten one end of the chain at P, and swing the chain or a convenient portion of it, around P as a center, marking two points, as m and n, on AB. Measure mn, and measure mQ or $nQ = \frac{1}{2}mn$.

(2) Stretch the chain from P to some point p on AB. Fasten the middle of the chain, as at C, and swing one end of the chain around C as a center until it comes to AB, as at Q.

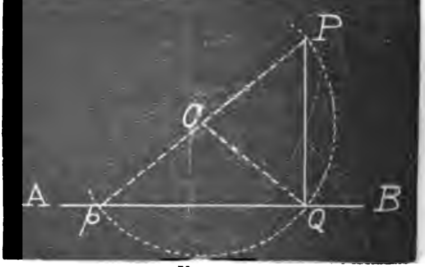

FIG. 11,

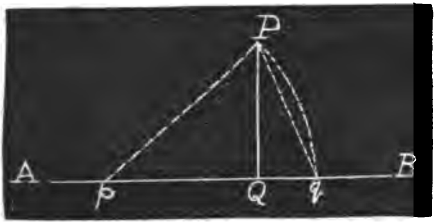

FIG. 12.

(3) Stretch the chain from P to some point p on AB. Fasten the end at p and swing the other end around till it comes to AB, as at q. Measure Pq, and measure from q toward p a distance $qQ = \frac{Pq^2}{2Pp}$.

CASE 2.—*When the distance of the point from the line is more than one chain.*

SUG.—(1) From any convenient point on AB, as p, measure pP. At a convenient point on pP, as C, let fall a perpendicular upon AB, as Cq. Measure on AB, from p, a distance $pQ = \frac{pq \times pP}{pC}$.

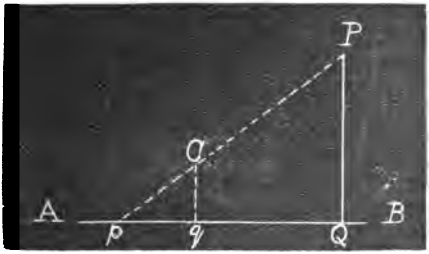

FIG. 13.

Cor.—The length of the perpendicular $= \frac{Cq \times pP}{pC}$.

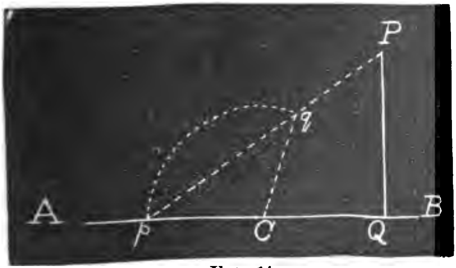

FIG. 14.

(2) From any convenient point p on AB, stretch the chain along AB toward the required perpendicular to some point, as C. Fasten the end of the chain at C and

swing the other end toward P to a point on pP, as q. Measure pq and pP.

Measure on AB a distance $pQ = \dfrac{pq \times pP}{2pC}$.

CASE 3. *When the point is inaccessible from the line.*

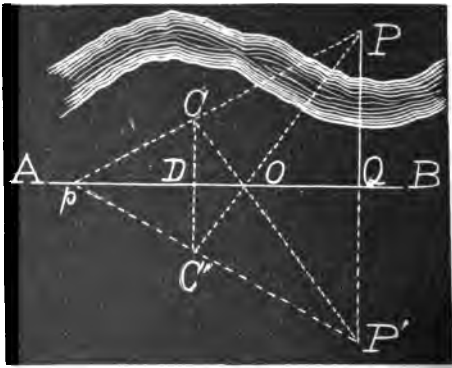

FIG. 15.

SUG.—(1) Let p be a convenient point in AB. From a convenient point in pP, as C, let fall a perpendicular CD upon AB and produce it to C', making $C'D = CD$. Mark, as O, the point where the line $C'P$ intersects AB. Find the point, as P', where the lines CO and pC' intersect.

Cor.—The length of the perpendicular $PQ = \dfrac{CD \times pQ}{pD}$.

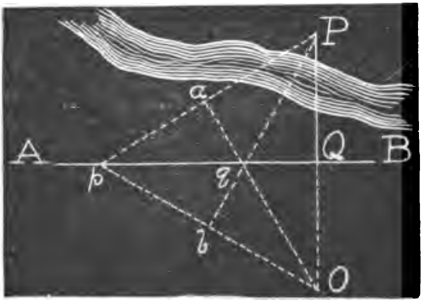

FIG. 16.

(2) Let p and q be two points on AB from which the point P may be seen. By one of the preceding methods, let fall a perpendicular from q upon the line pP and one from p upon the line qP. Let O be the point of intersection of the two perpendiculars.

CASE 4. *When the line is inaccessible from the point.*

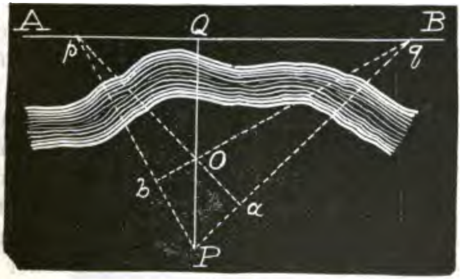

FIG. 17.

SUG.—Let p and q be two points on AB, visible from P. Let fall upon Pp a perpendicular from q, and upon Pq a perpendicular from p. Let O be the point of intersection of the perpendiculars.

48. Prob. 2 *To erect a perpendicular at a given point of a given line.*

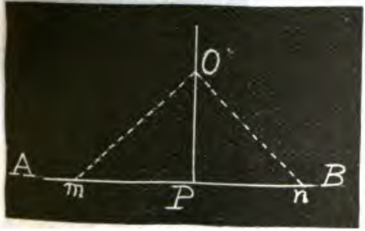

FIG. 18.

SUG.—(1) Let AB be the given line and P, the given point. Measure off equal distances of from one-half to two-thirds of a chain on opposite sides of P to points, as m and n. About the points m and n as centers and with the chain as a radius describe short arcs intersecting, as at O.

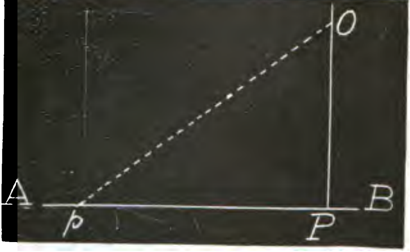

FIG. 19.

(2) Measure 40 links along the line from P to p. Fasten one end of the chain at p, and holding the 20th link from the other end at P, let an assistant take hold of the 50th link and draw the fold of the chain in the direction of

the required perpendicular, tightening each part alike.

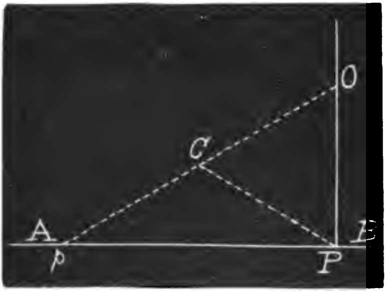

FIG. 20.

(3) Fasten one end of the chain at P and the other at any point, as C, on the side of AB from which the required perpendicular is to be set off. Swing the end at P to another point of AB, as at p. Drawing the chain on in the direction of pC, mark a point, as O, a chain's length from C.

(4) Choose some convenient point, as C, on one side of the required perpendicular. Measure PC and on AB, in the direction of C from the required perpendicular, measure $Pp = PC$. Measure from p in the line pC a distance $pO = \dfrac{2Pp^2}{pC}$.

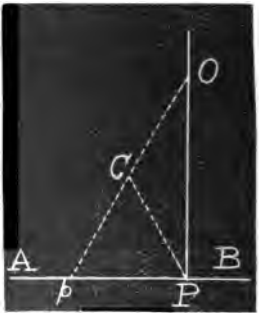

FIG 21.

49. Prob. 3. *To run a parallel to a given line from a given point.*

CASE 1.—*When the line and point are accessible from each other.*

SUGGESTIONS.—(1) Let AB be the given line and P, the given point.

Let fall from P a perpendicular upon AB, as PQ. At another point of AB, as p, taken as far from Q as convenient, erect a perpendicular to AB, as pq. Run a line from P, through the point q.

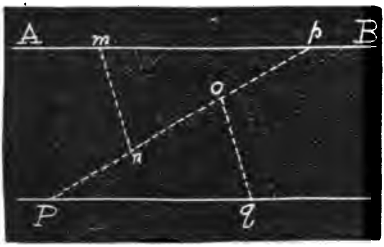

FIG. 22.

(2) Set out a line from P to some convenient point p of AB. Measure on AB and pP any equal distances pm and pn. Measure mn. From P measure on Pp a distance as $Po = pn$. Find a point q at a distance of Po from P and a distance mn from o.

(3) Set out and measure a line from P to a convenient point p on AB. Measure back $\frac{1}{2}Pp$ to o. Measure from a convenient point m of AB. Prolong mo to q, making $oq = mo$.

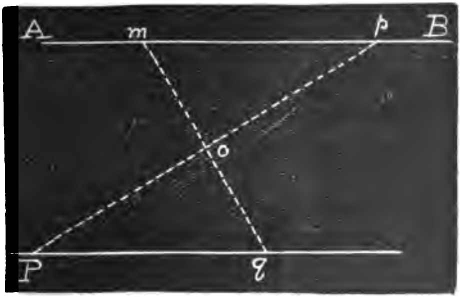

FIG. 23.

CASE 2.—*When the line is inaccessible from the point.*

SUG.—(1) By Prob. 1, let fall a perpendicular from P upon AB, as PQ. Erect a perpendicular to PQ at P.

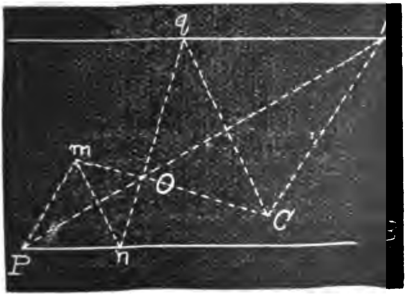

FIG. 24.

(2) In any line, as Pp, from P to AB, set a stake, as at O, and set one also at any convenient point, as C. From P run a parallel to Cp, meeting the line OC, as at m. From m run a parallel

to Cq—a line from C to a convenient point q on AB—meeting the line Oq, as at n.

50. Prob. 4 *To prolong a line beyond an obstacle obstructing the sight.*

SUG.—(1) Let AB be a line to be prolonged beyond a building or other object obstructing the sight. At m, four or five chains from the building, measure carefully a perpendicular mn of sufficient length to carry past the

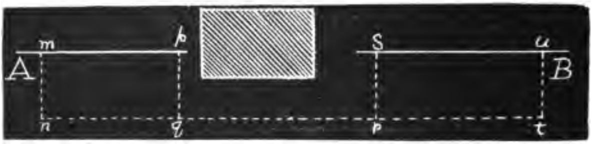

FIG. 25.

building. Near the building measure another perpendicular pq exactly equal to mn. Extend the line nq considerably past the building and set back upon AB by measuring the perpendiculars rs and tu exactly equal to mn.

(2) Fasten one end of the chain and also the end of the first link from the other end of the chain at B. Fasten the 33d link from the end at a point on AB, as at p. Taking hold of the 66th link and drawing the two parts of the chain from B and p equally tight, mark the end of the 66th link, as at q.

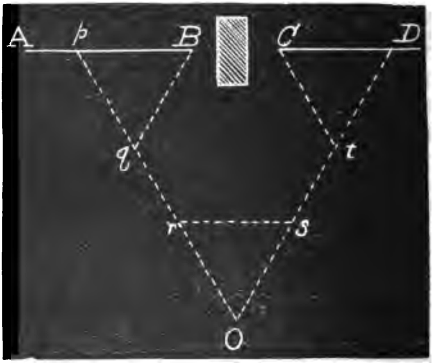

FIG. 26.

Extend the line pq sufficiently beyond the obstacle to some point, as O. Form another equilateral triangle, as rOs, as before. Extend Os to D, making $OD=pO$. Form the equilateral triangle CtD. The distance $pD = pO$.

(3) Suppose it required to extend a line from A, through a wood, to a given point C.

FIG. 27.

From B run a "random line," as Bp, as near in the required direction as can be conveniently ranged through. Measure Bp, marking points as m, n, o, etc., which offer opportunity of measuring from them to the required line, and noting their distances from B. Arriving at a point opposite C, as at p, erect a perpendicular to Bp passing through C. Measure pC, and at the points m, n, o, etc., set off perpendiculars to Bp, making them of lengths, as compared with pC, proportional to their distances from B.

Cor.—The distance $BC = \sqrt{Bp^2 + Cp^2}$.

A few cases of measuring inaccessible lines have been incidentally presented as corollaries of the foregoing problems. A connected and more systematic treatment of this work is given under the following:

51. Prob. 5. *To find the length of a line inaccessible to direct measurement.*

CASE 1.—*When each end of the line is accessible.*

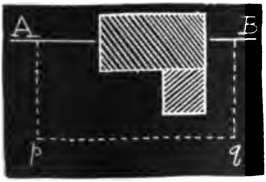

FIG. 28.

SUGGESTIONS.—Let AB be the line.

(1) Set off equal perpendiculars, as Ap and Bq, of sufficient length to pass the obstacle. Measure pq.

(2) Measure a line, as Ap, at a sufficient angle with AB to pass the obstacle. At p, erect a line perpendicular to Ap, and passing through B.

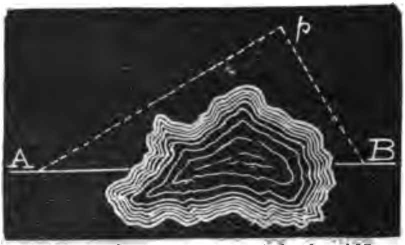

FIG. 29.

Measure pB, and compute $AB = \sqrt{Ap^2+Bp^2}$.

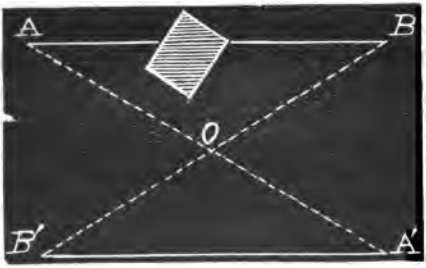

FIG. 30.

(3) **Measure from A and B, lines meeting, as at O.**

Extend BO to B', making $OB' = BO$.

Extend AO to A', making $OA' = AO$. Measure $B'A'$.

CASE 2.—*When one end of the line only is accessible.*

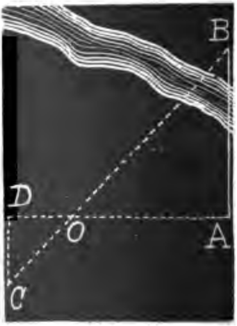

FIG. 31.

SUGGESTIONS.—(1) At A, set off a perpendicular, as AD, of convenient length, and at D, a perpendicular, as DC. Mark, as at O, the intersection of AD and BC. Measure CD, DO, and OA.

Then is $AB = \dfrac{AO \times CD}{OD}$.

(2) Set off at A a perpendicular, as AD, of convenient length. At D set off a perpendicular to DB meeting AB, as at C. Measure AD and AC.

Then is $AB = \dfrac{AD^2}{AC}$.

(3) From A, measure any convenient line, as AC. From a point, as D, in the line BC, measure a line parallel to CA, meeting AB, as at E. Measure EA.

Then is $AB = \dfrac{EA \times AC}{DE - AC}$.

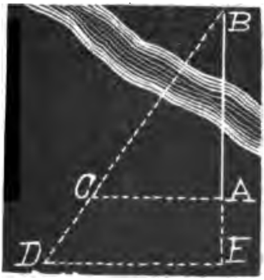

FIG. 32.

CASE 3.—*When each end of the line is inaccessible.*

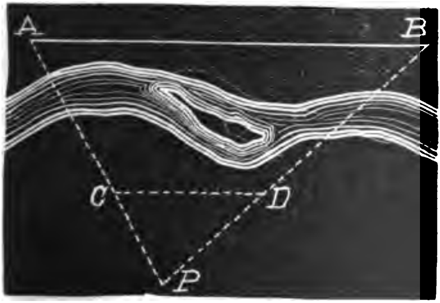

FIG. 33.

SUG.—(1) Choose any convenient point P, and as shown in Case 2 find the lengths of the lines PA and PB. On one of these lines, as on PA, measure from P to a convenient point C.

On PB, measure from P a distance $PD = \dfrac{PC \times PB}{PA}$.

Measure CD. Then is $AB = \dfrac{PA \times CD}{PC}$.

SCH.—The point P may be taken between CD and AB, if more convenient.

(2) Choose any point, as C, and find the distance CA. From C run a line of convenient length parallel to AB, (Prob. 3, Case 2), as CD, measuring it. Find the point, as

P, the intersection of the lines *AC* and *BD*. Measure *PC*.

Then is $AB = \dfrac{PA \times CD}{PC}$.

52. Prob. 6. *To set a stake in line between two points on opposite sides of a hill.*

SUGGESTION.—Let two men, each with a staff, go upon the hill and place themselves, as near as they can judge, in the required line and at a convenient distance apart, where each is able to see the point beyond the other. Now let each man in succession put the other in line between himself and the point beyond.

EXERCISES.

53. 1. Mark three points as the vertices of the angles of a triangle. Measure the longest side of the triangle, and also the perpendicular to it from the opposite angle.

Plat and find area of the triangle.

2. Set stakes marking the corners of an irregular four-sided field. Measure the longest diagonal and the perpendiculars upon it from the corners opposite.

Plat and find area of the field.

3. Measure the longest side of the field in the last exercise, and also the perpendiculars upon it from the opposite corners.

Find area and compare results.

Field Notes.—A simple and very convenient form of noting measurements is by writing them upon a rough sketch or diagram of the field.

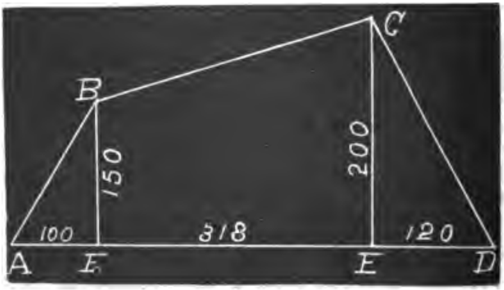

FIG. 34.

Thus, in a case like that of Ex. 3, the measurements as *AF*, *FB*, *FE*, etc., may be noted as shown in Fig. 34.

Another form is that in which the measurements on the principal line, called the **Chain Line**, are written in a column, as in the margin. A long narrow book* is best adapted to the purpose, the column occupying the middle of a page, so as to afford space for entries at the right and left. The point of beginning is denoted at the foot of the column, and the distances from the point of beginning to other points of the same line are written in the order of their measurement toward the top.

	538	to *D*
C 200	418	
B 150	100	
From *A*	0	

Measurements on perpendiculars to the chain line at the right or left of that line, regarding the direction in which the work proceeds, are written at the right or left of the column and opposite the distances, respectively, of the perpendiculars from the initial point of the line to which they are referred.

54. The Surveyor's Cross.—This is an instrument used in erecting perpendiculars. It consists of two pairs of sights placed at right angles to each other and supported upon a staff. To erect a perpendicular to a line at a given point, the instrument is set up in a vertical position at the point, and one of the pairs of sights directed toward a staff set up on the line. Being careful not to stir the instrument, a staff is then set in the direction of the other pair of sights.

To let fall a perpendicular to a line from a given point, the instrument is set as near as can be judged by the eye at the foot of the required perpendicular, then, observing how far out of the line of sight the given point may be, the instrument is moved readily to its proper place.

In practice, the Surveyor's Compass is usually employed for the purposes of the Cross.

---

* A book with paper ruled in small squares is convenient.

Plat and compute area of the following:

4. A rectangular field, length 8.50 ch., width 5.30 ch.
Area, 4 A. 80.8 sq. rd.

5. A square field, 7 ch. 15 l. on a side. Area, 5 A. 18 P.

6 The triangular field indicated by the following notes: (Measurements in links).
Area, 1.53 A.

	C	
	850	
	540	360 *B*
From	*A*	go East

7. A triangular field whose sides are 19 ch., 17.5 ch., and 18 ch.
Area, 21.18 A.

SUG.—Area $=[s(s-a)(s-b)(s-c)]^{\frac{1}{2}}$ in which a, b and c are the sides and $s=\frac{1}{2}(a+b+c)$.

Log area $= \frac{1}{2}[\log s + \log(s-a) + \log(s-b) + \log(s-c)]$.

8. A triangular field whose sides are 12.50 ch., 11.25 ch., and 9.42 ch.

9. Two triangular fields from the following notes:

A line across the column indicates a change of direction of measuring. The letters *R* and *L* indicate whether the change is toward the right or left.

The symbol o denotes a station.

(1)

	to o *A*
	12.00
	R o *C*
	to o *C*
	16.00
	R o *B*
	to o *B*
	14.00
From	o *A*

(2)

	to o *A*
	9.20
	L o *C*
	to o *C*
	8.50
	L o *B*
	to o *B*
	10.30
From	o *A*

10. A four-sided field $ABCD$ of which $AB = 8$ ch., $BC = 6$ ch., $CD = 9$ ch., $DA = 7$ ch., and $AC = 12$ ch.

11. A quadrangular field $ABCD$ in which $AB = 9$ ch. is an east and west line, $BC = 5.19$ ch. is a north and south line, and $AD = 8$ ch. is parallel to BC.

12. The fields indicated by the following notes:

(1)

	D	
	1250	
C 580	940	
B 325	415	
From	*A*	go E.

(2)

	D	
	1530	
	1000	512 *C*
	520	400 *B*
From	*A*	go N.

(3)

	D	
	1204	
	956	410 *C*
B 342	628	
From	*A*	go W.

SUG.—(1) Area $= \frac{1}{2}[(940 \times 325) + (1250 - 415) \times 580]$; and (3) area $= \frac{1}{2}[(342 + 410) \times 1204]$.

13. Set out an irregular four-sided field and measure it by each of the methods of Ex. 12. Make the plat, compute area and compare results.

14. Plat and find the area of a five-sided field *ABCDE* from the annexed notes.

	E	
	653	
D 225	300	
	R o *C*	
	C	
	1040	
B 430	750	
	420	205 *E*
	A	

```
 205            653
 430            225
 ---            ---
 635           3265
1040          1306
-----        1306
25400        -----
635          1.469
-----        6.604
6.604        -----
          2 ) 8.073
             -----
             4.037
```

Area, 4 A. 5.9 sq. rd.

In extended surveys, it is customary to measure one or more other lines than those required in platting and finding area, as a means of verifying the work.

Such lines are called **Proof Lines**, or **Checks**.

In selecting a proof line, care should be taken to choose one which would be most likely to detect any inaccuracy that may have occurred *anywhere* throughout the work. The following are examples of the use of proof lines:

15. Plat and find the area of the fields indicated by the following notes:

(1)

	579	to *C*
From	*F.S.*	on *AB*
	══	
	600	to *A*
	L o *C*	
	900	to *C*
	L o *B*	
	1000	to *B*
	500	*F. S.*
From	*A*	

(2)

Proof Line	*D*	
	560	
	B	
	══	
	C	
	850	
	625	210 *D*
B 315	430	
	A	

A point temporarily marked on a line, as at the middle of *AB*, with the intention of returning to it and measuring to some point, as *C*, is called a **False Station**, and is denoted by *F.S.*

Measurements not contiguous, but commencing at some other point in the field, are separated from other measurements by double lines across the column.

16. Plat and find area of two fields surveyed as shown by accompanying notes:

(1)

	B	
000	1000	
440	850	
200	600	
305	410	
115	270	
000	*A*	

(3)

	B	
000	1100	000
	930	212
400	650	
	570	320
115	410	
	A	228

Example (1) indicates a field bounded by a straight line, and by an irregular or broken line on one side of the straight line and joining the same points, *A* and *B*. The figures at the left of the column are the lengths of perpendiculars upon the chain line from the corners of the field, or prominent points of the broken boundary.

In Ex. 2, the chain line is a diagonal of the field, and the perpendiculars occur on opposite sides of it.

Perpendiculars employed in this manner are called Offsets.

17. Find the area of a field measured by equidistant offsets, as below:

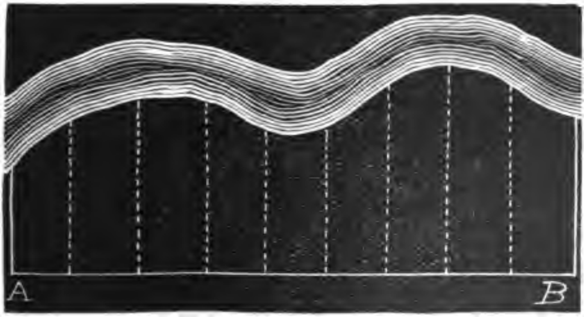

FIG. 35.

RULE.—*From the sum of the offsets, subtract the half-sum of the extreme ones and multiply the remainder by the common distance between them.*

18. Find the area of the fields given by the following notes:

312	500	to *B*	000	500	to *B*	240	600	000 *B*
560	400		250	400		425	500	170
425	300		120	300		322	400	317
350	200		342	200		150	300	208
105	100		470	100		216	200	344
230	000		000	000		370	100	126
From	*A*		From	*A*		210	*A*	320
	(1)			(2)			(3)	

19. Indicate manner of finding the area of a pond, or other inaccessible surface by offsets from the sides of a circumscribed rectangle.

If convenient, execute an actual survey of the kind. Plat and compute area.

20. To find the area of a triangular field ABC, there were made the following measurements:

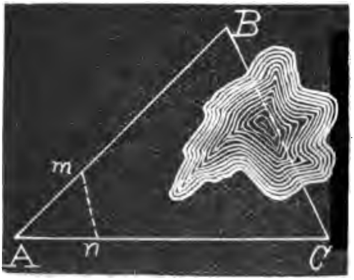

Fig. 36.

$Am = 5$ ch. and $AB =$ 14.50 ch.

$An = 4$ ch. and $AC =$ 17.25 ch.

Also, $mn = 4.50$ ch.

Required the area.

SUGGESTION.—Find area of the triangle Amn, (Ex. 7).

Then area ABC = area $Amn \times \dfrac{AB \times AC}{Am \times An}$.

The operation of finding area Amn is facilitated by making mn perpendicular to AB or AC.

If Am and An are taken equal, we have

$$\text{Area } Amn = \frac{mn}{4}\sqrt{4Am^2 - mn^2}.$$

Lines used as mn are called **Tie Lines.**

If $Am = An$, the angle subtended by mn is called a **Chain Angle.**

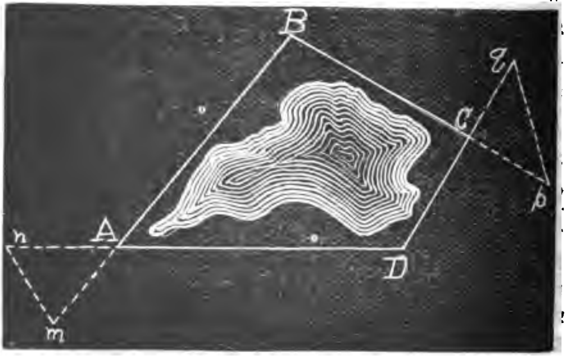

Fig. 37.

By prolonging the sides of a field externally, tie-li[…] or chain angles may be formed outside the field, wh[…] more convenient.

21. *Propose and execute a survey by tie-lines, of* [...] *irregular four-sided field whose sides may be measur[…] but whose diagonals are inaccessible.*

22. *Plat and compute area of a field from the followi[…] measurements:*

FIG. 38.

	476	to
n 240	226	m
	L o D	
	590	to
q 256	200	p
	L o C	
	343	to
q	143	
	L o B	
	723	to
n	250	
From	A	g

Area, 2 A. 1 R. 3[…]

SUGGESTIONS.—In platting the above survey, lay down the triangle *Amn* and produce the sides to *B* and *D*. With the points *B* and *D* as centers and the distances *BC* and *DC* as radii, describe arcs intersecting at *C*. Verify the plat by use of *pq* as a proof-line.

Take as the scale 2 or 3 chains to the inch, as the dimensions can then be measured on the plat with greater accuracy.

The scale of the above plat is 4 ch. to the inch.

If the plat is accurately laid down, the area may be found by measuring the diagonal *AC* and the perpendiculars to it from *B* and *D*, as drawn in the above plat. The area as above given was found, however, in the manner of Ex. 19, since in that way the accuracy of the result is made to depend upon the precision of the field work without the additional requirement of exactness in platting.

If, however, the plat is carefully constructed on a large scale, as suggested, the area may be found with sufficient closeness, usually, for practical purposes, from measurements made on the plat.

When the main lines of a survey are of considerable length, as from 30 to 40 chains, it is desirable to take the tie-lines at least 10 chains from the angles across which they are measured, since a small error in platting an angle with a short tie-line would cause the sides to deviate materially from their true position, when considerably prolonged. If in conseqence of obstructions, a long tie-line cannot be measured, a short one may be carefully measured, exact to even one-fourth of a link, and its length and distances from the angle multiplied by four to remove the fraction.

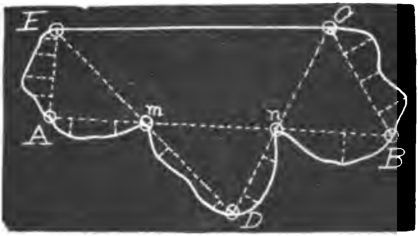

FIG. 39.

23. In a survey of the field *ABCDE*, the measurements were made along the dotted lines in the following order: *AB*, *BC*, *CD*, *DE*, *EA*, taking the offsets in passing, and noting the distances of the points of intersection, *m* and *n*. Required the proper form of the notes, the manner of constructing and proving the plat, and the procedure for area.

24. Plat and find area of a field from the following notes:

	A	
000	1760	
84	1440	
n	1364	348 *m*
96	920	
168	620	
86	332	
000	*R* o *C*	

	C	
	1280	000
	904	80
	660	112
	420	88
	R o *B*	000

	B	
000	2328	
100	1912	
50	1409	1053 *C*
90	1160	
60	920	
120	652	
	572	*m*
000	*A*	

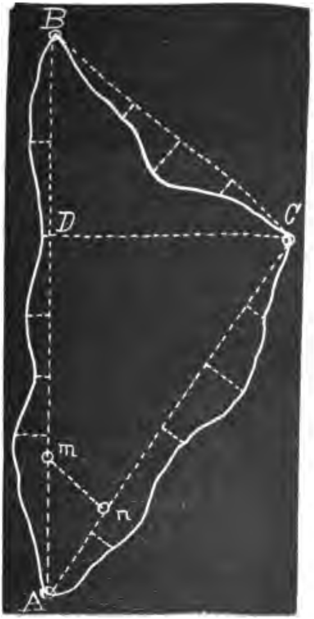

FIG. 40.

25. In a survey of the field *ABCD*, Fig. 41, the following measurements were made:

From *A* 304 | * 0, 276 | 209, 392 | 329, 344 | 539, 252 | 652, 374 | 802, 340 | 936, 366 | 1016, 316 | 1066, 250 | 1200, to *D*.

Required the notes in form, a plat of the field, and its area.

* In this notation, the number written at the left is the length of the offset, and the one at the right is the distance of the offset from the initial point of the line.

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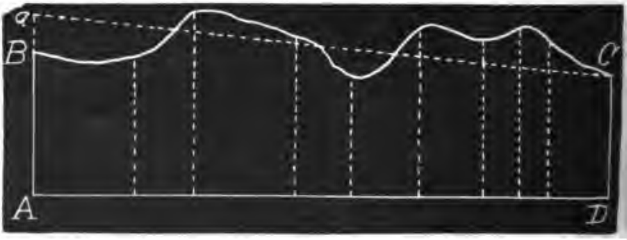


FIG. 41.

Having constructed the plat, stretch a fine thread along the irregular boundary, as from *C* to *a*, so as to make the areas added to the field, as near as can be judged by the eye, equivalent to those subtracted from it. Measuring *Aa* on the plat, the area of the field is given by the formula $AD\frac{(Aa+CD)}{2}$.

The above is called the method of **Equalizing** or of **Giving and Taking**, and is much used by practical surveyors. By experience, the surveyor is enabled to locate the equalizing line on the ground, which contributes to great saving of time and labor when area, only, is required.

26. Plan and execute a survey similar to that given in the last Example.

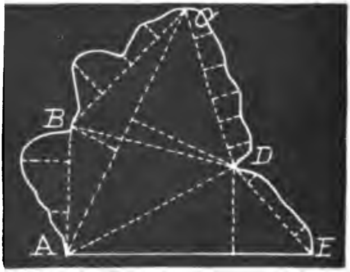


FIG. 42.

27. Find the area of the field *ABCDE*, from the accompanying notes.

000	1730	to *C*
250	1340	
200	960	
480	460	
000	*R* o *B*	
000	1450	to *B*
500	1000	
200	500	
From	*A*	

000	1212	to *E*
120	1018	
130	300	
000	*L* o *D*	
000	1720	to *D*
255	1478	
200	1238	
250	998	
150	760	
150	300	
000	*R* o *C*	

Check.	1776	to *D*
	B	
	3017	to *C*
	1805	1220 *D*
B 500	1361	
	R o *A*	
	2744	to *A*
	774	930 D
	R o *E*	

28. Find area of the field shown below, from the accompanying measurements.

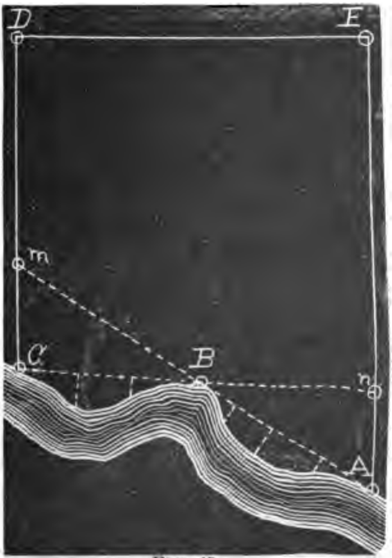


FIG. 43.

	F. S.	on *AE*
	786	
	420	to *B*
	348	60
	180	86
From	*C*	
	F. S.	on *CD*
	945	
	450	to *B*
80	316	
92	222	
64	86	
From	*A*	

	693	to *D*
	252	*F. S.*
From	*C*	

Check.	735	to *E*
	R o *D*	

792	to *A*
738	*F. S.*
R o *E*	

29. To find the area of a five-sided field *ABCDE*, measurements were made as follows:

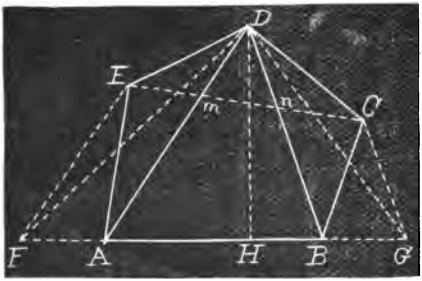


Fig. 44.

AB 8.64 ch., *BC* 5.10 ch., *CD* 6.30 ch., *DE* 5.58 ch., *EA* 6.54 ch., also the diagonals *AD* 10.80 ch. and *BD* 9.42 ch.

Required the area of the field from the above measurements, and also by measuring on the plat the base and altitude of the equivalent triangle.

Area, 6 A. 3 R. 20 P.

REMARK.—A much less laborious plan of measuring the above field would be to measure the diagonals *AD* and *DB*, noting on them the distances to the points *m* and *n* of intersection with the diagonal *EC*, and then measure *Em*, *mn* and *nC*, also a side, as *AB*, for use as a proof-line.

30. Plan and execute a survey like that of the last Example, pursuing either method, or each, at pleasure, keeping the notes in regular form, constructing the plat and computing area.

55. Rectangular Coordinates.—Let XX' and YY' be two lines intersecting each other at right angles, as at O.

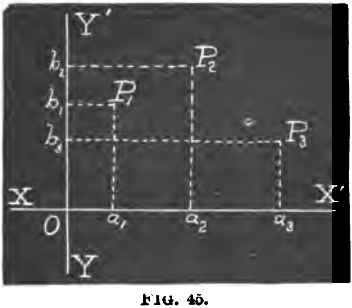


Fig. 45.

Let P_1, P_2, P_3 be any points in the plane of the lines.

Let P_1a_1, P_2a_2, P_3a_3, be perpendiculars from the points upon the axis XX', and P_1b_1, P_2b_2, P_3b_3 be perpendiculars from the points upon the axis YY'.

The distances Oa_1, Oa_2, Oa_3 are called **Abscissas** of the points P_1, P_2, P_3; and the distances Ob_1, Ob_2, Ob_3 are called **Ordinates** of the points.

The point O is called the **Origin**.

The abscissa and ordinate of a point are together called **Coordinates** of the points.

Coordinates at right angles with each other are called **Rectangular Coordinates**.

It is customary to denote abscissas by x and ordinates by y, coordinates of different points in connection with each other being distinguished by use of subscripts.

Thus, of the point P_1, the coordinates Oa_1 and Ob_1 or a_1P_1 may be denoted by x_1 and y_1; of the point P_2, the codinates Oa_2 and Ob_2 or a_2P_2 may be denoted by x_2 and y_2; and so on.

It will be seen that the coordinates of a point afford the means of locating it with respect to the axes.

The use of longitude and latitude in Geography is an illustration.

By use of the signs $+$ and $-$, the coordinates of any point in the plane of the axes are readily expressed.

EXERCISES.

56. 1. Construct the point of which $x = 4$ and $y = 7$.

2. Given $x = -5$ and $y = 3$, to construct the point.

3. Given $x = -3$ and $y = -6$, to construct the point.

4. Given $x = 6$ and $y = -4$, to construct the point.

5. Given $x = 0, y = 2$; $x = -5, y = 0$; $x = 0, y = 0$. Required the points.

57. **Application to Area.**—Let it be required to find

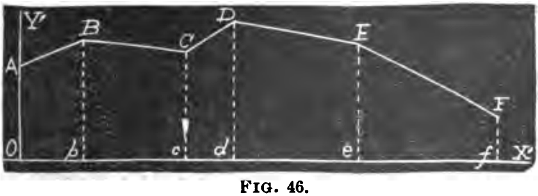


FIG. 46.

the area of a series of trapezoids included between perpendiculars from the points of a broken line upon a

straight line. Suppose the straight line, as OX', to be an axis of abscissas, and the first perpendicular at the left, as OA, to be an axis of ordinates.

Let x_1, x_2, x_3, etc., be the abscissas of the points A, B, C, etc., and y_1, y_2, y_3, etc., the corresponding ordinates.

Accordingly, the area of the several trapezoids is

$$\tfrac{1}{2}[x_2(y_1+y_2)+(x_3-x_2)(y_2+y_3)$$
$$+(x_4-x_3)(y_3+y_4)+---(x_n-x_{n-1})(y_{n-1}+y_n)],$$

in which n is the number of trapezoids plus one.

The above formula may be changed to the form

$$\tfrac{1}{2}[x_2(y_1-y_3)+x_3(y_2-y_4)+x_4(y_3-y_5)$$
$$+------x_{n-1}(y_{n-2}-y_n)+x_n(y_{n-1}+y_n)]. \quad (a).$$

Whence, for the area included between a straight line, as a base, and a broken line whose points are given by their coordinates upon the base, we have the following

RULE.—*From each ordinate subtract the second succeeding one and multiply the remainder by the abscissa corresponding to the intervening ordinate.*

Also, multiply the sum of the last two ordinates by the last abscissa.

Divide the algebraic sum of the products by 2.

The above formula and Rule have been deduced independently of any supposition as to the relative directions of the parts of the broken ine. They are therefore true whatever may be the form of the broken ine., That is, whether any part should be perpendicular to the base, either toward or from it, or whether any part should be turned backward respecting the preceding one.

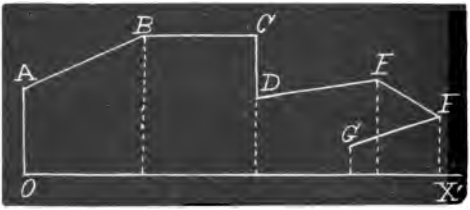


FIG. 47.

SUGGESTION.—Let the student verify the Rule in a case, for example, like the following, in which BC is represented as being parallel to the base, CD as perpendicular toward it, and FG as being turned backward from EF.

Find how it would be, if one or more of the ordinates were zero; if one or more were negative.

EXERCISES.

58. 1. Given $y_1=12$, $y_2=12$, $y_3=6$, $y_4=8$, and $y_5=6$, also, $x_1=10$, $x_2=18$, $x_3=24$, $x_4=30$, and $x_5=20$, to find area.

Given the following, to find area:

(2)

140	1000
435	812
250	725
200	500
360	450
320	000

(3)

000	950
240	844
306	530
640	325
415	200
000	000

(4)

1000	200
1150	317
828	420
650	305
460	524
000	250

59. As a second example of the application of coordinates in finding area, let there be taken an ordinary polygon, as *ABCDEF*.

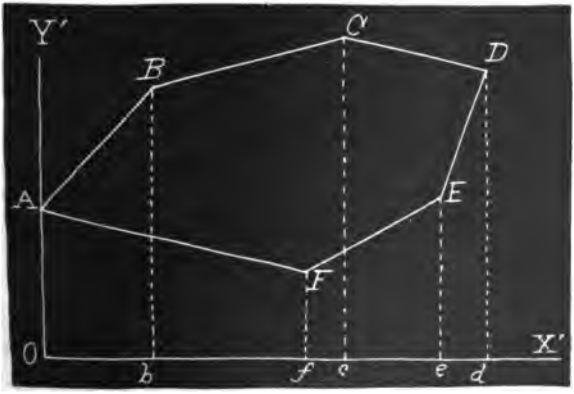


FIG. 48.

Let x_1, x_2, x_3, etc., be the abscissas of the points *A*, *B*, *C*, etc., and y_1, y_2, y_3, etc., the corresponding ordinates.

Now since formula (*a*) is true for any broken line, it holds for the case in which the broken line beginning, as at *A*, returns to the same point, forming thus a polygon as *ABCDEFA*.

In this case, the last term of (*a*) vanishes, and we have, as the area a polygon of n sides,

$$\tfrac{1}{2}[x_1(y_n - y_2) + x_2(y_1 - y_3) + x_3(y_2 - y_4) + x_4(y_3 - y_5) + \text{etc., to } n \text{ terms}]. \qquad (b)$$

or, factoring with respect to y, we have the form

$$-\tfrac{1}{2}[y_1(x_n - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4) + y_4(x_3 - x_5) + \text{etc., to } n \text{ terms}]. \qquad (c)$$

Whence, for the area of a polygon whose vertices are given by their coordinates, we have the following

RULE.—*From the ordinate of each vertex subtract the second succeeding one, and multiply the remainder by the abscissa of the intervening vertex;* or, *from the abscissa of each vertex subtract the second succeeding one, and multiply the remainder by the ordinate of the intervening vertex.*

Divide the sum of the products by 2.

SCH.—Formulas (*b*) and (*c*) will be seen to be in accordance with any situation of the coordinate axes, agreeably with convenience of field work. In particular cases, one or more terms will be found to disappear. Due attention to algebraic signs is important.

The formulas are easy to remember, and simple of application. With an instrument adapted to laying off right angles, they afford the most practical means of computing the content of irregular tracts.

EXERCISES.

60. 1. Required the area and a plat of a field the coordinates of whose corners are

$x_n = x_6 = 0$, $x_1 = 7$ ch., $x_2 = 12\frac{1}{2}$ ch., $x_3 = 18$ ch., $x_4 = 15$ ch., $x_5 = 10$ ch.; and

$y_n = y_6 = 6$ ch., $y_1 = 12$ ch., $y_2 = 20$ ch., $y_3 = 15$ ch., $y_4 = 8\frac{1}{2}$ ch., $y_5 = 0$ ch. Area, 16 A. 0 R. 28 P.

Find the area, supposing a different situation of the axes.

Required area and plat of the following:

(2)

Sta.	Abscissas.	Ordinates.
A	0000	9.30
B	7.25	16.12
C	15.18	20.05
D	28.10	12.00
E	22.14	5.40
F	10.20	0.00

Area, 31 A. 17.35 P.

(3)

Sta.	Abscissas.	Ordinates.
A	3.24	6.40
B	8.15	12.60
C	12.00	21.50
D	12.00	8.00
E	15.20	5.20
F	9.50	2.12

Area, 17 A. 28 P.

4 and 5. Plan and execute two surveys by the above method.

Plat and compute area.

SECTION IV.

COMPASS SURVEYING.

VERNIER COMPASS—6-INCH NEEDLE.

FIG. 49.

61. The foregoing pages have been occupied with the use of the *Chain*, that most important instrument of linear measure, in the development of a system of work called **Chain Surveying**.

We come now to the consideration more particularly of instruments of angular measurement, the simplest of which is the **Compass**.

The surveyor's compass consists of a circular box of brass usually about six inches in diameter, resting upon an arm of the same metal about fourteen inches in length. At the extremities of the arm are vertical attachments through which are fine slits terminated at intervals by circular apertures which serve as *sights* in directing the instrument upon any point. At the center of the box is a small vertical pin upon which is balanced a slender magnetized bar of steel called the **Needle**.

Turning with a free horizontal motion, the pointed ends of the needle traverse the graduated circumference of a circle which forms the bottom of the box. The plane of the sights cuts this circumference in two points marked N, and S, or otherwise distinguished as the north and the south points of the instrument. From these points the graduation of the circle runs 90° in each direction to the points marked E and W.

A circle of plate-glass forms the cover of the box. Two small spirit levels are placed at right angles to each other upon the arm, to aid in rendering the plane of the instrument horizontal.

The compass is mounted upon a three-legged support called a **Tripod** or upon a single staff called a **Jacob Staff**, with which it is so connected as to admit of being turned in any desired direction.

62. The magnetic needle, in almost all parts of the United States, points more or less to the east or west of a true meridian, or north and south line.

This deviation, which is called the **Variation or Declination** of the needle, is not constant, but increases or decreases by a very sensible amount in a series of years.

63. The Line of No Variation, as it is called, or that upon which the needle will indicate a true north and south direction, is not a fixed line. In 1870, its position was about that of Sault Ste. Marie, Cleveland, Raleigh, and Wilmington. It is now further west, nearly, if not quite, at the meridian of Lansing, as respects Michigan.

A compass-needle, therefore, placed east of this line would have a declination to the west, and when placed west of the line, the declination would be to the east, and in both cases the declination would increase as the needle was carried farther from the line of no variation.

Since the amount of the declination is constantly changing, a surveyor in running lines from field notes of some years' standing, as in the resurvey of a farm, for example, is obliged to make allowance for the changed direction of the needle, for each line that he runs.

To facilitate this, most compasses in use have attached a short graduated arc called a **Vernier**, by means of which the instrument may be set to any declination, as desired.

To do this, the surveyor places his instrument on some well defined line of the old survey, and turns the tangent-screw until the needle of his compass indicates the same bearing as that given in the old field-notes of the survey.

Then screwing up the clamping nut underneath the vernier, he can run all the other lines from the old field-notes without further alteration.

64. Diurnal Variation.—Beside the change in direction of the needle, above noted, the needle is subject to another change called **Diurnal Variation**, occurring from hour to hour during the day. In the northern hemisphere, the north end of the needle moves westward from about 8 o'clock in the morning till about 2 o'clock in the afternoon, from 10′ to 15′, and then gradually returns to its first position. In the southern hemisphere, the direction of the motion is reversed. This variation is about twice as great in the summer as in winter.

The direction of a line as indicated by the compass at

different hours of the day might therefore vary a quarter of a degree, which would cause a deviation of nearly half a link to a chain, or thirty-five links to a mile.*

65. **Using the Compass.**—In using the compass, the surveyor should keep the south end toward his person, and read the bearings from the north end of the needle. He will observe that the letters E and W on the face of the compass are reversed from their natural position, in order that the direction of sight may be correctly read.

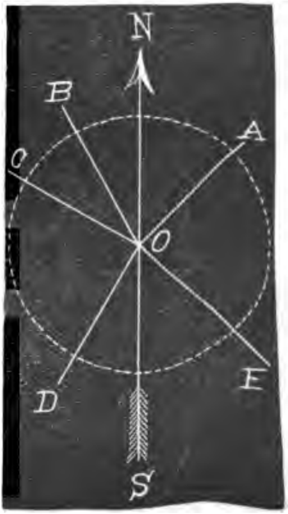
The compass-circle being graduated to half-degrees, a little practice will enable the surveyor to read the bearings to quarters, or even finer, estimating with his eye the space bisected by the point of the needle.

EXERCISES.

66. 1. *To measure an angle with the Compass.*

SUGGESTIONS.—Drive a peg marking the vertex of the angle, and set a staff on each side, at a distance of four or five chains from the vertex.

(1) Suppose AOB to be the angle thus marked. Set up the compass directly over the vertex of the angle. Level the instrument, and turn the sights in the direction of one of the sides of the angle, as OB.



Let NS represent the direction of the needle on coming to rest.

The line upon which the needle settles is called the **Magnetic Meridian**

The angle BON is the direction of the line OB from the magnetic meridian, and is called the **Bearing** of OB.

*For further information, see U. S. Coast and Geodetic Survey Report for 1879.

The bearing is indicated as to direction and amount by the division of the graduated circle of the compass to which the needle points. Thus, supposing the needle to point to the fifteenth degree from the north toward the west point of the circle, the bearing of OB would be read, "*North fifteen degrees West,*" and written N. 15° W.

Turning now the sights in the direction of OA, the angle AON, is the bearing of that line. Suppose the needle to indicate the bearing of OA, as N. 50° E.

Now the angle AOB is plainly the angle AON, plus the angle NOB, or $50° + 15° = 65°$, which is the measurement sought. Three other cases of the problem are presented in the following exercises which are left to the student to solve:

(2) Given the bearing of OC, N. 60° W., and of OB, N. 15° W., to find the angle BOC.

(3) Given the bearing of OA, N. 50° E., and of OE, S. 45° E., to find the angle AOE.

(4) Given the bearing of of OC, N. 60° W., and of OE, S. 45° E., to find the angle COE.

Calling N. and S. *meridianal* letters, we have for the angle between two lines from the same station, the following:

PRINCIPLES.—1. *When the meridianal letters are alike and the others unlike, the angle is the sum of the bearings,*

(2) *When the meridianal letters are unlike and the others alike, the angle is the supplement of the sum of the bearings.*

(3) *When both the meridianal and the other letters are alike, the angle is the difference of the bearings.*

(4) *When both the meridianal and the other letters are unlike, the angle is the supplement of the difference of the bearings.*

If the bearings of the sides of an angle are taken at different stations, one of the bearings must be reversed before applying the above principles.

2. Stake out a triangular field and take the bearing of each side in succession, going round the field.

Find the angles of the triangle and compare the sum with 180°.

3. Take the bearing of each side of a four-sided field. Find the angles and compare the sum with 360°.

4. Go round a five-sided field, taking the direct and the reverse bearings of the sides, observing whether the two agree in each case.

It is advisable in practice always to take the reverse bearings of lines as a means of detecting errors arising from local attraction.

The following problems are given as exercises for practice in the field. By their solution the student will acquire handiness in the use of the compass, and at the same time will obtain a knowledge of some important practical applications of the instrument.

67. Prob 1. *To let fall a perpendicular to a given line from a given point.*

CASE 1.—*When the line and point are accessible from each other.*

SUGGESTION.—Take the bearing of the line, and from the point run a line on a bearing differing 90° from the bearing of the given line.

CASE 2.—*When the point is inaccessible from the line.*

SUGGESTIONS.—Let AB be the given line, and P the given point. Take the bearings of P from the points A and B, and find the angles A and B. Measure AB.

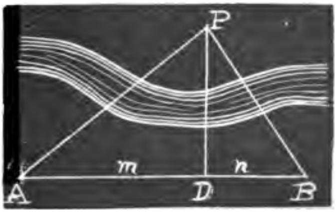


FIG. 51.

$$\frac{PD}{m} = \tan A, \text{ and } \frac{PD}{n} = \tan B. \text{ Whence, } m \tan A = n \tan B.$$

$$\text{or } \frac{m}{n} = \frac{\tan B}{\tan A}. \text{ Whence, } \frac{m}{m+n} = \frac{\tan B}{\tan A + \tan B}.$$

$$\text{or } m = \frac{AB \times \tan B}{\tan A + \tan B}. \text{ Similarly, } n = \frac{AB \times \tan A}{\tan A + \tan B}.$$

CASE 3.—*When the line is inaccessible from the point.*

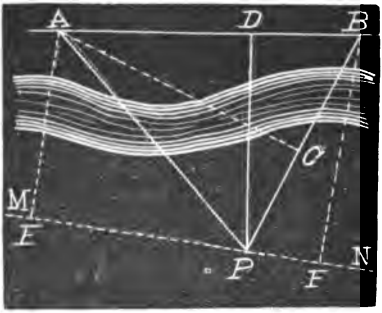


FIG. 52.

SUGGESTIONS.—At the given point take the bearings of PA and PB, and find the angle APB. Through P, upon any convenient bearing, run a line, as MN, upon which find the distances PE and PF to perpendiculars let fall from A and B upon MN, (Case 2).

Find PA and PB in the triangles PAE and PBF.

Suppose AC perpendicular to PB, whence angle BAC = angle BPD.

Now, $AC = PC \tan APB$, and $AC = BC \tan ABP = BC \cot BPD$.

$$\text{Whence, } \frac{\cot BPD}{\tan APB} = \frac{PC}{BC}.$$

Now, $PC = PA \cos APB$, and $BC = PB - PA \cos APB$.

$$\text{Whence, } \frac{\cot BPD}{\tan APB} = \frac{PA \cos APB}{PB - PA \cos APB},$$

$$\text{or } \cot BPD = \frac{PA \sin APB}{PB - PA \cos APB}.$$

(2) Measure on PA and PB, produced through P if necessary, distances PA' and PB' in the ratio of PA to PB. Take the bearing of the line $A'B'$ and direct the compass, at P, on a bearing differing by 90° from the bearing of $A'B'$, (Case 1).

Cor.—Length of perpendicular $PD = PB \cos BPD$.

68 Prob. 2. *To erect a perpendicular to a line at a given point.*

CASE 1.—*When the line and point are accessible from each other.*

SUGGESTION.—Set the compass at the given point, take the bearing of the line and then direct the instrument on a bearing differing 90° from the bearing of the line.

CASE 2.— *When the line is inaccessible.*

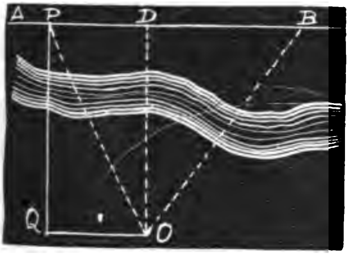


FIG. 53.

SUGGESTIONS. — Let AB be the given line and P the given point. Let O be any convenient point at which to work. Find the angle POD, (Prob. 1, Case 2), and the bearing of OD. Erect a perpendicular to OD at O, as OQ.

Measure on OQ a distance $= OP \sin POD$.

COR.—Length of $PQ = OP \cos POD$.

69. Prob. 3. *From a given point, to run a line parallel to a given line.*

SUGGESTION.—Let P be the given point and AB the given line.

From P run a line with the same bearing as AB.

When AB is inaccessible from P, so that its bearing cannot be taken, find the bearing of a perpendicular, from P to AB, as PD. (Prob. 1, Case 2), and at P erect a perpendicular to PD, (Prob. 2, Case 1).

70. Prob. 4. *To run a line between points not visible from each other.*

SUGGESTIONS.—Let it be required to run a line, as AB, from A.

FIG. 54.

Set the instrument at A and run a random, as AC, as

near the required direction of AB as convenient, to a point opposite some known point on AB, as P.

Measure CP perpendicular to AC.

Then $\tan CAP = \frac{CP}{AC}$, whence angle CAP is found. Knowing this angle and the bearing of AC, the bearing of AB becomes known.

Start now at A, on this bearing, and if no mistake has been made, the line will come out at B.

Having to set the instrument at points between A and B, the line should be continued by back-sights instead of by the direct bearing. *A general rule in such cases.*

The angle CAP, when it is quite small, may be found by multiplying 57.3° by PC, and dividing by AC. This is called the **Fifty-seven and three-tenths Rule.** The rule depends upon the fact that for small angles, AP differs insensibly from AC, and CP from the arc subtending the angle CAP.

Whence, angle $CAP : 360° :: CP : 2 \times 3.1416 \times AP$,

$$\text{or angle } CAP = \frac{CP}{AP} \times \frac{360°}{6.2832} = \frac{CP \times 57.3°}{AP}, \text{ or } \frac{CP \times 57.3°}{AC}.$$

The semi-circumference of a circle, with radius AP, is $3.14159265 \times AP$.

Whence arc $1' = 3.14159265 \times AP \div 10800$.

If $AP = 1$ ch., arc $1' = 0.00029088$ ch. $= 0.029088$ l.

If $AP = 1$ mi. $= 80$ ch., arc $1' = 0.029088$ l. $\times 80$ $= 2.327$ l. $= 2\frac{1}{3}$ l.

For a very small angle, the perpendicular PC differs insensibly from the arc of the angle. Hence, when angle $PAC = 1'$ and AP or $AC = 1$ mi., the perpendicular PC, without perceptible error, is $2\frac{1}{3}$ links. The line PC is called the departure of AC, for the distance AP or AC.

Taking $2\frac{1}{3}$ l. as the departure for 80 ch. at an angle of $1'$, the departure for 40 ch. would be $\frac{1}{2}$ of $2\frac{1}{3}$ l. $= 1\frac{1}{6}$ l. $= 1$ l. $+ \frac{1}{6}$ of 1 l.

For quite small angles, the departure varies directly as the angle. Whence, for 40 ch., the following:

$$\text{Dep. for } 1' = 1 \text{ l.} + \tfrac{1}{6} \text{ of } 1 \text{ l.};$$
$$\text{" " } 2' = 2 \text{ l.} + \tfrac{1}{6} \text{ of } 2 \text{ l.};$$
$$\text{" " } 3' = 3 \text{ l.} + \tfrac{1}{6} \text{ of } 3 \text{ l.};$$

and so on, practically true, to 60′ or 1°.

For any other distance, at the same angle, the departure varies directly as the distance. Accordingly,

Given minutes of angle, to find links of departure, we have the following

Rule.—*To the number of minutes, add its one-sixth and multiply the sum by the ratio of the distance to 40 ch.* (Good to 60 minutes.)

Or the following

General Rule.—*Multiply 0.0291 by the number of minutes, and multiply the product by the number of chains in the distance.* (Good to 240 minutes.)

Example.—Given angle = 30′ and distance = 23.20 ch., to find the departure.

Since for 40 ch., 1′ of angle gives $1\frac{1}{6}$ l. of departure, we may say, without sensible error for a small angle, that 1 l. of departure gives $\frac{6}{7}$ of 1′ of angle, for the same distance.

Or, as it may be written,

$$\text{Dep. of } 1 \text{ l.} = 1' - \tfrac{1}{7} \text{ of } 1'.$$
Similarly, $$\text{" " } 2 \text{ l.} = 2' - \tfrac{1}{7} \text{ of } 2',$$
$$\text{" " } 3 \text{ l.} = 3' - \tfrac{1}{7} \text{ of } 3',$$

and so on, practically true to 60′ or 1°.

For any other distance with the same departure, the angle varies inversely as the distance. Accordingly,

Given links of departure, to find minutes of angle, we have the following

Rule.—*From the number of links of departure, subtract its one-seventh and divide the remainder by the ratio of the distance to 40 ch.* (Good to 60 minutes.)

General Rule.—*Multiply 0.0291 by the number of chains in the distance, and divide the number of links of departure by the product.* (Good to 240 minutes).

Example.—Given the distance = 100 ch. and the departure = 1.30 ch. = 130 l., to find the angle.

71. Prob. 5. *To pass an obstacle in line.*

SUGGESTIONS.—Let AB be a line in which an obstacle is met at a. Different methods may be applied in passing the obstacle, some of which are the following:

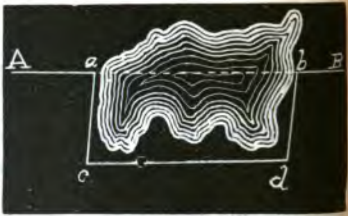


FIG. 55.

(1) Make an offset ac, in any convenient direction, and of sufficient length to pass the obstacle. Run cd parallel to Aa a sufficient distance, and return to the line AB by an offset db, parallel and equal to ac. Then $ab = cd$.

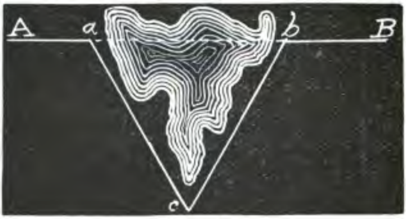


FIG. 56.

(2) Make at a, an angle bac, of 60°. Make at c an angle $acb = 60°$.

Measure $cb = ac$.

Then $ab = ac$.

(3) Make an offset at a, at right angle, measuring it a convenient distance to C. Measure at C the angle bCa, to the point b in the line AB, and beyond the obstacle. Go forward from b.

The distance $ab = aC \times \tan C$.

If the distance ab is made 100 or 1000, the distance ab may be taken directly from the Table of Tangents, with a proper placing of the decimal point.

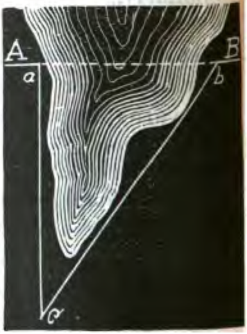


FIG. 57.

If b is not visible from a, it

may be found by measuring a distance $Cb = \dfrac{ac}{\cos C}$.

Examples.—1. A, a point on left bank of river running south-west; B, a point in line, north of A, on the right bank. Sighted a point C, west of A, on the right bank. Crossed river and measured north from C 10.18 ch. to a point D west of B. Required the distance AB.

2. A, a point on right bank of river running S. E.; B, a point in line, north of A, on left bank. From A run W., 3.33 ch. to a point, as C, from which the point B bears N. 16° 30′ E. Required the distance AB.

3. A, a point on left bank of river running N. W.; B, a point in line north ~~east~~ of A, on right bank. From A run ~~S.~~ W 5.90 ch. to a point, as C, from which the point B bears N. 17° E.; continued W ~~east~~ from C to D, a point 9 ch. from A, from which the point B bears N. 25¼° E.

Required the mean of results of the two observations, as the distance AB.

4 and 5. To be planned and solved by the student.

72. Prob 6. *To find a true north and south line by means of the North Star.*

The north star appears to describe a small circle about the true north point or pole as a center. The radius of this circle is called the **Polar Distance** of the star. This polar distance is not a constant quantity, but becomes about ⅓ of a month ~~second~~ of arc less every year. On the first January, 1880, it was about 1° 19′ 51′′, and in 1890 it will be about 1° 16′ 41′′.

When in its revolution, the star is farthest from the meridian, it is said to be at its **greatest eastern or western elongation.**

The times of the elongations as given by a correct clock, for latitude from 38° N to 60° N and to the year 1890, are approximately as shown in the following tables:

EASTERN ELONGATIONS.

Day.	Apr.	May.	June.	July.	Aug.	Sept.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 37 A.M.	4 39 A.M.	2 37 A.M.	12 39 A.M.	10 37 P.M.	8 36 P.M.
7	6 14 "	4 16 "	2 14 "	12 16 "	10 14 "	8 12 "
13	5 50 "	3 52 "	1 50 "	11 52 P.M.	9 50 "	7 48 "
19	5 26 "	3 28 "	1 26 "	11 29 "	9 27 "	7 25 "
25	5 03 "	3 05 "	1 03 "	11 05 "	9 03 "	7 01 "

WESTERN ELONGATIONS.

Day.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 27 A.M.	4 25 A.M.	2 28 A.M.	12 26 A. M.	10 24 P. M.	8 30 P.M.
7	6 04 "	4 02 "	2 04 "	12 02 "	10 00 "	8 06 "
13	5 40 "	3 38 "	1 40 "	11 39 P. M.	9 36 "	7 43 "
19	5 17 "	3 15 "	1 17 "	11 15 "	9 13 "	7 19 "
25	4 53 "	2 51 "	12 53 "	10 51 "	8 49 "	6 55 "

To find the meridian of a place by means of an elongation of the north star requires the arrangement of the following preliminaries:

Set two posts firmly in the ground about 8 feet apart east and west, and saw them off to a level about 3 feet from the ground.

Lay upon the posts a plank 3 or 4 feet long and 6 or 8 inches wide, planed smooth on the upper surface, and nail or pin it securely to the supports, forming a sort of table.

To the north of the table, at a distance of 10 or 12 feet, set in the ground a stiff pole 12 or 15 feet high, having a cross bar nailed to its top, in an east and west direction, from which to suspend a plumb-line nearly reaching the ground, and having a bob weighing 1 or 2 pounds, which may be caused to hang in a pail of water, to insure steadiness.

Provide also a block or piece of plank 8 or 10 inches long, and smooth on the under side. Let one of the compass sights be fastened at right angles with the upper surface of the block and even with the side which is to be toward the south.

Everything being thus got in readiness, the observer, a few minutes before the time of an elongation as given in the above Table, should be at his post and begin moving the block, even with the south edge of the table, keeping the plumb-line and star, as seen through the vertical slit, constantly in range with each other. A light will generally be needed near the plumb-line, to render it visible. As the star approaches its elongation, it will appear to move nearly vertical for several minutes, so as to be seen without moving the sight. When it is certain that the star has reached its elongation, confine the block carefully, by sticking a few tacks along its edges. Project the vertical slit to the ground by means of a plumb-line and mark the point by setting a substantial stake with its top a little below the surface of the ground.

Being still careful not to move the the block, let an assistant take one of the iron-pointed rods, or a stake, with a light, and go a hundred feet or more toward the star, and having found the point, as directed by the observer, in range with the plumb-line as seen through the slit, let him mark it by driving a stake.

Having now two stakes in range of the elongation, the remainder of the operation may be deferred till morning.

To find the angle which the line as above determined makes with the meridian of the point of observation, requires a trigonometrical computation.

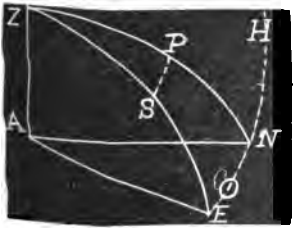


FIG. 58.

Let A be the point of observation, Z, the zenith of that point, HO, an arc of the northern horizon, N, the north point of that arc, S, the north star at its eastern elongation, PS, the polar distance of the star, AN, the meridian of the point of observation, and AE, the line of the two stakes.

The angle sought is NAE = angle PZS = arc NE.

Now, in the spherical triangle PZS, PZ is the co-latitude of the point A, which must be known. Solving this triangle, we have $\sin Z = \frac{\sin PS}{\sin ZP}$, or $\sin Z = \frac{\sin \text{polar dist.}}{\cos \text{lat.}}$.

From this, the angle Z becomes known, and, accordingly, it may be formed on the west side of the line AE, and thus the direction of the meridian AN determined.

On AN, thus found, let a substantial stake be set a hundred yards or more from A, and we have a permanent meridian with which we may compare the magnetic meridian at any time, and thus determine the declination of the needle.

The declination of the needle is the angle which the magnetic meridian makes with the astronomical meridian.

Every surveyor should have a standard meridian, as a means of knowing as near as possible the declination of the needle in his locality.

SCH.—For the purpose, simply, of finding the declination of the needle, it is sufficient to lay out on the ground the line of direction of the star at one of its elongations, and then, knowing the bearing of this line as shown by the needle, and the corresponding azimuth of the star, the declination of the needle is readily computed.

Thus, let $\pm a$ = azimuth, $\pm b$ = bearing, and $\pm d$ = declination, accordingly as they are *east* or *west*.

Then $\pm d = \pm a - (\pm b)$.

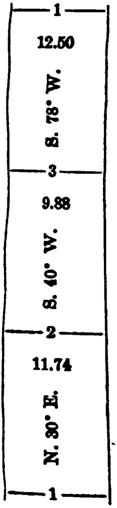
RULE.—*Subtract the bearing from the azimuth.*

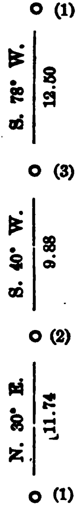
In applying the Rule, due regard is to be had to the algebraic signs.

73. Field Work.—This is the same as in Chain Surveying, with the additional matter of observing and recording the bearings of lines.

Lines traversed on bearings are called **Courses**.

The following are examples of forms of Field Notes:





Sta.	Bearing.	Distance
1	N. 30° E.	11.74
2	S. 40° W.	9.88
3	S. 78° W.	12.50

The first form admits of writing offsets at the right and the left of the column as heretofore done. In practice, the bearings in this form are written obliquely across the column, so as to be easily seen without turning the paper.

74. The following is a specimen of the form of Field Notes used in the United States Survey:

THIRD STANDARD PARALLEL NORTH

through

Range No. 21 East

of the

PRINCIPAL BASE AND MERIDIAN

in the

TERRITORY OF MONTANA

as surveyed by

JAMES M. PAGE,

U. S. Deputy Surveyor,

On the night of August 22, 1880, I took an observation on Polaris in accordance with instructions contained in the Manual of Surveys, and drove pickets on the line thus established.

Survey commenced August 23, 1880, with a Burt's improved solar compass.

Before commencing this survey, I test my compass on the line established last night, and find it correct.

I begin at the standard cor. to townships 13 north, ranges 20 and 21 east, which is a post 4 inches square, marked

S. C. T. 13 N. on N.;

R. 21 E., S. 31 on E.; and

R. 20 E., S. 36 on W. faces, with 6 notches on N., E., and W. faces, and pits N., E., and W. of post, 6 ft. distant, and mound of earth around post.

Thence I run

Chains	
	East on S. boundary Sec. 31. Va. 20¼° E. Ascend
18 . 00	A point about 200 ft. above township cor., top of ridge.
40 . 00	Set a sandstone 18 x 8 x 5 in., 12 in. in the ground, for standard ¼ sec. cor. marked S. C. ¼ on N. face, dug pits 18 x 18 x 12 in. E. and W. of stone, 5½ ft. dist., and raised a mound of earth 1½ ft. high, 3½ ft. base, alongside; thence over high, rolling prairie.
67 . 00	Enter pine timber.
80 . 00	Set a sandstone 24 x 10 x 7 in., 18 in. in the ground, for a standard cor. to Secs. 31 and 32, marked S. C., with 5 notches on E. and 1 notch on W. edges; from which A pine 12 in. diam. bears N. 77 E., 41 lk. dist., marked T. 13 N., R. 21 E., S. 32 B. T. A pine 18 in. diam., bears N. 50° W., 20 lk. dist., marked T. 13 N., R. 21 E., S. 31 B. T. A pine 7 in. diam., bears S. 30° W., 119 lk. dist., marked T. 13 N., R. 21 E., S. 5 B, T.
	Land high, mountainous, hilly and rolling. Soil sandy, gravel, and rocky; 4th rate. Timber pine, 23 ch.; mostly dead and fallen.

As will be noticed, this form consists of two columns, reading from the top. In the first, are entered the distances from the point of beginning; and in the second, the direction and description of the line run, the monuments planted, with the witnesses of the same, the features of the ground, timber, soil, etc.

It is a commodious and convenient form for use, and ought to be adopted by surveyors throughout the country.

Finding Area.—The manner of finding the area of fields surveyed with the Compass is developed in the solution of the following problems.

75. Prob. 1. *Given two sides and the included angle of a triangle, to find the area.*

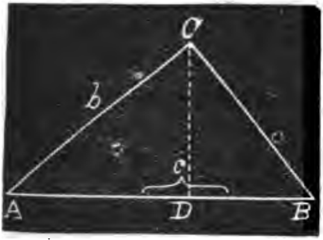


FIG. 59.

Solution.—Let ABC be a triangle in which are known the sides b and c and the angle A. Let CD be a perpendicular upon AB, from the opposite angle.

Geometrically, the area $ABC = \frac{c \times CD}{2}$; but $CD = b \sin A$, whence area $ABC = \frac{1}{2} bc \sin A$

Example.—To find the area of a triangular field, there were measured the bearings and lengths of two sides, from the same corner, as follows: (1) N. $20\frac{1}{2}°$ E., 12 ch.; (2) N. $72\frac{3}{4}°$ E., 30.10 ch.

Solution.—Let $b = 12$ and $c = 30.10$. The included angle $A = 72\frac{3}{4}° - 20\frac{1}{2}° = 52\frac{1}{4}°$.

Applying logarithms	$b = 12$	log	1.079181
in above formula:	$c = 30.10$	log	1.478566
	$A = 52\frac{1}{4}°$	log sin	9.898006
	2 ar. co.	log	9.698970
	Area = 142.8	log	2.154723

or Area = 14.28 A. = 14 A. 1 R. 4.8 P.

Let the student present an example from the "field."

76. Prob. 2. *Given two angles and the included side of a triangle, to find the area.*

Solution.—Let A and B be the given angles, and c the given side, (Fig. 59).

Now, $C = 180° - (A + B)$, whence

$$\sin C = \sin (A + B).$$

We also have, $\sin B : \sin C :: b : c$.

Whence, $b = \dfrac{c \sin B}{\sin (A+B)}$.

Again, $CD = b \sin A = \dfrac{c \sin A \sin B}{\sin (A+B)}$.

Whence, area $ABC = \dfrac{c \times CD}{2} = \dfrac{c^2 \sin A \sin B}{2 \sin (A+B)}$.

Example.—To find the area of a triangular field one of whose corners was marked by a stake in a pond, there were measured the opposite side 12.50 ch. and at one end were taken the bearings of sides N. 30° E. and N. 78° E., and at the other end, the bearing of the third side N. 40° W.

Solution.—Let A be the angle at which the first two bearings were taken, and B the angle at which the third was taken.

Then, $A = 48°$, and $B = 180° - (78° + 40°) = 62°$.

Applying logarithms	$c = 12.50$	log	1.096910
in above formula:	$c = 12.50$	log	1.096910
	$A = 48°$	log sin	9.871073
	$B = 62°$	log sin	9.945935
	2 ar. co.	log	9.698970
	$A + B = 110°$ "	log sin	0.027014
	Area = 54.55 sq. ch.,	log	1.736812

77. Prob. 3. *Given two angles and an opposite side of a triangle, to find the area.*

Solution.—Let A and B be the given angles, and a the given side.

Now, $\sin C = \sin (A+B)$, and $c : a :: \sin C : \sin A$,

whence, $c = \dfrac{a \sin (A+B)}{\sin A}$. We also have $CD = a \sin B$.

Whence, area $ABC = \dfrac{c \times CD}{2} = \dfrac{a^2 \sin B \sin (A+B)}{2 \sin A}$.

Example.—Given $a = 10$ ch., $A = 25\frac{1}{2}°$, and $B = 40\frac{1}{4}°$, to find the area of the triangle.

Solution.—Applying logarithms in the above formula,

$a = 100$	log	2.000000
$B = 40\frac{1}{4}°$	log sin	9.810316
$A + B = 65\frac{3}{4}°$	log sin	9.959882
2	ar co. log	9.698970
$A = 25\frac{1}{2}°$ "	log sin	0.366016
Area = 68.42	log	1.835184

or, Area = 6.842 A. = 6 A. 3 R. 14.7 P.

78. Prob. 4. *Given the diagonals of a quadrilateral and an angle formed by their intersection, to find the area.*

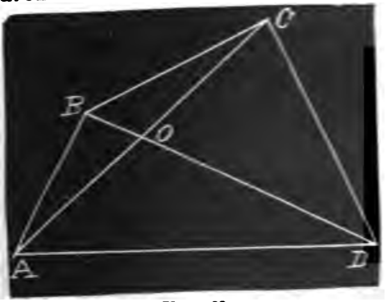


FIG. 60.

Solution.—Let $ABCD$ be the quadrilateral, m and n its diagonals, and O an angle at which the diagonals intersect.

By Prob. 1, area $AOB = \frac{1}{2}\, AO \times BO \sin O$
" $AOD = \frac{1}{2}\, AO \times DO \sin O$
" $DOB = \frac{1}{2}\, CO \times DO \sin O$
" $BOC = \frac{1}{2}\, CO \times BO \sin O.$

Whence, by addition, area $ABCD = \frac{1}{2}(AO + CO) \times (BO + DO) \sin O$,

$$\text{or, area } ABCD = \frac{mn \sin O}{2}.$$

Example.—The diagonals of a four-sided field were found to measure 18 ch. and 24 ch. Setting a compass at their intersection, the bearings of two adjacent corners of the field were found to be N. 30½° E. and S. 50° E. Required the area of the field.

Solution.—Applying logarithms in the above formula, having found $O = 99\frac{1}{2}°$, we have

$m = 18$	log	1.255273
$n = 24$	log	1.380211
$O = 99\frac{1}{2}°$	log sin	9.994003
2	ar co. log	9.698970
Area = 213.03	log	2.328457

or, Area = 21.303 A. = 21 A. 1 R. 8.5 P.

Let the student solve an example from his own practice.

79. Prob. 5. *Given the lengths and bearings of the sides of a polygonal field, to find the area.*

Solution.—Let *AB CDE* represent the field. Let *NS* denote the meridian of the most westerly station. This line, which may be assumed as passing through any station at pleasure, but more conveniently the extreme western or the eastern one, is called the **Principal Meridian.**

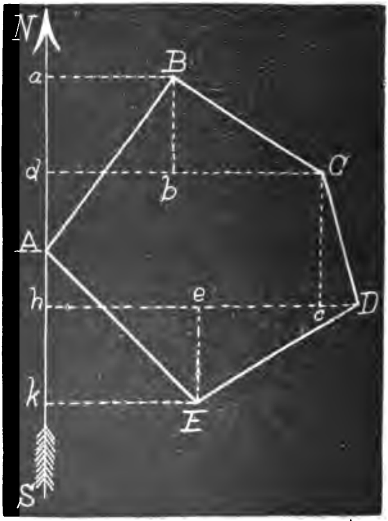


Fig 61.

To the principal meridian let there be drawn from the several stations the perpendiculars *Ba*, *Cd*, *Dh* and *Ek*, and upon *Cd* and *Dh* let there be drawn the perpendiculars *Bb*, *Cc*, and *Ee*.

These perpendiculars are, respectively, the bases and the altitudes of trapezoids composing a portion of the field.

Now, if from the sum of the areas of the trapezoids

the sum of the areas of the triangles ABa and AEk be subtracted, the remainder will be the area sought.

That is, clearing of fractions,

$$2 \times \text{area pol.} = (aB + Cd)\,Bb + (dC + Dh)\,Cc + (hD + Ek) \times Ee - aB \times aA - Ek \times kA.$$

It is now to be considered how the dimensions of the trapezoids and triangles depend upon the lengths and bearings of the sides of the field.

80. Latitude and Departure.—For convenience of description, let it be supposed that a survey of the field above represented was made "with the land on the right," beginning at A.

In going from A to B, there was made a distance Aa *north*, and a distance aB, *east*; in going from B to C there was made a distance Bb, *south*, and a distance bC, *east*; finally, in going from E to A, there was made a distance kA, *north*, and a distance Ek, *west*. Distances made *north* are called **Northings**; and *south*, **Southings**; distances made *east* are called **Eastings**, and west, **Westings**. Northings and southings are together called **Latitudes**; and eastings and westings are called **Departures**.

It will be seen that the length of a course is the hypotenuse of a right triangle of which the latitude of the course is the side adjacent to the bearing, and the departure, the side opposite the bearing. Whence,

Latitude = length of course $\times$ cosine of bearing, and
Departure = length of course $\times$ sine of bearing.

From these fundamental formulas, several others expressing relations of either of the four quantities to two others are easily derived.

Thus, denoting the latitude by l, the departure by d, the length of course by c, and the bearing by b, is obtained the following

TABLE OF CASES.

No.	Given.	Required.	Formulas.	
1	b, c	l, d	$l = c \cos b$,	$d = c \sin b$.
2	b, l	c, d	$c = \dfrac{l}{\cos b}$	$d = l \tan b$
3	b, d	c, l	$c = \dfrac{d}{\sin b}$	$l = \dfrac{d}{\tan b}$.
4	c, l	b, d	$\cos b = \dfrac{l}{c}$	$d = \sqrt{c^2 - l^2}$.
5	c, d	b, l	$\sin b = \dfrac{d}{c}$	$l = \sqrt{c^2 - d^2}$.
6	l, d	b, c	$\tan b = \dfrac{d}{l}$	$c = \sqrt{l^2 + d^2}$.

The Traverse Table.—This table, which is given with others in the back part of the book, shows the latitude and departure for any bearing to each quarter degree for any distance from 1 to 10. For other distances, the latitude or departure is found by adding the latitudes or the departures of the partial distances, as shown in the following

EXERCISES.

81. 1. Find the latitude and the departure for a bearing of 24°, for a distance of 7 ch.; for a distance of 5 ch.; for a distance of 10 ch.

2. Find the latitude and the departure on a bearing of 37½°, for a distance of 12 ch.

OPERATIONS.

For 37¼°, distance	10,	lat. = 7.9600	dep. =	6.0529
" " "	2,	" = 0.7960	" =	0.6053
" " "	12,	" = 8.7560	" =	6.6582

3. Find the latitude and departure on a bearing of 40¾° for a distance of 17.23 ch.

OPERATIONS.

For 40¾°, distance	10,	lat. = 7.5756,	dep. =	6.5276
" " "	7,	" = 5.3030,	" =	4.5693
" " "	.2,	" = 0.15151,	" =	0.13055
" " "	.03,	" = 0.022727,	" =	0.019583
" " "	17.23	" = 13.053	" =	11.247

ANOTHER FORM OF WORK.

Bearing.	*Distances.*	*Latitudes.*	*Departures.*
40¾°	1000	07576	06528
	700	53030	45693
	20	15151	13055
	3	22727	19583
	1723	1305.3237	1124.7433

We take the distance in links, and write the latitude and departure for the first figure of the number, omitting the decimal point; we write under them the latitude and departure for the second figure, setting them down one place farther toward the right; under them, the latitude and departure for the third figure, setting them one place farther toward the right, and so on.

We then add the separate latitudes and separate departures, and point off four figures from the right. The results thus obtained are the latitude and departure sought, as expressed in links.

4. Find the latitude and departure corresponding to a distance of 2643 links on a bearing of 75°.

OPERATIONS.

Bearing.	*Distances.*	*Latitudes.*	*Departures.*
75°	2000	5176	19319
	600	15529	57956
	40	10353	38637
	3	07765	28978
	2643	684.0195	2552.9948

Notice that bearings from 45° upward are found in the right hand column of the table, and the columns of latitude and departure are denoted at the foot of the page. Care needs to be taken here to avoid mistakes of latitudes for departures and departures for latitudes.

5. Find the latitudes and departures for the following bearings and distances:

(1) Bearing 52½°, Distance 437.
(2) Bearing 65¼°, Distance 3669.
(3) Bearing 21¾°, Distance 2030.
(4) Bearing 40°, Distance 506.
(5) Bearing 81½°, Distance 12.34 ch.

82. Meridian Distance. The distance of a station or any point from the principal meridian is called its **Meridian Distance.** The meridian distance of a line is the meridian distance of its middle point. By reference to the Figure (Art. 79), it will be seen that the bases of the trapezoids and triangles are the meridian distances of the several stations, except the first.

The triangles may be regarded as trapezoids, one of whose bases is the meridian distance of the first station.

It will be observed also that the sum of the bases of the trapezoids is the double meridian distance of the corresponding course, and that the altitudes of the trapezoids are the latitudes of the corresponding courses.

Having already connected latitude and departure with distance and bearing, it remains now only to connect meridian distance with departure.

We observe readily the following relations:

Merid. dist of $A=0$. (1).

"	"	" $B=$merid. dist. of $A+$depart. of AB.							(2)
"	"	" $C=$	"	"	" $B+$	"	"	BC.	(3)
"	"	" $D=$	"	"	" $C+$	"	"	CD.	(4)
"	"	" $E=$	"	"	" $D+(-$")		"	DE.	(5)

Adding the 1st and 2d, the 2d and 3d, and so on, of the above meridian distances, observing that the sum of the meridian distances of consecutive stations is the double

meridian distance of the included course, we obtain the following relations:

$2\times$merid. dist. of AB=departure of AB.

$2\times$ " " " BC=

$2\times$merid. dist. of AB+dep. of AB+dep. of BC.

$2\times$merid. dist. of CD=

$2\times$merid. dist. of BC+dep. of BC+dep. of CD.

$2\times$merid. dist. of DE=

$2\times$merid. dist. of CD+dep. of CD+dep. of DE.

$2\times$merid. dist. of EA=

$2\times$merid. dist. of DE+dep. of DE+dep. of EA.

Whence the following

PRINCIPLE—*The Double Meridian Distance of any course equals the double meridian distance of the preceding course, plus the departure of that course, plus the departure of the course itself.*

This general form of statement will be seen to be consistent with the fact that the double meridian distance of the first or the last course is equal to the departure of that course, and also with the usage of employing the term plus in the enunciation of an algebraic sum.

Referring to the expression of the double area of the polygon, (Art. 79), we observe that the factors of its several terms are (1) double meridian distances of the sides of the polygon and (2) latitudes. It will be noticed also that the subtractive terms are those in which the latitude is a northing.

Had the principal meridian been assumed to pass through the most easterly station, or had the measurements been supposed to be made with the "land on the left," the subtractive terms would have been those in which the latitude was a southing.

We are now prepared to find the area of fields by the method of double meridian distances.

Example.—Find the area of the field given in the following Field Notes:

Station.	Bearing.	Distance.
		ch.
1	N. 26½° E.	12.00
2	N. 59° E.	9.80
3	S. 66° E.	19.60
4	S. 35° W.	15.68
5	S. 66° W.	13.12
6	N. 46° W.	14.72

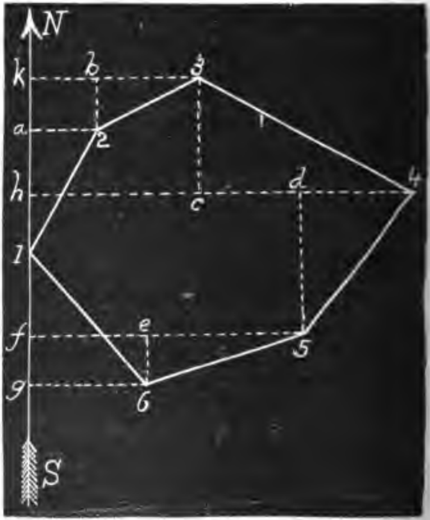


FIG. 62.

Finding from the Traverse Table the latitudes and departures to the nearest link, we have

Bearing	26½°	Dist.	12.00	Lat.	10.74 N.	Dep.	5.35 E.
"	59°	"	9.80	"	5.05 N.	"	8.40 E.
"	66°	"	19.60	"	7.97 S.	"	17.91 E.
"	35°	"	15.68	"	12.85 S.	"	8.99 W.
"	66°	"	13.12	"	5.34 S.	"	11.99 W.
"	46°	"	14.72	"	10.23 N.	"	10.59 W.

Obviously, in going entirely around a field there should be made the same southing as northing, and the same westing as easting. But from unavoidable lack of precision in the use of instruments, this is practically seldom found to have been done, according to the figures used. The error, however, can usually be made very small. Finding it large, the entire field work should be reviewed.

It is not a settled point among surveyors how great an error of latitude or departure may be allowed without resurveying the lot. Some would admit a difference of one link for every three chains in the sum of the distances, others for every five chains, and again others would require it to be within one link for every ten chains.

As a check against errors of bearing, a back sight should be taken at every station, and the reverse bearing compared with the corresponding direct bearing of that station. If the two are found to differ considerably, both should be reviewed. Let us now see how small an error of latitude and of departure we have in the present case.

Sum of northings $=10.74+5.05+10.23=26.02$.
" " southings $=7.97+12.85+5.34=26.16$.

Difference of latitudes $=00.14=$ error of latitude.

Sum of eastings $=5.35+8.40+17.91=31.66$.
" " westings $=8.99+11.99+10.59=31.57$.

Difference of departures $=00.09=$ error of departure.

The above errors may be considered reasonably small for a field of the size of the present one.

83. Balancing—The next work is to distribute the errors among the several courses in proportion to their lengths, in accordance with the following

PRINCIPLE.—*As the sum of the lengths of all the courses is to the length of each course, so is the total error to the error of that course.*

This operation is called **Balancing**.

In practice, exceptions to the application of the above principle may occur as follows:

(1) Some of the courses may have been measured over rough or uneven ground and, accordingly, such courses should bear a larger proportion of the error.

(2) Some of the bearings may have been taken with an indistinct sight, which would dictate the allotment of more than a proportionate amount of the error to them

The application of such exceptions is left to the judgment of the surveyor.

Distances as measured over uneven ground are liable to be too long. In such cases, the length of a course may be diminished when such change would favor the balancing. Similarly, a doubtful bearing may be changed, if the error should appear to be attributable to it.

Applying the above principle, we divide the errors by the sum of the lengths of all the courses and multiply the quotients by the length of each course, indicating the products as positive or negative, accordingly as they are to be added or subtracted in making the required correction.

Thus, $00.14 \div 84.92 = 00.00165$; and $00.09 \div 84.92 = 00.00106$; $00.00165 \times 12 = 00.0198$ or $+00.02$; and $00.00106 \times 12 = 00.01272$ or -00.01, to the nearest link.

In the same manner, by multiplying the above quotients by the lengths of the other courses, the correction for them is readily obtained.

Collecting results thus found, we have the following

TABLE I.

Sta.	Latitude.		Departure.		Cor.L	Cor.D	Balanced.			
	N.	S.	E.	W.			N	S.	E.	W.
1	10.74		5.35		+.02	—.01	10.76		5.34	
2	5.05		8.40		+.02	—.01	5.07		8.39	
3		7.97	17.91		—.03	—.02		7.94	17.89	
4		12.85		8.99	—.03	+.02		12.82		9.01
5		5.34		11.99	—.02	+.01		5.32		12.00
6	10.23			10.59	+.02	+.02	10.25			10.61

The Principle of Double Meridian Distance (Art. 82) can now be applied as follows.

Double merid. dist. of 1 - - - 2= 5.34.
" " " " 2 - - - 3= 5.34+ 5.34+ 8.39=19.07.
" " " " 3 - - - 4=19.07+ 8.39+17.89=45.35.
" " " " 4 - - - 5=45.35+17.89— 9.01=54.23.
" " " " 5 - - - 6=54.23— 9.01—12.00=33.22.
" " " " 6 - - - 1=33.22—12.00—10.61=10.61.

Thus, the double meridian distance of the last course comes out equal to the departure of that course, which verifies the numerical correctness of the preceding work.

We may now form the following

TABLE II.

Sta.	Latitude. N.	Latitude. S.	D.M.D.	Double + Areas.	Double — Areas.
1	10.76		5.34		57.4584
2	5.07		19.07		96.6849
3		7.94	45.35	360.0790	
4		12.82	54.23	695.2386	
5		5.32	33.22	176.7304	
6	10.25		10.61		108.7525
				1232.0380	262.8958
				262.8958	
				2)969.1422	
				484.5711 sq. ch.—48 A. 1 R. 33.14 P.	

The student will see that the areas in the above table correspond to the terms of the expression of the double area of a polygon as found in Art. 79. North and south latitudes are often distinguished by the use of the signs + and —, the former denoting north and the latter south. So doing, the latitudes may be written in a single column preceded by their signs. Again, the sign of the double area may be considered as agreeing with the sign of the latitude used as a multiplier, regardless of whether the product corresponds to an additive or subtractive term in the expression of the double area of the polygon. Accordingly, the numbers denoting double areas in the above table would exchange columns.

SUGGESTION.—It will repay the student to review carefully the solution of the above problem, before going forward to other examples.

The following is given as an example of a compact form of solution to be used by the student:

Sta.	Bearing.	Dist.	Latitude. N.	Latitude. S.	Departure. E.	Departure. W.	D. M. D.	N. Area.	S. Area.
1	S.40°W.	7.00		5.36		4.50	17.08		91.5488
2	N.45°W.	8.90	6.29			6.29	6.29	39.5641	
3*	N.36° E.	12.50	10.12 10.11		7.35		7.35	74.3820	
4	N.	5.40	5.40				14.70	79.3800	
5	S.81° E.	18.60		2.90 2.91	18.36 18.37		33.06		95.8740
6	S. 8°W.	13.70		13.56 13.57		1.92 1.91	49.50		671.2200
7	W.	13.00	00.01 00.00			13.00	34.58	00.3458	
								193.6719	858.6428
									193.6719
									2)664.9709
									10)332.4854 sq.ch.
									33.24854 A.

Remarks.—1. The latitudes and departures as found in the Traverse Table are written on the same line as the corresponding bearing, the latitudes and departures as corrected being written just above the former.

The corrected latitudes and departures should be written in red ink, and the original latitudes and departures crossed out with red ink.

2. The error of latitude was found to be 0.04, and the error of departure, 0.02. The sum of the courses is 79.10.

Multiplying the errors per chain by the length of the courses, and using the products to the nearest link, the several corrections are obtained, which are added to the latitudes and departures which are too small and subtracted from those which are too large.

The total error is so small as not to affect the latitudes and departures of the short courses, perceptibly. They are accordingly left unchanged.

3. The most westerly station, marked with a star, is readily found by making a rough plat of the survey, drawing the lines of proportionate length, as near as can be judged by the eye, and in the directions required by the bearings.

In finding the Double Meridian Distances in the above example, (Art. 82, Prin.). we began at station 3 and, going forward, finished at station 2, where the D. M. D. obtained was found to be equal to the departure of the second course, thus verifying the work.

84. Platting. — Compass surveys are conveniently platted by laying down with the protractor the bearing of each course from the meridian of the station at which the bearing was taken, and drawing the lengths of the courses to some assumed scale.

Or, having found the corrected latitudes and departures, a very expeditious mode of platting is the following: Take as an example the survey treated in Art. 83, and write out from Table I a table of total latitudes and departures as shown on next page.

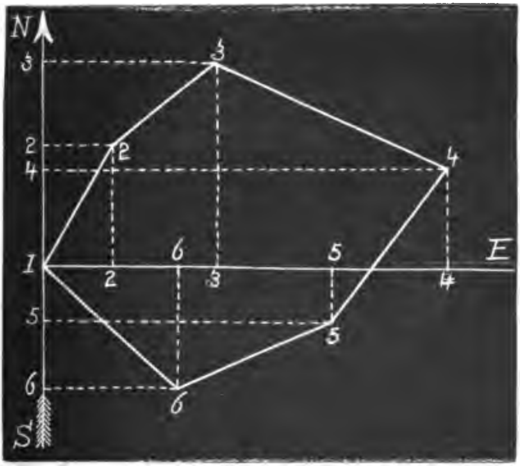


FIG. 63.

Draw a line, as *NS*, to denote the principal meridian, and from any point on it, as 1, draw a principal east and west line, as 1*E*. Such lines are called **Axes**.

On these lines, respectively, lay off from 1 the total latitudes and departures, in the directions indicated by their signs; marking the points with figures corresponding to the number of the station in each case.

Sta.	Total Lat.	Total Dep.
1	00.00	00.00
2	+10.76	5.34
3	+15.83	13.73
4	+ 7.89	31.62
5	— 4.93	22.61
6	—10.25	10.61
1	00.00	00.00

With a parallel ruler, draw lines from the points parallel to the axes. The intersections locate the corresponding stations, which have only to be joined, to form the plat.

EXERCISES.

85. Plat and find area from the following notes:

(1)

(1)
4 . 24
N. 47° W.
(4)
12 . 41
S. 21° W.
(3)
5 . 86
N. 83¾° E.
(2)
8 . 25
N. 12° E.
(1)

Area 4 A. 2 R. 37 P.

(2)

(1)
7 . 82
East.
(4)
14 . 35
N. 16⅓° E.
(3)
11 . 07
N. 86⅓° W.
(2)
14 . 45
S. 3⅓° W.
(1)

Area 13 A. 1 R. 19.39 P.

(3)

(1)
23 . 00
S. 14° E.
(4)
11 . 00
N. 83° E.
(3)
17 . 00
N. 23° E.
(2)
23 . 66
N. 77° W.
(1)

Area 32 A. 1 R. 18 P.

(4)

Sta.	Bearing.	Distance.
1	S. 20° W.	5.30
2	S. 70° W.	10.90
3	N. 31° W.	9.40
4	N. 45° E.	9.30
5	S. 60° E.	11.85

(5)

Sta.	Bearing.	Distance.
1	S. 34¼° W.	35.30
2	N. 56½° W.	32.00
3	N. 31¼° E.	27.30
4	N. 85° E.	12.80
5	S. 56¾° E.	22.00

Area 106 A. 3 R.

6, 7 and 8. Three original problems made by the student in the field.

SUGGESTION.—Work closely, and see how near the latitudes and the departures may be caused to balance.

(9)

Sta.	Bearing.	Distance.
1	S. 34° W.	5.10
2	N. 56° W.	5 81
3	S. 29½° W.	2.52
4	N. 48¼° W.	8.73
5	N. 35° E.	6.49
6	S. 56¼° E.	14.15

(10)

Sta.	Bearing.	Distance.
1	N. 50½° E.	16.50
2	S. 68¼° E.	14.20
3	S. 9¾° E.	8.45
4	S. 21° W.	6.84
5	S. 73½° W.	12.31
6	N. 78¼° W.	9-76
7	N. 17° W.	11.64

86 If in formula (*c*), Art. 59, the abscissas of the vertices of the polygon be replaced by the departures of the courses represented by the sides of the polygon, the area of the polygon will be found to be given by the following

RULE.—*Multiply the total latitude of each station by the sum of the departures of the adjacent courses, and divide the sum of the products by 2.*

Example.—To illustrate the above Rule, we may take the example of Art. 83.

Sta.	Bearing.	Dist.	Latitude.		Departures		Total Latitude.	Adjacent Dep.	Double Area.
			N.	S.	E.	W.			
1	N. 26¼° E.	1200	1076		534		0000		
2	N. 59° E.	980	507		839		1076	1373	1477318
3	S. 66° E.	1960		794	1789		1583	2628	4160124
4	S. 35° W.	1568		1282		901	789	888	700632
5	S. 66° W.	1312		532		1200	— 493	—2101	1035793
6	N. 46° W.	1472	1025			1061	—1025	—2261	2317525

2) 9691422

48.45711 A.

SUGGESTIONS.—Station 1 being taken as the origin, its latitude is zero. The total latitude of any station is found by adding, algebraically, the latitude of the preceding course to the latitude of the preceding station. The total latitude of the last station is the latitude of the last course with its sign changed. (*Check*).

Let the student solve some of the Exercises of Art. 85 by this method.

87. Problem. *Given the bearings of the sides of a field, to find the bearings when the field is supposed to be revolved so as to cause one of the sides to coincide with a meridian.*

EXAMPLES.

1. The bearings of the sides of a field are 1st N. 12° E., 2d N. 83½° E., 3d S. 21° W., and 4th N. 47° W. What will the bearings be, if the field be supposed to be revolved so as to cause the 1st side to be on a meridian?

Ans.—1st N., 2d N. 71½° E., 3d S. 9° W., and 4th N. 59° W.

SUGGESTION.—Suppose the field to be revolved toward the left, through an angle of 12°. Accordingly, each bearing would be changed by that amount. The readings of the new bearings are readily determined by inspection.

2. The bearings of the sides of a field are 1st S. 3½° W., 2d N. 86½° W., 3d N. 16½° E., and 4th E. Required the new bearings when the first side is made to coincide with the meridian.

Ans.—1st S., 2d W., 3d N. 13° E., and 4th N. 86½° E.

3. The bearings of the sides of a field are 1st S. 20° W., 2d S. 70° W., 3d N. 31° W., 4th N. 45° E., and 5th S. 60° E. Required the new bearings when the third side is made to coincide with the meridian.

Ans.—1st S. 51° W., 2d N. 79° W., 3d N.. 4th N. 76 E., and 5th S. 29° E.

4. The bearings of the sides of a field are, 1st N. 45° E., 2d S. 30° W., 3d S. 5° E., 4th W., and 5th N. 20° E. What will the bearings become, if the field be revolved so as to bring the 3d side to the meridian?

Ans.—1st N. 50° E., 2d S. 35° W., 3d S., 4th N. 85° W., 5th N. 25° E.

5. The bearings of the sides of a field are, 1st E., 2d N. 9° E., 3d S. 69° E., 4th S. 66° E., 5th S. 42° W., 6th S. 75° W., 7th N. 39° W., and 8th N. 42° E. What will the bearings become, if the field be revolved so as to cause the 4th side to coincide with the meridian?

Ans.—1st S, 24° E., 2d N. 75° E., 3d S. 3° E., 4th S., 5th, N. 72° W., etc.

Additional exercises may be formed from the above by requiring different sides to be brought to coincide with the meridian.

RULE.—*Change each bearing agreeably with the direction in which the field is supposed to be revolved by an amount equal to the bearing of the side which is brought to the meridian, and express the result in accordance with the proper form of denoting bearings.*

6. What were the bearings of the sides of a field which are now N. 16½ E., E., S. 3½° W., and N. 86½° W., the variation of the needle having changed 2½° toward the west since the former survey?

Supplying Omissions.—From inaccessibility of lines, and sometimes from accident, omissions may occur in the field notes of a survey. In a closed survey, any two omissions may, in general, be supplied by computation. It is, however, desirable to avoid as far as possible the necessity of supplying omissions in this manner, since it in-

fringes upon the tests which otherwise serve to verify the work.

The several cases which may occur are presented in the following problems:

88. Prob. 1. *To find an omitted bearing and distance.*

CASE 1.—*When the omissions pertain to the same course.*

In a closed survey, the sum of the northings should equal the sum of the southings; and the sum of the eastings should equal the sum of the westings. The defect of these equalities in the present case, must be on the one hand the latitude, and on the other the departure of the omitted course.

Example.—

Sta.	Bearing.	Dist.	Lat.	Dep.
A	N. 31° W.	9.40	+ 8.057	— 4.841
B	N. 45° E.	9.30	+ 6.576	+ 6.576
C	Omit	ted.		
D	S. 20° W.	5.30	— 4.980	— 1.813
E	S. 70° W.	10.90	— 3.728	—10.243

Solution.—Sum of

northings = 14.633
of southings = 8.708

Diff. = CG = 5.925

Sum of

westings = 16.897
of eastings = 6.576

Diff. = GD = 10.321

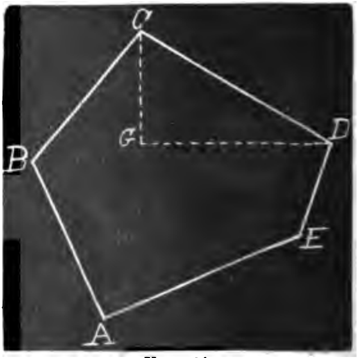


FIG. 64.

The latitude of the omitted course is thus a southing and its departure, an easting. Its bearing is therefore S—° E.

To find the bearing or angle GCD, we have

$$\tan GCD = \frac{GD}{CG} = \frac{10.321}{5.925} \doteq 1.74194.$$

Whence $GCD = 60° 8'$, or the required bearing is S. 60° 8′ E.

To find the distance CD, we have

$$CD = (5.925^2 + 10.321^2)^{\frac{1}{2}} = 12.00.$$

REMARK.—It will be noticed that a plat of the field may be made, and the area found without supplying the omissions.

CASE 2.—*When the omissions pertain to different courses.*

If the field be supposed to be revolved until the side whose length is omitted becomes a meridian, the given bearings being changed accordingly, (Art. 87, Prob.), then, since the departure of the side made a meridian is 0, the difference between the sums of the eastings and westings of the other courses is the departure, in its new position, of the side whose bearing is omitted.

Knowing the length and the departure of this side, its latitude and bearing may be found, (Art. 80).

The difference between the sums of the northings and southings of the courses in their new positions, is the length of the side which was made a meridian.

Example.—

Sta.	Bearing.	Changed Bearing.	Distance.	Lat.	Dep.
A	N. 20° E.	North.	Omitted.		0.0000
B	N. 45° E.	N. 25° E.	8.00	+ 7.2505	+ 3.3809
C	S. 30° W.	S. 10° W.	5.00	— 4.9240	— 0.8682
D	Omitted.		7.20		
E	West.	S. 70° W.	5.92	— 2.0248	— 5.5630

Solution.—Sum of eastings = 3.3809

" " westings = 6.4312

Difference = 3.0503 (an easting).

Latitude of $DE = (7.20^2 - 3.0503^2)^{1/2} = 6.5219$ (a southing).
Sine of changed bearing of $DE = 3.0503 \div 7.20 = 0.42365$

Whence " " " DE is S. 25° 4′ E.
Whence original " " DE was S. 5° 4′ E.
Sum of northings = 7.2505
" " southings = 13.4707

Difference = 6.22 = length of AB.

REMARK.—It is sometimes doubtful whether the latitude of the course whose bearing is omitted is a northing or a southing.

In the present case, the question is determined by a simple inspection of the latitudes, since the sum of the southings is less than the sum of the northings, without considering the northing of the first course.

In other cases, there may be two sets of values of the omitted parts, with either of which the problem is satisfied.

Practically, however, the ambiguity is removed by a general knowledge which the surveyor has of the directions of the lines.

89. Prob. 2. *To find the omitted lengths of two courses.*

CASE 1.—*When the courses are consecutive.*

The bearing and length of a line which would close a survey, leaving out the unknown sides, may be found by Prob. 1, Case 1. This line and the unknown sides form a triangle in which the angles, as found from the given bearings, and the length of one side are known. The lengths of the other sides may therefore be computed.

The procedure will be readily worked out by the student, without illustration.

CASE 2. *When the courses are not consecutive.*

This case may be treated in the same manner as the preceding.

Or, we may suppose the field to be revolved so as to make one of the sides whose length is omitted, a meridian, the bearings of the other sides being changed, accordingly.

We may then find the difference of the sums of the eastings and westings, which will be the departure, in its new position, of the other side whose length is wanting. Having the bearing of that side and its departure, its

length and latitude may be found. Finding the difference between the sums of the northings and southings, we obtain the length of the side which was made a meridian.

Example.—

Sta.	Bearing.	Changed Bearings.	Distance.	Lat.	Dep.
A	N. 15° E.	N. 30° W.	5.00	+ 4.33	− 2.50
B	N. 45° E.	North.	Omitted.		0.00
C	S. 55° E.	N. 80° E.	10.05	+ 1,75	+ 9.90
D	S. 15° W.	S. 30° E.	12.25	−10.61	+ 6.12
E	S. 75° W.	S. 30° W.	Omitted.		
F	N. 33¾° W.	N. 78¾° W.	9.96	+ 1.95	− 9.77

Sum of eastings = 16.02
" " westings = 12.27

Difference = 3.75 = Dist. × sin 30°.
Whence, length of *EF* = 3.75 ÷ 0.5 = 7.50.
Lat. *EF* = 3.75 ÷ tan 30° = 6.50.
Sum of northings = 8.03
" " southings = 17.11

Difference = 9.08 = length of *BC*.

REMARK.—If the sides whose lengths are omitted are parallel, the problem is indeterminate.

90. Prob. 3. *To find the omitted bearings of two courses.*

We find, (Prob. 1, Case 1), the bearing and length of a line which would close a survey, having the lines whose bearings are given as the other sides.

The line thus found and the two lines whose bearings are omitted form a triangle. The lengths of the sides of the triangle being known, its angles may be found; and from the angles and the bearing of one of the sides the bearings of the other sides may be found.

The closing line and the triangle are illustrated by the diagram accompanying the following

Example.—

Sta.	Bearing.	Dist.	Lat.	Dep.
A	N. 15° E.	5.00	+ 4.8296	+ 1.2941
B	Omitted.	9,08		
C	S. 55° E.	10.05	— 5.7645	+ 8.2325
D	S. 15° W.	12.25	— 11.8327	— 3.1705
E	Omitted.	7.50		
F	N. 33¾° W.	9,96	+ 8.2814	— 5.5334

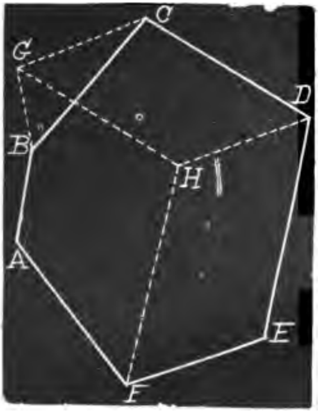


FIG. 65.

The side *EF*, without change of bearing, is represented by *CG*. *BG* is the closing line of the field *ABGHF*, in which we have

Sum of northings = 13.1110
" " southings = 17.5972

Difference = 4.4862
(a northing).

Sum of eastings = 9.5266
" " westings = 8.7039

Difference = 0.8227
(a westing).

Whence, (Prob. 1), bearing *BG* is N. 10° 23′ 30′′ W., and length *BG* is 4.56.

In the triangle *BGC*, *BC* = 9.08 and *CG* = *EF* = 7.5b. Solving the triangle, (Art. 29), we find angle *GBC* = 55° 25′ 40′′ and angle *BGC* = 94° 31′ 49′′.

Whence bearing *BC* is N. 45° 2′ 10′′ E.; and bearing *EF* is S. 75° 4′ 41′′ W.

REMARK.—The problem may possibly have two solutions, accordingly as the triangle may fall on either side of the closing line. The ambiguity is, however, practically unimportant.

Exercises.—To be made by the student in the field.

LAYING OUT LAND.

91 Prob. 1. *To lay out a given quantity of land in the form of a square.*

Solution.—Let a be the given area in square links, and x the length of a side of the required square.

$$\text{Then } x^2 = a, \text{ whence } x = \sqrt{a}.$$

Measure a distance $= \sqrt{a}$, and at its extremities erect perpendiculars, each equal to $\sqrt{a}$.

Examples.—1. Lay out 3 A. 4 sq. rd. in the form of a square.

2. Lay out a square acre.

92. Prob. 2. *To lay out a given quantity of land in the form of a rectangle.*

CASE 1.—*When one side of the required rectangle is given.*

Solution.—To be found by the student.

Examples.—1. Lay out a rectangle containing 3 A. 3 R. 30.72 P., whose length shall be 7.30 ch.

2. Lay out a rectangle of $4\frac{1}{2}$ A. whose width shall be 6 ch.

CASE 2.—*When the ratio of the sides is given.*

Solution.—Let a be the area of the rectangle, x its length, y its width, and $m : n$ the ratio of x to y.

Then $xy = a$ and $\dfrac{x}{y} = \dfrac{m}{n}$. Whence, $x = \left\{\dfrac{am}{n}\right\}^{\frac{1}{2}}$

and $y = \left\{\dfrac{an}{m}\right\}^{\frac{1}{2}}$

Examples.—1. Lay out $6\frac{1}{4}$ A. in a rectangular form, the ratio of the length and breadth of which shall be $2\frac{1}{2}$.

2. Lay out a rectangle of 5 A., the distance around which shall be 28.50 ch.

3. Lay out a rectangle whose length shall be twice its width, and which shall contain 3 A.

93. Prob. 3 *To lay out a given quantity of land in the form of a rhomboid.*

CASE 1.—*When the base and an adjacent angle are given.*

Solution.—Let a be the area, b the base, and A the angle. Let x be the other side adjacent to A.

Then $bx \sin A = a$. Whence, $x = \dfrac{a}{b \sin A}$.

Let the student complete the solution.

Example.—Lay out 2 A. in the form of a rhomboid, the length of whose base shall be 8 ch. and the adjacent angle 30°.

CASE 2.—*When the adjacent sides are given.*

Solution.—Let a be the area, and b the base, as before, and c the adjacent side.

Let X be the angle between b and c.

Then $bc \sin X = a$. Whence, $\sin X = \dfrac{a}{bc}$.

Let the student finish the work.

When is the parallelogram a rectangle? When, impossible?

Another Solution.—Let x be the altitude of the required rhomboid.

Then $x = \dfrac{a}{b}$, and $\sqrt{c^2 - x^2} =$ projection of c on b.

Let the student complete the solution.

Example.—The area of a rhomboid is to be 6 A., its base 12 ch., and the adjacent side 6.25 ch. Find the altitude, and lay out the rhomboid.

94. Prob. 4. *To lay out a given area in the form of a triangle.*

CASE 1.—*When a side of the triangle is given.*

Solution.—To be found by the student.

CASE 2.—*When the base and adjacent angle are given.*

Solution.—To be found by the student.

CASE 3.—*When the base and adjacent side are given.*

Solution.—To be found by the student.

Examples.—1. Lay out a triangle containing 2 A. on a base of 8 ch.

2. Lay out a triangle containing 2 A. on a base of 8 ch., and adjacent angle 45°.

3. Lay out a triangle containing 2 A. on a base of 8 ch., and adjacent side 5 ch.; the adjacent side 7 ch.

4. Lay out an equilateral triangle containing 2 A.

DIVISION AND PARTITION OF LAND.

It is often required to divide areas into portions having a given ratio to each other, or to part off from a field a given number of acres, by a line fulfilling some specified condition of position, direction, etc., with respect to given points, corners or sides of the tract divided.

It will be impossible, of course, to consider every case which may practically arise, but the following problems will be sufficient to suggest the general scope of the subject, and their solution will serve to indicate the procedure in some of its most useful applications.

95. Prob. 1 *To divide a triangle into parts having a given ratio.*

CASE 1.—*By lines from an angle.*

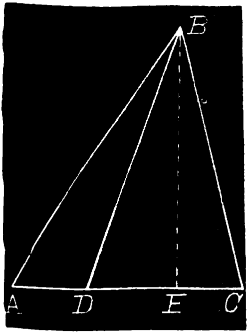


FIG. 66.

Solution.—Let ABC be any triangle, and suppose it is required to divide it by a line from B, into two parts having the ratio of m to n.

Let BD be the line of division, so that $ABD : DBC :: m : n$ (1). But $ABD : DBC :: AD : DC$ (2).

Combining (1) and (2), we have

$$AD : DC :: m : n,$$

whence, $AD : AC :: m : m + n,$

whence, $AD = \dfrac{m \times AC}{m + n}$. Similarly, $DC = \dfrac{n \times AC}{m + n}$.

Measure the distance AD thus found, and run the line BD.

If the triangle were to be divided into three parts in the ratio of $m : n : p$, we should have

$$AD = \frac{m \times AC}{m + n + p} \text{ and } DE = \frac{n \times AC}{m + n + p}.$$

Cor.—To part off by a line, as BD, a given area a, we have $AD : AC :: a :$ area ABC, whence $AD = \dfrac{a \times AC}{\text{area } ABC}$.

Examples.—1. Find the measurements required to divide a triangular field by lines from an angle to a side whose length is 12.30 ch., into parts to each other as 2, 3 and 4.

2. Find the measurement required to part off 3 A. 2 R. 10 P. from a triangular field a side of which is 18.50 ch., and a perpendicular thereupon from the opposite angle is 10.40 ch.

CASE 2.—*By lines parallel to a side.*

Solution.—Let D be the point in the side AB from which a line parallel to BC shall divide ABC so that $ADE : DECB :: m : n$. Then

$$ADE : ABC :: m : m + n.$$

But $ADE : ABC :: AD^2 : AB^2$. Whence $AD^2 : AB^2 : m \,.\, m{+}n$, giving $AD = AB \left\{\dfrac{m}{m + n}\right\}^{\frac{1}{2}}$

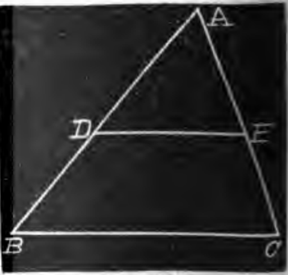


FIG. 67.

Measure the distance AD thus found, and run DE parallel to BC, (Art. 69, Prob. 3).

If the triangle is required to be divided into three parts in the ratio of $m : n : p$, we should have

$$AD = AB \left\{\frac{m}{m+n+p}\right\}^{\frac{1}{2}} \text{ and } AF = AB \left\{\frac{m+n}{m+n+p}\right\}^{\frac{1}{2}}$$

Cor. 1.—To part off a triangle, as ADE, of given area, a we have $AD = AB \left\{\dfrac{a}{\text{area } ABC}\right\}^{\frac{1}{2}}$.

Cor. 2.—To part off a quadrilateral, as $DECB$, of given area, a', we may find by Cor. 1 the distance AD required to part off a triangle of the area $ABC - a'$ and measure $BD = BA - AD$.

Examples.—1. Find the measurement for dividing a triangular field of 12 A. into parts in the ratio of 4 to 5, by a parallel run from a point in a side whose length is 10.35 ch.

2. Find measurements for dividing by parallels, the above field into three equivalent parts.

3. Find measurement for parting off from the same field by a parallel, a triangle of 5 A.; a quadrilateral of 7½ A.

CASE 3.—*By lines perpendicular to a side.*

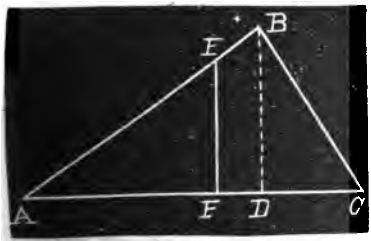


FIG. 68.

Solution.—Let ABC be a triangle required to be divided by a perpendicular to AC, into parts having the ratio of m to n.

Let EF be the line of division, so that

$AEF : EBCF :: m : n$,

or $AEF : ABC :: m : m+n$, (1).

Let BD be a perpendicular upon AC.

Then $AEF : ABC :: AF \times EF : AC \times BD :: m : m+n$, (2).

From similar triangles, $AF : EF :: AD : BD$,

$$\text{whence, } EF = \frac{AF \times BD}{AD}.$$

Substituting this value of EF in (2), we have

$$\frac{AF^2 \times BD}{AD} : AC \times BD :: m : m + n,$$

$$\text{or } AF^2 : AC \times AD :: m : m + n,$$

$$\text{whence, } AF = \left\{\frac{AC \times AD \times m}{m + n}\right\}^{\frac{1}{2}}.$$

Find AD and then AF. Measure the distance AF and run FE perpendicular to AC, (Art. 68, Prob. 2).

Similarly, may be found the distances to perpendiculars dividing the triangle into three or more parts having a given ratio.

Cor.—To part off a triangle, as AEF, of given area, a, we have $AF = \left\{\frac{AC \times AD \times a}{\text{area } ABCD}\right\}^{\frac{1}{2}}.$

The distance AF to a perpendicular which shall part off a triangle $AEF = a$, may be found otherwise, as follows: triangle $AEF = \frac{1}{2} AF \times EF = a$, and $EF = AF \times \tan A$. Whence, $AF = \left\{\frac{2a}{\tan A}\right\}^{\frac{1}{2}}.$

Examples.—1. The bearings and lengths of two sides of a triangular field from the same corner are N. 20° E., 15 ch., and N. 50° E., 20 ch. Required the measurement from that corner to a perpendicular upon the longer side which shall divide the field into two parts having the ratio of 2 to 3.

2. Required the measurement to a perpendicular which shall divide the above field into two equivalent parts; into three equivalent parts.

3. Required the measurement to a perpendicular which shall part off from the same field a triangle of 4 A.; a quadrilateral of 5 A.

Case 4.—*By lines from a given point in a side.*

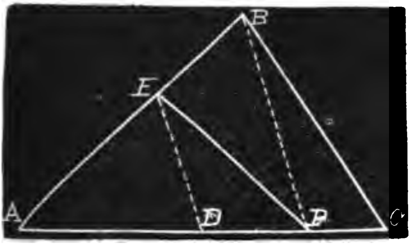


FIG. 69.

Solution.—Let P be the given point. We divide AC, as at D, so that

$AD : DC :: m : n$. Take the bearing of B from P; run DE parallel to PB; and run PE. It will divide the triangle ABC into the parts AEP and $PEBC$ to each other as m to n.

Let the student demonstrate.

Similarly, ABC may be divided by lines from P into any number of parts having a given ratio.

To find the point E by measurement from A, we have $AEP : ABC :: m : m + n$. (1).

But since triangles having an angle in common are to each other as the products of the sides including the angle, we have $AEP : ABC :: AE \times AP : AB \times AC$. (2).

Combining (1) and (2), we have

$AE \times AP : AB \times AC :: m ; m + n$.

Whence, $AE = \dfrac{AB \times AC \times m}{AP\,(m + n)}$.

Cor. 1.—To part off a triangle as AEP of given area, a, we have $AE = \dfrac{AB \times AC \times a}{AP \times \text{area } ABC}$.

Cor. 2.—Since area $ABC = \dfrac{AB \times AC \times \sin A}{2}$, the formula of Cor. 1 becomes $AE = \dfrac{2a}{AP \times \sin A}$.

Examples.—1. The bearings and lengths from the same corner of a triangular field are S. 30° E., 18.75 ch., and S. 70° E., 12.50 ch. From the same corner, the distance on

8*

the latter side to a certain point is 10 ch. Required the measurement for running a line from that point which shall divide the field into two parts having the ratio of 1 to 2.

2. Required the measurements for a division of the same field, by lines from the given point, into parts to each other as 3, 4 and 5.

3. Required the measurement for parting off from the same field, by a line from the given point, a triangle of 3 A.; a quadrilateral of 4 A.

CASE 5.—*By a line through a given point within the triangle.*

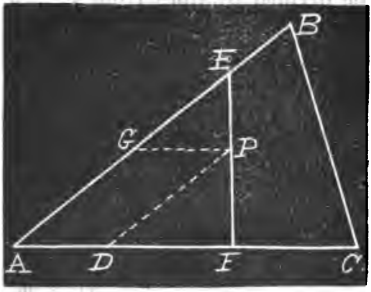


FIG. 70.

Solution.—Let ABC be the given triangle and P be the given point.

Draw PD parallel to AB, and PG parallel to AC.

Since the point P is given in position, the distances PD and PG are known.

Let EF be the required line of division, so that

$$AEF : ABC :: m : m + n.$$

Now, $AEF : ABC :: AE \times AF : AB \times AC.$

Whence, $AE \times AF : AB \times AC :: m : m + n.$ (1).

For convenience of writing, let $AE = y$, $AF = x$, $PG = b$, and $PD = c$.

From the similar triangles AEF and DPF, we have

$$y : x :: c : x - b, \text{ whence } y = \frac{cx}{x - b}.$$

Substituting in (1), $\dfrac{cx^2}{x - b} : AB \times AC :: m : m + n.$

Whence, $\dfrac{cx^2}{x-b} = \dfrac{AB \times AC \times m}{m+n} = q$, for brevity.

Solving, we find $x = \dfrac{q \pm \sqrt{q^2 - 4\,bcq}}{2c}$.

Cor.—To part off a triangle AEF of given area a, we have $q = \dfrac{2a}{\sin A}$, whence $x = \dfrac{a \pm \sqrt{a^2 - 2\,abc \sin A}}{c \sin A}$.

If in solving the above Case, $PD = c$ and $PG = b$ are taken perpendicular to AC and AB, respectively, the formula for parting off a triangle of given area a becomes $x = \dfrac{a}{c} \pm \left\{\dfrac{a^2}{c^2} - \dfrac{2\,ab}{c \sin A}\right\}^{\frac{1}{2}}$.

Examples.—To divide a triangular field ABC into two equivalent parts by a line running through a certain point P within the field, there were taken the following measurements:

From the corner A, AB N. 70° W., length 12 ch.
" " " AC N. 10° W., length 15 ch.
From the point P, S. 70° E., distance 5 ch. to AC.
" " " S. 10° E., distance 3 ch. to AB.

Required the distance on AC from A to the line of division sought.

2. Required the measurement on AC to a line passing through the given point and parting off from the above field a triangle of 4 A.

3. The bearings and lengths of the sides AB and AC, being as above, and the bearing and length of AP being N. 50° W., 8 ch., it is required to find the measurement for parting off a triangle of 3 A. by a line through P.

96. Prob. 2. *To divide a parallelogram into parts having a given ratio by a line parallel to a side.*

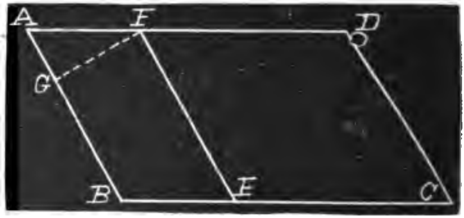


FIG. 71.

Solution.— Let $ABCD$ be the parallelogram which is to be divided by a line parallel to AB into parts to each other as m to n.

Let EF be the required line of division, so that

$$ABEF : FECD :: m : n,$$

whence, $ABEF : ABCD :: m : m + n$. (1).

But $ABEF : ABCD :: AF : AD$. (2).

Combining (1) and (2), $AF : AD :: m : m + n$,

$$\text{whence } AF = \frac{AD \times m}{m + n}. \quad FD = \frac{AD \times n}{m + n}.$$

Similarly, the parallelogram may be divided into three parts having the ratio of m to n to p.

Cor. 1.—To part off a parallelogram, area a, by a line parallel to AB, we should have $AF = \dfrac{AD \times a}{\text{area } ABCD}$.

Cor. 2.—To part off a parallelogram, area a, on AB, from an indefinite parallelogram, we should have,

$$AF = \frac{a}{AB \times \sin A}.$$

Examples.—1. The bearings and lengths of the sides of a field in the form of a parallelogram are N. 5° E., 8.20 ch., and N. 80½° E., 20.40 ch. Required the measurements on the longer side to points from which parallels should be run to divide the field into parts as 3 to 5; into parts as to 4 to 7 to 9.

2. Find the measurement for parting off adjoining the shorter side of the above field an area of 10 A., in the form of a parallelogram.

3. Two men unite in buying an undivided quarter section of land, each contributing \$1000. When they came to divide the land between them, it appears that there is a swamp on one corner, containing 25 acres, which they estimate as worth \$8 an acre.

They agree that one shall take the swamp and so much of the good land as necessary to make the two shares of equal value.

Required the measurement for division.

97. Prob. 3. *To divide a trapezoid into parts having a given ratio.*

CASE 1.—*By lines dividing the bases proportionally.*

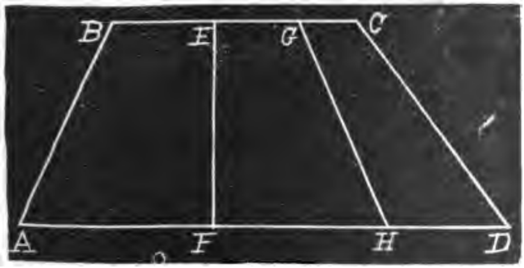


FIG. 72.

Solution.—Let $ABCD$ be any trapezoid required to be divided into parts having the ratio of m to n to p.

This is done in the easiest manner by dividing each base into parts having the ratio to each other as m, n and p, and joining the corresponding points of division. The measurements necessary to find the points of division are:

$$BE = \frac{m \times BC}{m+n+p}, \quad EG = \frac{n \times BC}{m+n+p}, \quad AF = \frac{m \times AD}{m+n+p},$$

and $FH = \dfrac{n \times AD}{m+n+p}$.

Cor.—To part off a given area a by a line, as EF, which shall divide the bases proportionally, we have

$$BE = \frac{a \times BC}{\text{area } ABCD} \text{ and } AF = \frac{a \times AD}{\text{area } ABCD}.$$

Examples.—1. Given AD, N. 80° E., 12.60 ch., AB, N. 10½° E., 8.12 ch., and BC, N. 80° E., 10.34 ch., to find the measurements required in dividing the field into parts having the ratio of 4 to 7, by a line dividing the parallel sides proportionally.

2. Find the measurements for parting off from the above field an area of 5 A. by a line dividing the parallel sides proportionally.

CASE 2.—*By lines parallel to the bases.*

Solution.—Let $ABCD$ be a trapezoid to be divided into parts in the ratio of m to n, by a line parallel to BC.

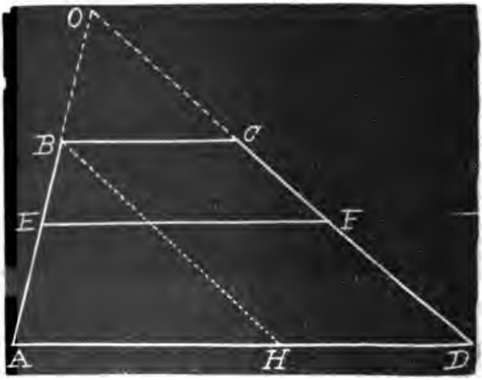


FIG. 73.

Suppose EF to be the required line of division, so that $EBCF : AEFD :: m : n$.

Regarding the sides AB and DC as prolonged to meet at O, we have $OAD : OBC :: AD^2 : BC^2$,

whence $OAD - OBC$,

or $ABCD : OBC :: AD^2 - BC^2 : BC^2$. (1).

Similarly, we have $EBCF : OBC :: EF^2 - BC^2 : BC^2$. (2).

Combining (1) and (2), $ABCD : EBCF :: AD^2 - BC^2 : EF^2 - BC^2$,

or $m + n : m :: AD^2 - BC^2 : EF^2 - BC^2$.

$$\text{whence } EF = \left\{\frac{m \times AD^2 + n \times BC^2}{m+n}\right\}^{\frac{1}{2}} \qquad (a)$$

Supposing BH to be parallel to CD, the triangles ABH and EBG give $AB : AH :: EB : EG$,

$$\text{or} \quad AB : AD - BC :: EB : EF - BC.$$

$$\text{Whence, } EB = \frac{AB\,(EF - BC)}{AD - BC}. \quad (b).$$

Thus, first finding EB by formula (a), we can then find EB by formula (b), and measuring that distance from B, we may run EF parallel to BC, dividing the trapezoid as required.

Similarly, a trapezoid may be divided in three or more parts having a given ratio. Indeed, the above formulas may be directly applied to that purpose by making a simple substitution.

Cor.—To part off a trapezoid of given area a, adjoining BC, we obtain from formula (a)

$$EF = \left\{\frac{a \times AD^2 + (\text{area } ABCD - a)\, BC^2}{\text{area } ABCD}\right\}^{\frac{1}{2}}$$

The distance BE is then found from formula (b).

Examples.—1. Given a trapezoidal field $ABCD$ in which AB is an east and west line, 9 ch., BC a north and south line, 5.19 ch., and AD a north and south line, 8 ch., it is required to run a north and south line dividing the field so that the parts on BC and AD shall have the ratio of 2 to 3.

2. Find the measurement from A to part off from the above field by a north and south line an area of 3 A. adjoining AD.

CASE 3.—*By lines perpendicular to the bases.*

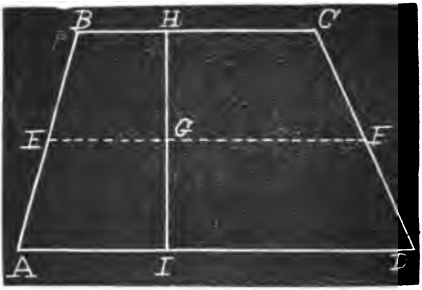


FIG. 74.

Solution.—Let $ABCD$ be a trapezoid to be divided into parts in the ratio of m to n by a line perpendicular to AD.

Let EF be the line joining the middle points of the non-parallel sides AB and CD. We divide EF, as at G, into two parts having the ratio of m to n, and through G run HI perpendicular to AD.

To find the point G on the ground, we have the formula $EG = \frac{m\,(BC + AD)}{2\,(m + n)}$. Whence, measuring from E the distance EG on the bearing of BC, we have the point sought.

Cor.—To part off a given area a, by a line perpendicular to the bases, we have $EG = \frac{a\,(BC + AD)}{2 \times \text{area } ABCD}$.

Or, denoting the altitude of the trapezoid by h, we have $EG = \frac{a}{h} = \frac{a}{AB \times \sin A}$.

The point I or H may be found by the formula $AI = EG + AE \times \cos A$, or $BH = EG - EB \times \cos A$.

Examples.—1. Given AD, E. 20 ch., AB, N. 15° E., 9.50 ch., and BC, E. 12 ch., required the measurement for dividing the field by a perpendicular to AD into two parts having the ratio of m to n.

2. Required the measurement for parting off from the above field, by a perpendicular to AD, an area of 4 A. adjoining AB.

CASE 4.—*By lines parallel to one of the non-parallel sides.*

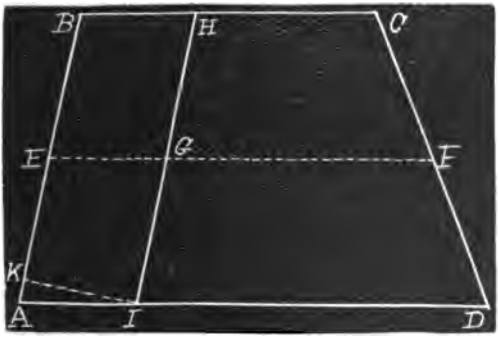


FIG. 75.

Solution.—We divide the line joining the middle points of the non-parallel sides, as EF, into parts, as at G, in the ratio of m to n, and through the point of division we run HI parallel to AB.

The distance $EG = AI = \dfrac{m\ (BC + AD)}{2\,(m+n)}$.

COR.—To part off a parallelogram of given area a, adjoining AB, we have $AI = \dfrac{a\,(BC + AD)}{2 \times \text{area } ABCD}$.

We may part off a parallelogram of given area a, upon one of the non-parallel sides of a trapezoid, independently of the area of the trapezoid, by use of the formula $AI = \dfrac{a}{AB \times \sin A}$.

Examples.—1. Given AB, N. 40° W., 10 ch., AD, N. 15° E., 24 ch., and BC, N. 15° E., 16 ch., required the measurement to divide the field into two parts in the ratio of 4 to 5, by a line parallel to AB.

2. Find the measurement required to part off from the above field a parallelogram of 10 A. adjoining AB.

SUGGESTION.—Solve by different methods and compare results.

CASE 5.—*By lines from a given point in one of the parallel sides.*

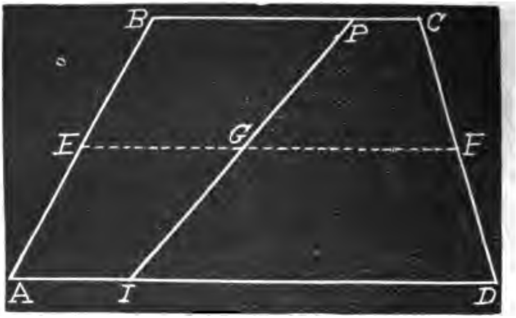


FIG. 76.

Solution.—Let P be the given point. We find the point G as before, and run the line PG, continuing it to I. The line PI is the one required.

Otherwise, the line PI may be obtained as follows:

$$ABPI : ABCD :: BP + AI : BC + AD :: m : m + n.$$

Whence, $$AI = \frac{m\,(BC + AD) - (m + n)\,BP}{m + n}$$

$$= \frac{m\,(BC + AD)}{m + n} - BP.$$

Accordingly, we may measure off the distance AI, as thus found, and then run the line PI.

Cor.—To part off a given area a by a line from a given point, as P in BC, we have $AI = \dfrac{a\,(BC + AD)}{\text{area } ABCD} - BP.$

It will be observed that in case BP is greater than $\dfrac{m\,(BC + AD)}{m + n}$, the distance AI is negative. In such cases the above solution comes a step short of the result sought. Thus, let PI be the line of division, as above

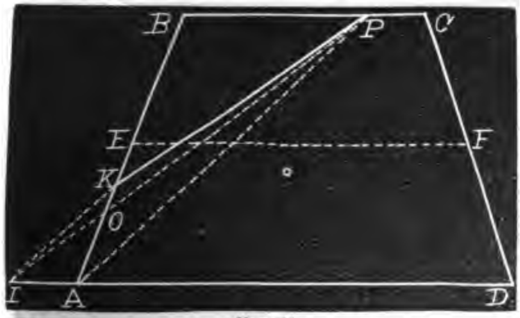


FIG. 77.

obtained. The parts of the trapezoid *ABCD* are, as before, *ABPI* and *IPCD*, the areas of which are in the ratio of m to n. But the area of *ABPI* = area of the triangle *OBP* -- area of the triangle *OAI*.

Accordingly, this subtraction is required to be performed. This is done by noting the bearing of the line *AP* and running from *I* a line on the same bearing to a point *K* in *AB*. The line *PK* divides *ABCD* as required.

Examples.—1. The parallel sides of a trapezoidal field are 15 ch. and 25 ch.

Required the measurement on the latter to the point to which a line being run from a point 10 ch. from an extremity of the former, the field shall be divided into parts in the ratio of 4 to 5.

2. The same as the above, except as changed by the supposition that the given point is in the longer base.

3. The distance between the given sides of the above field being 12 ch., it is required to find the measurement for parting off an area of 15 A. by a line from the given point.

98. Prob. 4. *To divide a trapezium into parts having a given ratio.*

CASE 1.—*By lines from an angle.*

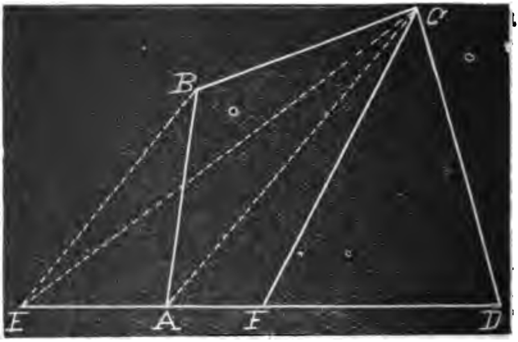


FIG. 78.

Solution.—Let $ABCD$ be a trapezium to be divided into two parts having the ratio of m to n, by a line from C.

We draw AC, and from B draw a line parallel to AC meeting DA produced at E. We then divide ED, as at F, into the parts EF and FD, having the ratio of m to n. The line CF divides the trapezium as required. That is, $ABCF : FCD :: m : n$, or $ABCF : ABCD :: m : m + n$.

SCH.—The above solution is readily executed on the ground.

In a similar manner a trapezium may be divided into any number of parts having a given ratio.

When a dividing line intersects a side of the trapezium, the solution involves an additional step, as shown under Case 5 of the preceding problem.

The point F may be otherwise found as follows:

The triangle $DCF = \dfrac{n \times ABCD}{m + n}$,

and again, $DCF = \dfrac{DC \times \sin D \times DF}{2}$.

Whence, $DF = \dfrac{2n \times ABCD}{DC(m + n) \sin D}$.

Cor.—To part off a triangle, as DCF, of given area a, we have $DF = \dfrac{2a}{DC \times \sin D}$.

Examples.—1. Given *AB*, N. 8° W., 7.60 ch., *BC*, N. $76\frac{1}{2}$° E., 10.21 ch., *CD*, S., 11.40 ch., and *DA*, N. $80\frac{1}{2}$° W., 9.00 ch. Required the measurement for locating a line *CF* which shall divide *ABCD* into the parts *ABCF* and *FCD* to each other, respectively, as 2 to 3.

SCH.—The area of the trapezium will be most readily found by adding the half products of two sides multiplied by the sine of their included angle.

2. Required the measurement for parting off from the above field a triangle *DCF* of 10 A,

CASE 2.—*By lines from a given point in one of the sides.*

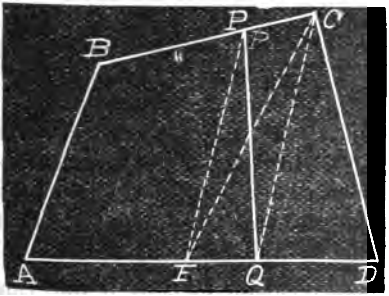


FIG. 79.

Solution.—Let *P* be the given point.

From an extremity, as *C*, of the side containing *P*, suppose a line to be run to a point, as *F*, parting off a triangle *CFD*, equal to one of the proportional parts, as $\dfrac{m \times ABCD}{m+n+p}$, into which the given trapezium is required to be divided, (Case 1). At *F* take the bearing of *P*. From *C*, run *CQ* on reverse bearing of *FP*. *PQ* is one of the lines of division sought.

Another Solution.—Suppose *CB* and *DA* to be produced to *O*. The triangle *OBA* will have two angles and included side known

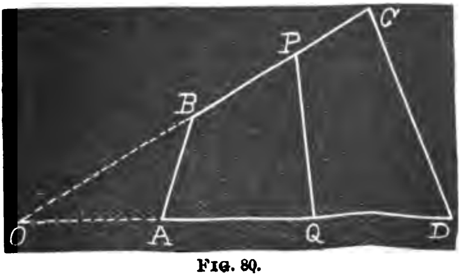


FIG. 80.

from the data of the given trapezium. The sides OA and OB, and the area of the triangle may be computed.

Call its area t.

The distance BP is known. Call OP, d.

Let PQ be a line of division sought.

Let OQ be x. The quadrilateral $ABPQ$ being a given proportional part of the given trapezium $ABCD$, its area is known. Call it a.

Now, the area of the triangle $OPQ = t + a$.

But area $OPQ = \dfrac{OP \times OQ \times \sin O}{2} = \frac{1}{2} dx \sin O = t + a$.

Whence, $x = \dfrac{2t + 2a}{d \sin O}$. Whence, $AQ = x - OA$ becomes known.

Examples.—1. The field being the same as given in Ex. 1, Case 1, it is required to find the measurement for locating a line which shall divide the field into two parts having the ratio of 2 to 3, the line being required to run from a point P in BC at a distance of 5 ch. from B.

2. Required the measurement for locating a line running from P which shall part off from the above field an area of 10 A. adjoining AB.

CASE 3.—*By lines parallel to a side.*

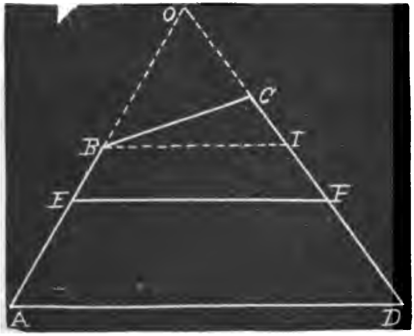


FIG. 81.

Solution.—Let it be required to divide a trapezium, as $ABCD$, by a line, as EF, parallel to AD, into two parts $EBCF$ and $AEFD$, to each other as m to n.

Suppose the sides including the parallel to be pro-

duced to meet at O. The triangle BOC may be regarded as known. Call its area a. The trapezium $EBCF$ is known as to area, being $\dfrac{m \times ABCD}{m+n}$. Call this area b.

The area of the triangle AOD is known. Call it c. Its side AO is also known.

Now, (Art. 95, Prob. 1, Case 2), $OE = AO \left\{\dfrac{(a+b)}{c}\right\}^{\frac{1}{2}}$

Whence, $BE = OE - OB$.

Measure this distance and run EF parallel to AD.

Another procedure is to draw BI parallel to AD, forming the triangle BCI, whose area and side BI may be found; whence the ratio of the trapezoid $EBIF$ to the trapezoid $ABID$ is obtainable, and accordingly the distance BE, (Art. 97, Prob. 3, Case 2).

SCH.—The problem of parting off a given area from a trapezium by a line parallel to a side, is essentially the same as the above.

Examples.—1. The field being as given in Ex. 1, Case 1, it is required to find the measurement for locating a parallel to BC that shall divide the field into two parts in the ratio of 3 to 4.

2. Find the measurement required to part off from the same field an area of 10 A. by a line parallel to BC.

3. Given the following Field Notes, it is required to divide the field by north and south lines, into three equivalent parts.

Sta.	Bearing.	Dist.
A	N.	21.50
B	N. 71° E.	9.78
C	S. 3½° E.	24.72
D	W.	10.65

Area, 23.08 A.

SUGGESTIONS.—Let EF be the first line of division from AB. Let $AE = x$.

Find $EF = AB + x \tan 19° = 21.50 + 0.34\,x$.

Whence, $21.50\,x + 0.17\,x^2 = \frac{1}{3} \times 23.08$.

The next line of division may be located in the same manner.

CASE 4.—*By lines perpendicular to a side.*

Solution.—Let $ABCD$ be a trapezium to be divided into two parts having the ratio of m to n, by a line perpendicular to AD.

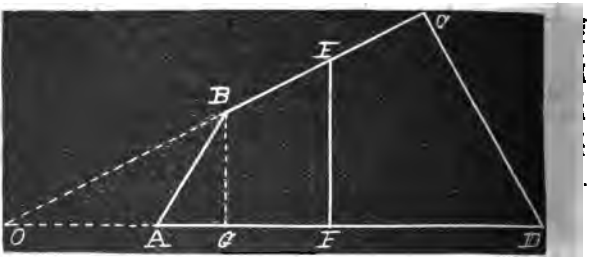


FIG. 82.

Let EF be the line of division, so that $ABEF : FECD :: m : n$, or $ABEF : ABCD :: m : m + n$, whence,

$$ABEF = \frac{m \times ABCD}{m + n} = b.$$

Produce CB and DA to meet at O. The area of the triangle OBA may be computed. Call it a. The side OA may also be found. Draw BG perpendicular to AD. The area of $ABG = \frac{1}{2}AG \times BG = \frac{1}{2}AB^2 \times \cos A \sin A = \frac{1}{4}AB^2 \times \sin 2A$. Call it $\pm a'$, accordingly as the angle A is acute or obtuse.

Now, from similar triangles OBG and OEF we have

$$OF^2 : OG^2 :: a + b : a \pm a', \text{ whence } OF = OG\left\{\frac{a + b}{a \pm a'}\right\}^{\frac{1}{2}}.$$

$AF = OF - OA$.

Obviously, the trapezium may be divided in a similar way into any number of parts having a given ratio.

Examples.—1. In a field $ABCD$, given AB, N., 10 ch.; BC, S. 85¾° E., 9.25 ch.; CD, S. 4½° E., 7.60 ch.; and DA, S. 76¾° W., 10.40 ch.; area of field, 8 A. 2 R. 7 P.

Required the distance from B to a perpendicular upon AB which shall divide the field into two parts having the ratio of 1 to 2.

2. Find the distance from B to a perpendicular upon AB which shall part off from the same field 5 A. adjoining BC.

CASE 5.—*By lines fulfilling no given condition respecting sides or angles.*

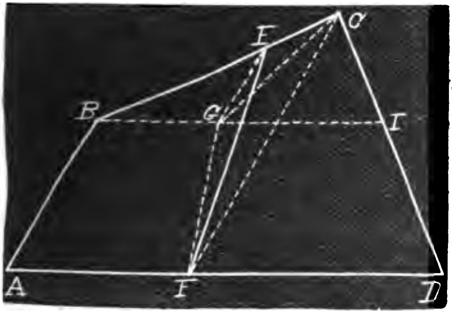


FIG. 83.

Solution.—Let it be required to divide the trapezium $ABCD$ into two parts having the ratio of m to n by a line between AD and BC.

Draw BI parallel to AD and divide BI and AD, as at G and F, into parts to each other as m to n. Thus, $AF = \dfrac{m \times AD}{m + n}$ and $BG = \dfrac{m \times BI}{m + n}$.

Draw CG and GF. The broken line CGF divides the trapezium into the parts $ABCGF$ and $FGCD$, having the ratio of m to n. From G draw GE parallel to FC, and draw EF, as the required line of division.

Examples.—To be proposed and solved by the student in the field.

99 Prob. 5. *To divide a polygon into parts having a given ratio.*

CASE 1.—*By lines from an angle.*

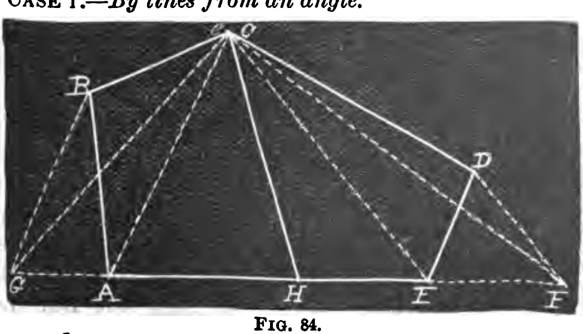


FIG. 84.

Solution.—Let it be required to divide the polygon $ABCDE$ into two parts in the ratio of m to n by a line from C.

From B draw a parallel to CA, and from D, a parallel to CE, meeting AE produced at the points G and F.

Divide GF, as at H, into two parts to each other as m to n. Draw CH as the line sought.

Similarly, a polygon may be divided into any number of parts having a given ratio.

Cor.—To part off a given area a, we may find

$$GH = \frac{a \times GF}{\text{area } Pol}.$$

To part off a given area, independently of the area of the polygon, we proceed as follows: Let $ABCDE$ represent a many-sided field. From the corner C, run a random as near in the direction of the required line of division as can be determined by the eye, conveniently to an opposite corner, as A. Find the area of the part, as CAB, thus parted off, and find the difference between it and the area required to be parted off. Let this difference be denoted in the figure by the triangle $ACH = d$. We shall then have $AH = \dfrac{2d}{AC \times \sin CAE}$.

CASE 2.—*By lines from a given point in a side.*

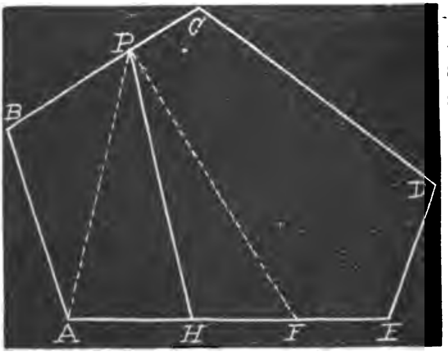


FIG. 85.

Solution.—Let $ABCDE$ represent a many-sided field that is required to be divided into two parts having the ratio $m : n$, by a line from a given point P, in the side BC. Let PH be the dividing line required, so that $ABPH : Pol. :: m : m + n$.

Run a random, as PA or PF, parting off an area which is to be computed. Call it a.

Find the difference between a and $\dfrac{m \times Pol.}{m+n}$.

Call the difference d, and apply the formula of Case 1, Cor., to find AH or FH.

The consideration whether a is greater or less than required will indicate the direction from A or F in which the distance found by the formula should be measured.

Examples.—1 Given the following Field Notes, it is required to locate a line from Station 1, which shall divide the field into two equivalent parts.

Sta.	Bearing.	Dist.
1	N. 20½° E.	5.83
2	S. 79¾° E.	10.15
3	S. 27½° W.	9.45
4	N. 61¼° W.	8.28
5	N. 15½° W.	1.04

2. Required a line which shall part off from the same field 8 A. adjoining the first course.

3. Required a line running from the middle of the first course, which shall divide the same field into parts having the ratio of 2 to 3.

4. Two men own land situated between a road XX' and a line YY', and divided by a line AA'.

It is required to run a line BB', at right angles with the road which shall part off areas of equal value from the two portions.

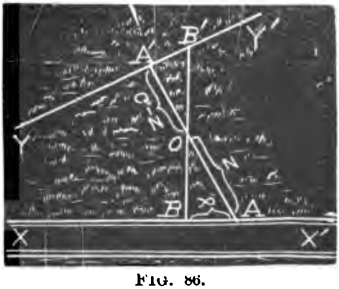


FIG. 86.

*Solution.**— Let T be the triangle AOB, and T', the triangle $A'OB'$.

Let v = value per acre of T, and v' = value per acre of T'.

* By George Key, a pupil of the author.

Let angle $OAB = A$, and angle $OA'B' = A'$ be known; and let $AB = x$, $AA' = c$, and $AO = z$.

We shall then have

$$\text{area } T = \frac{z^2 \sin A \cos A}{2} \qquad (1)$$

$$\text{and area } T' = \frac{(c - z)^2 \sin A' \cos A}{2 \cos (A' - A)} \qquad (2)$$

By conditions of the problem, $Tv = T'v'$.

Whence, $T : T' :: v' : v$. Let the ratio $v' : v = r$.

Then $T = T'r$. Whence, from (1) and (2),

$$\frac{z^2}{(c - z)^2} = \frac{r \sin A'}{\sin A \cos (A' - A)}$$

$$\text{or } \frac{z}{c - z} = \left\{ \frac{r \sin A'}{\sin A \cos (A' - A)} \right\}^{\frac{1}{2}} = n.$$

Whence, $z = \dfrac{cn}{n + 1}$, and $x = z \cos A = \dfrac{cn \cos A}{n + 1}$.

SECTION V.

TRANSIT SURVEYING.

FIG. 87.

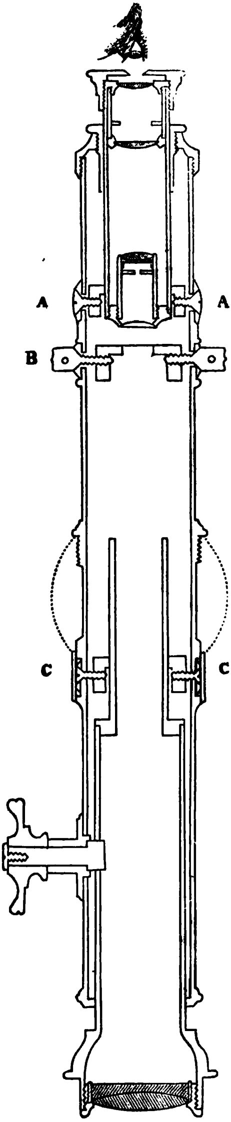


FIG. 88.

100. The essential parts of the Transit, as shown in the cut, are the *telescope* with its axis and two supports, the *circular plates* with their attachments, the *sockets* upon which the plates revolve, the *leveling head*, and the *tripod* on which the whole instrument stands.

The *telescope* is from ten to eleven inches long, firmly secured to an axis having its bearings nicely fitted in the standards, and thus enabling the telescope to be moved in either direction, or turned completely around if desired.

The different parts of the telescope are shown in Fig. 88.

The object-glass, composed of two lenses, so as to show objects without color or distortion, is placed at the end of a slide having two bearings, one at the end of the outer tube, the other in the ring *CC*, suspended within the tube by four screws, only two of which are shown in the cut.

The object-glass is carried out or in by a pinion working in a rack attached to the slide, and thus adjusted to objects either near or remote as desired.

The eye-piece is made up of four plano convex lenses, which, beginning at the eye-end, are called respectively the

eye, the field, the amplifying, and the object lenses, the whole forming a compound microscope having its focus in the plane of the cross-wire ring *BB*.

The eye-piece is brought to its proper focus usually by turning its milled end, the spiral movement within carrying the eye-tube out or in as desired; sometimes a pinion, like that which focuses the object-glass, is employed for the same purpose.

101. The Cross-Wires, (Fig. 89), are two fibres of spider-web or very fine platinum wire, cemented into the cuts on the surface of a metal ring, at right angles to each other, so as to divide the open space in the center into quadrants.

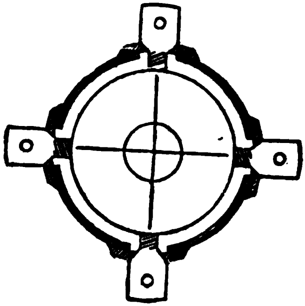


FIG. 89.

102. Optical Axis—The intersection of the wires forms a very minute point, which, when they are adjusted, determines the optical axis of the telescope, and enables the surveyor to fix it upon an object with the greatest precision.

The imaginary line passing through the optical axis of the telescope, is termed the **Line of Collimation**, and the operation of bringing the intersection of the wires into the optical axis is called **Adjusting the Line of Collimation.** This will be hereafter described.

103. The Vertical Circle firmly secured to the axis of the telescope is 4½ inches diameter, plated with silver, divided to half degrees, and with its vernier enables the surveyor to obtain vertical angles to single minutes.

104. The Level on Telescope consists of a brass tube about 6½ inches long, each end of which is held between two capstan-nuts connected with a screw or stem attached to the under side of the telescope tube.

105. **The Magnetic Needle** is four to five inches long in the different sizes of transits, its brass cup having inserted in it a little socket or center of hardened steel, perfectly polished, and this resting upon the hardened and polished point of the center pin, allows the needle to play freely in a horizontal direction, and thus take its direction in the magnetic meridian. The needle has its north end designated by a scallop or other mark, and on its south end has a coil of fine brass wire, easily moved, so as to bring both ends of the needle to the same level. The needle is lifted from the pin by a concealed spring underneath the upper plate, actuated by a screw shown above, thus raising the button so as to check the vibrations of the needle, or bring it up against the glass when not in use, to avoid the unnecessary wear of the pivot.

106. **The Lower Plate**, called the **Limb**, is divided on its upper surface—usually into degrees and half-degrees—and figured in two rows, viz., from 0 to 360, and from 0 to 90 each way; sometimes but a single series is used, and then the figures run from 0 to 360 or from 0 to 180 on each side.

107. **The Verniers**, of which there are two placed opposite each other against the limb, are auxiliary scales used in measuring smaller portions of the limb than are shown by its graduations. Thirty divisions on the vernier correspond precisely with twenty-nine half degrees on the limb. Hence one division on the limb exceeds one division on the vernier by one-thirtieth of one-half of a a degree, that is, by one minute.

Accordingly, the number of any division of the vernier, *on the side toward which the vernier is moved*, which coincides with a division of the limb is the number of minutes of arc intercepted by the zero of the vernier and the last preceding division of the limb.

Thus, by the device of a vernier we are enabled to measure angles to within one minute, although the limb of the transit is graduated only to half-degrees.

Adjustments.—The principal adjustments of the Transit are—

(1) *The Levels.*
(2) *The Line of Collimation.*
(3) *The Standards.*

108. To Adjust the Levels.—Set up the instrument upon its tripod as nearly level as may be, and having unclamped the plates, bring the two levels above and on a line with the two pairs of leveling screws; then with the thumb and first finger of each hand clasp the heads of two opposite, and, turning both thumbs in or out, as may be needed, bring the bubble of the level directly over the screws, exactly to the centre of the opening. Without moving the instrument proceed in the same manner to bring the other bubble to its centre; after doing this, the level first corrected may be thrown a little out; bring it in again; and when both are in place, turn the instrument half-way around; if the bubbles both come to the centre, they would need no correction, but if not, with the adjusting pin turn the small screws at the end of the levels until the bubbles are moved over half the error; then bring the bubbles again into the centre by the leveling screws, and repeat the operation until the bubbles will remain in the center during a complete revolution of the instrument, and the adjustment will be correct.

109. To Adjust the Line of Collimation.—To make this adjustment—which is, in other words, to bring the intersection of the wires into the optical axis of the telescope, so that the instrument, when placed in the middle of a straight line, will, by the revolution of the telescope, cut its extremities—proceed as follows:

Set the instrument firmly on the ground and level it carefully; and then having brought the wires into the focus of the eye-piece, adjust the object-glass on some well-defined point, as the edge of a chimney or other object, at a distance of from two hundred to five hundred

110. To Adjust the Standards.—In order that the wires may trace a vertical line as the telescope is moved up or down, it is necessary that both the standards of the telescope should be of precisely the same height.

To ascertain this and make the correction, if needed, proceed as follows:

Having the line of collimation previously adjusted, set up the instrument in a position where points of observation, such as the point and base of a lofty spire, can be selected, giving a long range in a vertical direction.

Level the instrument, fix the wires on the top of the object and clamp to the spindle; then bring the telescope down, until the wires bisect some good point, either found or marked at the base; turn the instrument half around, fix the wires on the lower point, clamp to the spindle, and raise the telescope to the highest object.

If the wires bisect it, the vertical adjustment is effected; if they are thrown to either side this would prove that the standard opposite that side was the highest, the apparent error being double that actually due to this cause.

To correct it, one of the bearings of the axis is made movable, so that by turning a screw underneath the sliding-piece, as well as the screws which hold on the cap of the standard, the adjustment is made with the utmost precision.

111. To Adjust the Vertical Circle.—Having the instrument firmly set up and carefully leveled, bring into line the zeros of the circle and vernier, and with the telescope find or place some well-defined point or line, from one hundred to five hundred feet distant, which is cut by the horizontal wire.

Turn the instrument half way around, revolve the telescope, and fixing the wire upon the same point as before, note if the zeros are again in line.

If not, loosen the capstan-head screws which fasten the vernier, and move the zero of the vernier over half the error; bring the zeros again into coincidence, and proceed

precisely as at first, until the error is entirely corrected, when the adjustment will be complete.

It is not always convenient to make this adjustment so as entirely to eliminate the index error. In this case, the error should be noted and the proper correction made in measuring a vertical angle.

To find the index error we have the following

RULE,—*Level the instrument and direct the telescope upon some well defined point. Note the reading of the circle.*

Reverse the telescope and turn the vernier plate 180°. Direct the telescope upon the point and note the reading of the circle.

Subtract the first reading from the second, and divide the remainder by 2.

EXERCISES.

It will be pleasant to pass now from verbal description of the transit, or even from examination of the instrument in the class-room, to its actual use in the field.

112. Prob. 1. *To measure a horizontal angle.*

SUGGESTIONS.—Select any two well-defined objects between which the angle is to be measured. Drive a peg in the ground to mark the vertex of the angle.

Set up the instrument by means of the plumb-line precisely over the center of the peg, observing at the same time to have the lower leveling plate as near as possible horizontal.

Loosen the lower clamp and turn the instrument on the spindle till the level tubes are each parallel to an opposite pair of the leveling screws.

Observing whether a bubble is to be caused to move toward the right or the left in being brought to the middle of the tube, turn the parallel pair of screws both inward or outward until the bubble comes to the center. Each level being treated in this way, the limb of the instrument is caused to be parallel to the horizon.

The leveling screws should be kept bearing equally against the plates. If they should seem to turn too hard, ~~relieve the stress by turning the other pair.~~ The instrument having been leveled, unclamp the vernier plate and set the zero of the vernier to coincide with the zero of the limb. Clamp the plates in this adjustment.

Unclamp the spindle, and turn the instrument carefully in the direction of the left hand side of the angle so that the intersection of the cross-wires shall be quite near in range with the object selected. Tighten the lower clamp, and with the tangent screws bring the intersection of the cross-wires precisely upon the center of the object.

Unclamp the vernier and turn the telescope along the other side of the angle, bringing the intersection of the wires as near as convenient upon the object in that direction. Clamp the vernier, and, by use of the upper tangent screw, bring the intersection of the wires precisely upon the object.

Read the angle which has thus been turned off.

Instead of setting the zeros in coincidence, it is as well or better to note the reading as the telescope is turned upon the first point, then turn off the angle, and again noting the reading, take the difference between the two readings as the angle sought.

The operation may be repeated in this way, successively adding the angle to the last reading, as many times as desired. Then subtracting the first reading from the last increased by 360° for each time the point of a complete revolution of the vernier has been passed, and dividing the difference by the number of repetitions, the quotient may be taken as a more precise value of the angle than would be obtained by a single observation.

With a few things more, which are of the nature of cautions, we leave the student to his practice.

(1) Be constantly on guard against any accident to your instrument, either in standing or being carried from place to place. Boys coming around must observe "*hands off.*"

(2) Do not support yourself by taking hold of the tripod. Never "ride" your instrument.

(3) Do not turn the screws up too tightly. Bring them simply to bear well, that's all.

(4) Learn to shut either eye without covering it with your fingers.

113. Prob. 2. *To let fall a perpendicular from a given point upon a given line.*

SUGGESTIONS.—Let P denote the given point, and AB the given line.

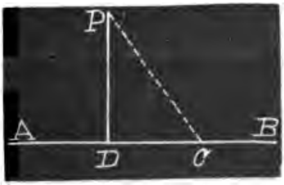


FIG. 91.

Set the instrument at any point, as C, of the given line. Measure the angle ACP. Set the instrument at P, and turn off the angle $CPD = 90° -$ angle ACP.

SCH.—When the line is inaccessible from the point, a procedure similar to that of Art. 67, Prob. 1, Case 3, may be employed.

Let the student develop it.

114. Prob. 3. *To erect a perpendicular to a given line at a given point.*

Left to the student without suggestion.

115. Prob. 4. *To run a line from a given point, parallel to a given line.*

Let the student work out a solution.

116. Prob. 5. *To measure an angle of elevation or depression.*

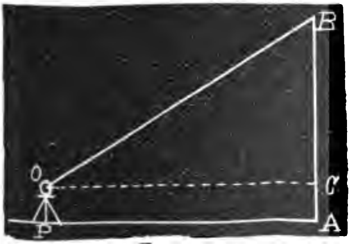


FIG. 92.

SUGGESTIONS.—Set the instrument at the vertex of the angle and level the horizontal limb.

Revolve the telescope upward or downward as the case may require, and adjust the line of sight to the inclined side of the angle. Take the reading of the vertical circle, applying the proper correction for index error, (Art. 111).

Otherwise, taking the reading of the circle, repeat the observation with the telescope and vernier plate reversed, and find the mean of the two readings for the angle sought.

Measurement of Hight and Distance.—Most of the following problems may be easily duplicated by examples, for practice in the field.

117. Prob. 1. *To find the hight of an accessible object on a horizontal plane.*

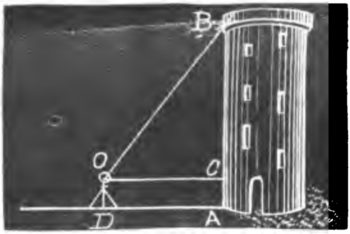


FIG. 93.

SUGGESTIONS.—Let AB be the hight required. Measure any convenient distance AD and, with the instrument at D, measure the angle of elevation BOC. Then $CB = AD \times \tan BOC$ and $AB = CB + OD$.

Example.—Given $AD = 200$ ft., angle $BOC = 40° \ 30'$, and $OD = 5$ ft. 6 in., to find AB.

118. Prob 2. *To find the hight of an inaccessible object on a horizontal plane.*

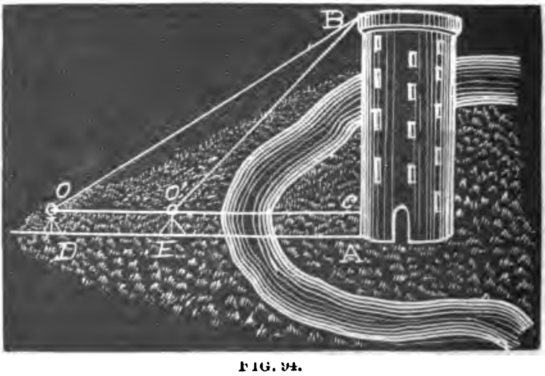


FIG. 94.

SUGGESTIONS.—Let AB be the required hight. Measure the angle of elevation, as at D. Find a point, as E, in the

vertical plane DBA, at which measure the angle of elevation. Measure DE.

Then $CB = DE \times \dfrac{\sin BOC \sin BO'C}{\sin (BO'C - BOC)}$, and $AB = BC + OD$:

also $EA = DE \times \dfrac{\sin BOC \cos BO'C}{\sin (BO'C - BOC)}$

119. Prob. 3. *To find the hight of an object situated above a horizontal plane, and its hight above the plane.*

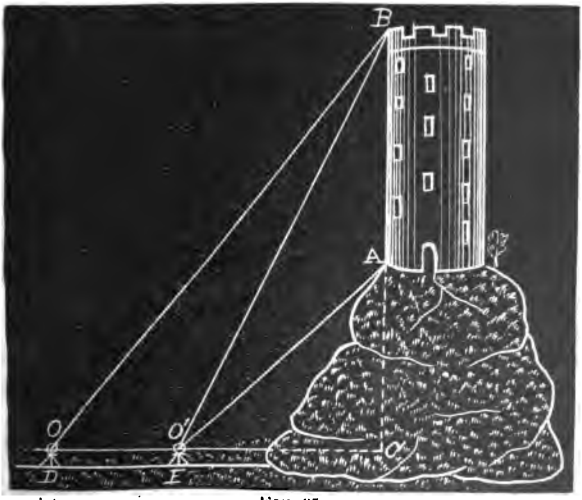


FIG. 95.

SUGGESTIONS.—Let AB be the required hight. Measure angle of elevation as at D. Measure DE, and at E measure the angles of elevation of the points A and B.

Let the student complete the work, (Prob. 2).

Example.—Given angle $BOC = 16° 28'$, $DE = \frac{1}{4}$ mile, angle $BO'C = 52° 14'$, and angle $AO'C = 48° 38'$, to find AB and CA.

REMARK.—The hight of the point A above the horizontal plane is found by adding the hight of instrument at E to the hight CA.

In practice, it will be found difficult to make the hight of instrument the same at the two points of observation.

The following problem presents a more general case.

120. Prob 4. *To find the hight of an object from observations made on sloping ground.*

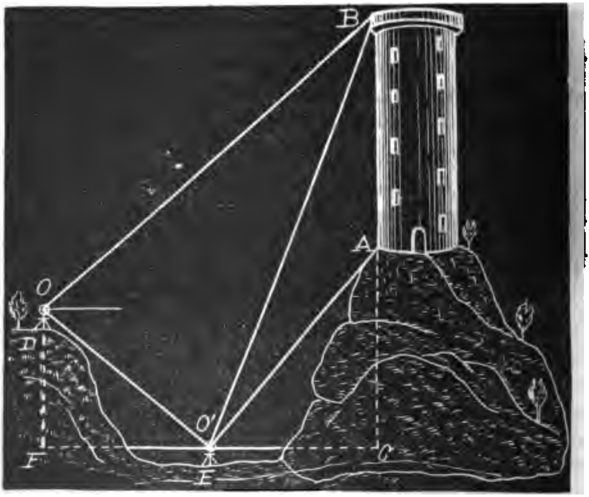


FIG. 96.

SUGGESTIONS.—Let AB be the required hight. Set the instrument as at D, and measure the angle of elevation of the point B. Revolve the telescope, finding a convenient point, as E, in the vertical plane DBA.

Set a stake at D, and on it mark the hight of instrument, as DO.

Remove to E, and take the angles of elevation of the points A and B.

Take also the angle of elevation of the point O, and measure the horizontal distance between E and D.

Let the student now show how, from the data thus obtained, the required hight may be computed.

QUERY.—How would the solution be affected by the point D being below the point E?

Example.—Given angle of elevation of B at the point $D = 34°$, the elevations of A and B at the point $E = 38°$ and $56° 30'$, respectively, the elevation of O at $E = \pm 20°$ and the horizontal distance $ED = 56.38$ ft., to find AB.

121. Prob 5. *To find the distance to an inaccessible point.*

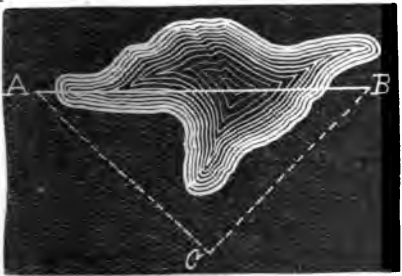


FIG. 97.

SUGGESTIONS.—Let AB be the required distance.

Setting the instrument at A, measure the angle BAC to some convenient point C. (Prob. 1). Measure AC and the angle ACB.

Then $AB = \dfrac{AC \times \sin C}{\sin (A + C)}$. If $C = 90°$, $AB = AC \times \sec A$.

If $A = 60°$ and $C = 90°$, $AB = 2 \times AC$.

Example.—Given angle $A = 40° \; 15'$, side $AC = 15$ ch., and angle $C = 110° \; 32'$, to find the side AB of the triangle ABC.

Another Method.—Choose the point C, conveniently, and measure CA, CB, and the angle C.

To find AB from these data, we may compute the angles A and B by use of formula 56, Art. 23, and then find AB by formula 55.

A different procedure is as follows:

Let $BC = a$, $AB = b$, and $AB = c$.

Then $c^2 = a^2 + b^2 - 2ab \cos C$. Substituting for $\cos C$ its value $2 \cos^2 \frac{1}{2} C - 1$, we have

$$c^2 = a^2 + b^2 - 2ab\,(2 \cos^2 \tfrac{1}{2} C - 1)$$

$$\text{or} \quad c^2 = (a + b)^2 - 4ab \cos^2 \tfrac{1}{2} C$$

$$\text{or} \quad c^2 = (a + b)^2 \left\{ 1 - \frac{4ab \cos^2 \frac{1}{2} C}{(a + b)^2} \right\}$$

Assume an angle x such that $\cos^2 x = \dfrac{4ab \cos^2 \frac{1}{2} C}{(a + b)^2}$, from which compute x.

Then $c = (a + b) \sin x$.

Example.—Given $CA = 12$ ch., $CB = 8.12$ ch., and $C = 100° \; 20'$, to find AB.

122. Prob. 6. *To find the distance between two inaccessible points.*

CASE 1.—*When a third point in line is accessible.*

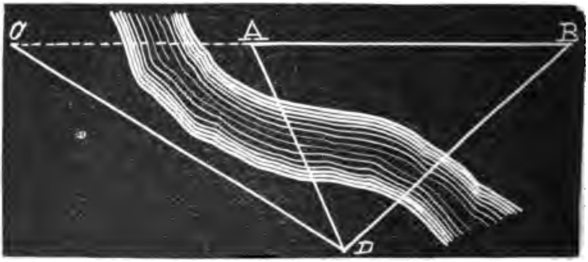


FIG. 98.

SUGGESTIONS.—Let A and B be the two points, and C a third point in the line AB.

Find a point D from which the points A, B, and C are visible, and measure the angles CDA and ADB.

Measure DC and the angle C.

Find logarithm of AD in triangle CAD, and then, since angle $BAD =$ angle $ACD +$ angle ADC, the side AB of the triangle ABD may be found.

CASE 2.—*When the points are each visible from two other points.*

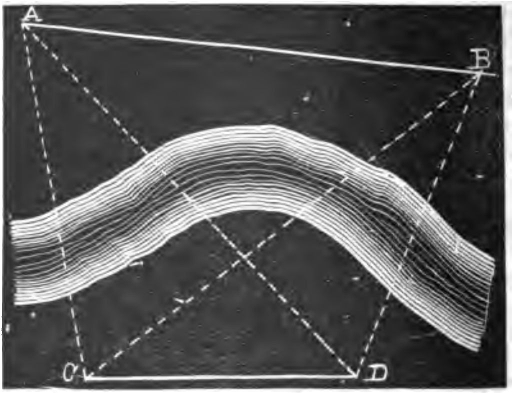


FIG. 99.

SUGGESTIONS.—Let C and D be two points from which A and B are each visible.

Measure the angles ACB, and BCD, the line CD, and the angles ADC and ADB.

Then in the triangles ACD and BCD the sides AC and BC may be computed, and from them the side AB of the triangle ACB may be found.

Check.—Find AB in the triargle ADB.

Example.—Given CD = 10 ch. and the angles ACB = 60° 15′, BCD = 50° 15′, CDA = 40° 20′, and ADB = 60° 10′, to find AB = 17.78 ch.

CASE 3.—*When the points are each visible from a single point.*

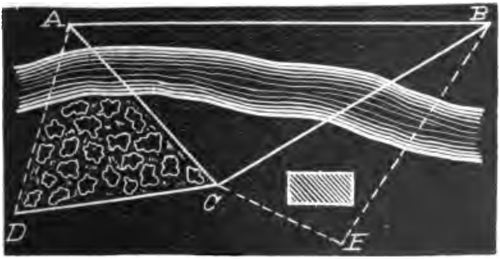


FIG. 100.

SUGGESTIONS.—Let C be a point from which A and B are each visible.

Find CA in the triangle CAD and CB in the triangle CBE, (Prob. 5),

Measure the angle ACB, and compute AB.

CASE 4.—*When no point can be found from which the two points are visible.*

SUGGESTIONS.—Let C and D (Fig. 101) be points visible from each other, and from which the points A and B, respectively, may be seen.

Measure CD. Find CA and DB, as in Case 3. Measure the angles ACD and BDC.

Compute BC and the angle BCD in the triangle BCD.

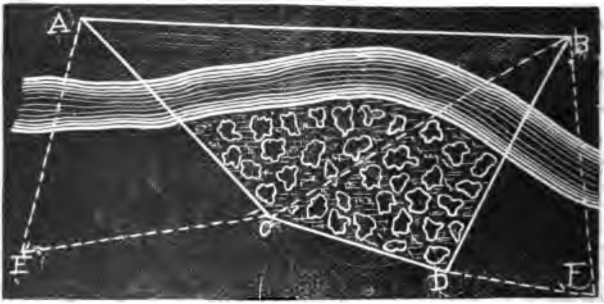


FIG. 101.

Find angle ACB = angle ACD — angle BCD. Compute AB in triangle ABC.

123. Prob 7. *To find the distances of a point from three points at given distances from each other.*

SUGGESTIONS.—Let A, B and C be the given points, and P the point whose distances from A, B and C are rerequired.

At P measure the angles APB and BPC. Suppose a circumference through the points A, C and P intersecting the line PB at D. Draw AD and CD. The angles of the triangle ABC may be computed from the sides by formula 59, Art. 23.

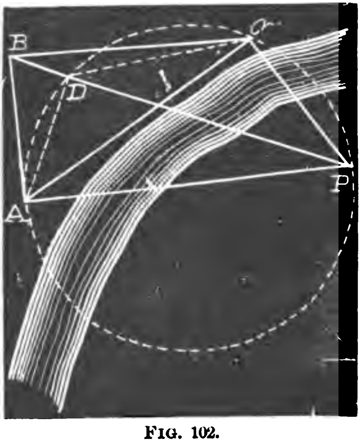


FIG. 102.

In the triangle ADC, since the angle DAC = angle BPC and the angle DCA = angle APB, the sides AD and CD may be computed.

In the triangle ABD, the angle BAD = angle BAC — angle DAC.

Whence the angle ABD may be found. Similarly, the angle CBD may be found, as a check.

In the triangle ABP, knowing its angles and a side, the sides PA and PB may be computed. Similarly, in the triangle CBP may be computed the side PC and the side PB, as a check.

QUERY.—How would the solution be affected by the supposition that the point B is on the opposite side of the line AC? That it is on the line AC?

Example.—Given $AB = 2.8$ miles, $BC = 3.47$ miles, $AC = 5.35$ miles, the angle $APB = 12° 15'$, the angle $BPC = 15° 30'$, to find the distances from P to the points A, B and C.

124. Prob. 8. *To range out a line with the transit.*

SUGGESTIONS.—Set the instrument and carefully level it over the first station. Direct the telescope to a somewhat distant point of the line. Let stakes or rods be set in line as far as can be seen.

Carry the instrument forward beyond the first two or three stakes, if possible, and set it over a point in the line, as at first. Direct the telescope in line by sighting points already marked, and continue the line as desired.

A few suggestions to the rodman may be allowed in this connection:

(1) Learn to get the range of natural objects in the line, and keep close to it.

(2) In running long lines, select high ground for your stations, from which long sights ahead may be obtained. Do not require the instrument to be set up twice where once would do as well.

(3) When ready to set up your flag or rod, be as near as you can judge for yourself in the line; face the instrument, hold the rod plumb and directly in front of you. Move the rod steadily to the right or left as directed till it is in place, then press it down. After setting it, look a second time to see if it has to be corrected.

(4) When axmen are cutting a line through the wood, keep near them, setting the flag on line at short intervals, so as to keep them in line, and thus save needless cutting.

125. Prob. 9. *To find a meridian with the transit.*

SUGGESTIONS.—Choose an open space toward the north as nearly level as possible. At the south end set firmly

in the ground a long stone or a sound oak stake, marking the top, which should be near the surface of the ground, with a well-defined cross cut into the material.

Provide a reflector from which the light of a lantern may be thrown into the telescope to illuminate the cross-wires.

A disk of tin slightly concave, having a hole through the center, and soldered to a ring fitted to the object end of the telescope, forms a convenient arrangement. Other devices will suggest themselves to the mind of the ingenious student.

About half an hour before the time of an elongation of Polaris as given in the Table of Elongations, (Art. 72), set the transit precisely over the cross in the stone, and carefully level the instrument.

Set the variation plate at zero, and set the vernier plate at the angle given in the Azimuth Table, which corresponds nearest to your latitude. (See Table VIII.)

Direct the telescope upon Polaris, focus the object glass on the star, and adjust the eye-piece so as to bring the cross-wires in focus.

Seeing now *again* that everything is in perfect adjustment, follow the star by the slow motion screw which turn the instrument upon the spindle, until it no longer moves away from the vertical wire. Then, being careful not to jar the instrument, unclamp the vernier plate and bring the zeros together.

Turn the telescope down and plant a stone or stake in exact line, and at a distance of five or six hundred feet. Cut a well-defined cross in the top of the stone marking a precise point in line. If the work has been accurately done throughout, the line indicated by the crosses on the stones will be a true meridian.

The work should be verified by repeated observations upon the star at the same and the opposite elongation, and the mean of results taken as indicating a reliable line.

Another Method.—Set the transit over a well-defined point, as above. Direct the telescope upon any bright star in the south and at the east of the meridian. Bring the intersection of the cross-wires precisely upon the star, with the vertical circle securely clamped.

Note the reading of the horizontal limb. Being careful not to disturb the vertical circle, unclamp the vernier plate, turn the instrument to the west of the meridian, and wait the coming of the star.

After a sufficient interval, turning the instrument a little to the right or left, the star will be seen entering the field of the object glass.

Now clamp the plate, and with the slow motion screw turn the instrument so as to catch the star upon the intersection of the cross-wires.

Note again the reading of the limb. Find half the difference of the two readings, and set back this half angle toward the east.

126. Prob. 10. *To find the angles which the parts of a broken line form with any given line.*

SUGGESTIONS.—Let $ABCDEF$ be a broken line, and suppose it is required to find the angles which the parts BC, CD, DE and EF form with the line AB.

Set the transit at B, with the vernier set at zero. Loosen below and direct the telescope to A. Clamp the limb, revolve the telescope on its horizontal azis, unclamp the vernier and direct the telescope to C. The reading of the instrument will be the angle bBC which the line BC forms with the line AB.

Remove to C; and, leaving the vernier clamped, unclamp below, revolve the telescope, and direct it to B.

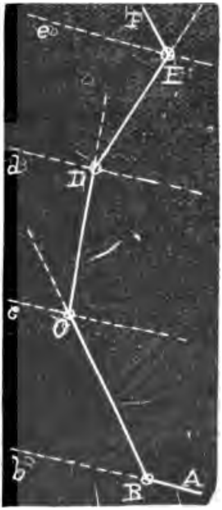


FIG. 103.

The limb remaining securely clamped, revolve the telescope, unclamp the vernier, and direct to *D*. The reading will now be the angle *cCD* which the line *CD* forms with the line *Cc* or its parallel *AB*.

The work goes on in this manner to its close.

Let the student further describe it.

If the broken line enclose a field, the reading of the instrument when set as at *A* and directed to *B*, having gone

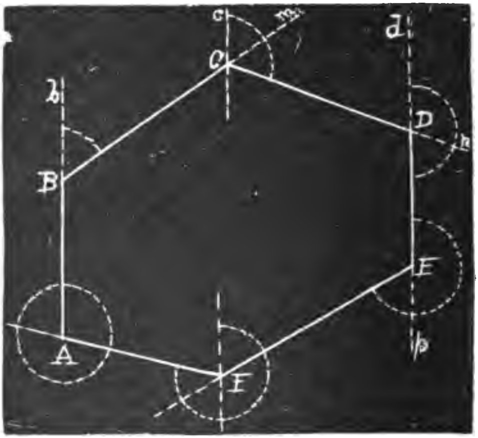


Fig. 104.

entirely around the field, should be 360°. This constitutes a check against errors occurring anywhere in the work.

The line, as *AB*, with respect to which the directions of the other lines are found, is called the **Meridian of the Survey**; and the angles, described in right-hand rotation, which the lines, as *BC*, *CD*, etc., form with the meridian, are called the **Azimuths** of the lines, respecting that meridian. They are indicated by the dotted arcs in the above figure.

The operation of measuring the azimuths of the successive parts of a broken line is called **Traversing.**

An angle, as bBC, mCD, etc., which a line forms with the prolongation of another through the vertex is called an **Angle of Deflection.**

By inspection, are discovered the following relations:

Azimuth of $BC =$	Azimuth	of AB	$= 0°$	+	Deflec.	of BC.
" " $CD =$	"	" BC		+	"	" CD.
" " $DE =$	"	" CD		+	"	" DE.
" " $EF =$	"	" DE		+	"	" EF.
" " $FA =$	"	" EF		+	"	" FA.
" " $AB = 360°$	"	" FA		+	"	" AB.

That is, *The azimuth of any course equals the azimuth of the preceding course plus the deflection of the course itself.*

Exercises.—Let the student plan and work in the field two or three examples of traversing, also measuring the lines.

The notes may be kept in the form shown in the margin.

Or a single column may be employed, as in the 1st and 2d forms, Art. 73.

Sta.	Azimuths with AB.	Distances.
A	0°	10.24
B	27° 40′	9.17
etc.	etc.	etc.

127. Prob. 11. *Given the azimuth of a line with reference to any assumed meridian, to find the bearing of the line respecting the same meridian.*

Suggestions.—Observe that it is neither the true nor the magnetic bearings that are required, but the bearings respecting a line *assumed* as a meridian.

FIG. 105.

Thus the line used as the meridian of the survey is regarded as extending from south to north, as *SN*, without regard to its real direction.

Observe also that the azimuths are turned off from left to right, beginning at 0°.

A simple inspection of things will show the following connection between azimuths and bearings:

1. For azimuths 0°, 90°, 180° and 270° the bearings are, respectively, N., E., S. and W.

2. For azimuths between 0° and 90°, the bearing is the same as the azimuth, marked N. E.

3. For azimuths between 90° and 180°, the bearing is the supplement of the azimuth, marked S. E.

4. For azimuths between 180° and 270°, the bearing is the azimuth minus 180°, marked S. W.

5. For azimuths between 270° and 360°, the bearing is 360° minus the azimuth, marked N. W.

Exercises.—Express the bearings corresponding to the following azimuths: 73° 40′; 294° 45′; 200° 15′; 127°.

State how the true and the magnetic bearings would be found, if such were required.

Example.—Given the magnetic bearing of *AB*, N. 5° 30′ W., declination of needle 15′ E., azimuth of *CD* with *AB*, 130° 24′, to find the magnetic and the true bearing of *CD*.

128. Prob. 12. ***Given the azimuths and the lengths of the sides of a field, to find the area.***

Solution.—Convert the azimuths into bearings, (Art. 127, Prob. 11), and compute the area, (Art. 79, Prob. 5).

Example:

Sta.	Azimuths with AB.	Bearings with AB.	Dist.	Latitudes. N.	Latitudes. S.	Dep. E.	Dep. W.	D.M.D.	N.Ar.	S.Ar.
A	0° 00′	North.	12,00	12.00		0.00		0.00	0.00	
B	120° 15′	S 59° 45′ E.	8.30		4.18	7.17		7.17		29.97
C	152° 36′	S. 27° 24′ E.	10.20		9.05	4.68		19.02		172.13
D	224° 54′	S. 44° 54′ W.	6.60		4.67		4.66	19.04		88.92
E	309° 24′	N. 50° 36′ W.	9.30	5.90			7.19	7.19	42.42	

42.42 291 02
42.42

2) 248.60

10) 124.30 sq. ch.

12.43 A.

The Traverse Table not being adapted to the above bearings, the latitudes and departures are computed from the formulas of Art. 80.

SCH.—Instead of the method of Double Meridian Distances, that by Total Latitudes, (Art. 86), may be employed with less labor.

Let the student verify the above area in that manner.

Another Solution.—Let ABC, etc., be any field. Let AY be any convenient line with respect to which the azimuths of the sides beginning at A are taken.

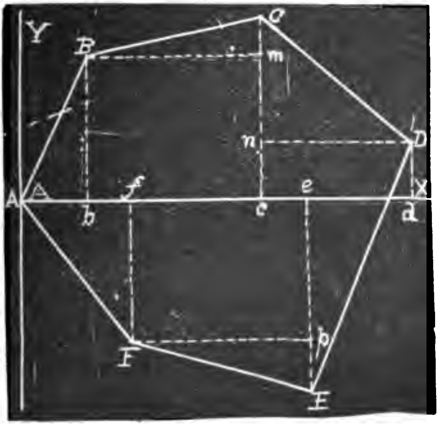


FIG. 106.

Let AX be an axis of abscissas and AY that of ordinates.

Let d be any side, the particular sides AB, BC, CD, etc., being d_1, d_2, d_3, etc.

Let A be the azimuth of any side, those at A, B, C, etc., being A_1, A_2, A_3, etc.

Let $s = d \sin A$ and $c = d \cos A$, particular values of s and c being distinguished by subscripts corresponding to those of d and A.

Let x_1, x_2, x_3, etc., be the abscissas of the points B, C, D, etc., and y_1, y_2, y_3, etc., their corresponding ordinates.

We shall then have

$$x_1 = d_1 \sin A_1 = s_1,$$
$$x_2 = x_1 + d_2 \sin A_2 = x_1 + s_2 = s_1 + s_2,$$
$$x_3 = x_2 + d_3 \sin A_3 = x_2 + s_3 = s_1 + s_2 + s_3,$$
$$\text{etc.,} \qquad \text{etc.,} \qquad \text{etc.,}$$

and
$$y_2 = d_1 \cos A_1 = c_1,$$
$$y_2 = y_1 + d_2 \cos A_2 = y_1 + c_2 = c_1 + c_2,$$
$$y_3 = y_2 + d_3 \cos A_3 = y_2 + c_3 = c_1 + c_2 + c_3,$$
$$\text{etc.,} \qquad \text{etc.,} \qquad \text{etc.}$$

Substituting the values of y in (b) and of x in (c), (Art. 59), we have

$$-\tfrac{1}{2}[x_1 (c_1 + c_2) + x_2 (c_2 + c_3) + x_3 (c_3 + c_4) + x_4 (c_4 + c_5) + \text{etc., to } n \text{ terms}], \qquad (m)$$

and
$$\tfrac{1}{2}[y_1 (s_1 + s_2) + y_2 (s_2 + s_3) + y_3 (s_3 + s_4) + y_4 (s_4 + s_5) + \text{etc., to } n \text{ terms}]. \qquad (n)$$

These formulas may be transformed into

$$-\tfrac{1}{2}[c_1 (x_0 + x_1) + c_2 (x_1 + x_2) + c_3 (x_2 + x_3) + c_4 (x_3 + x_4) + \text{etc., to } n \text{ terms}], \qquad (o)$$

and
$$\tfrac{1}{2}[s_1 (y_0 + y_1) + s_2 (y_1 + y_2) + s_3 (y_2 + y_3) + s_4 (y_3 + y_4) + \text{etc., to } n \text{ terms}], \qquad (p)$$

in which $x_0 = x_n = 0$ and $y_0 = y_n = 0$ are the coordinates of the origin.

In applying the above formulas, due regard must be had respecting algebraic signs. This is done by observing the signs of the sine and cosine of the azimuths, in finding s and c. The factor d is regarded as positive.

Whence for

azimuths between	$0°$	and	$90°$	we have	$+s$	and	$+c$
"	"	$90°$	"	$180°$	"	$+s$	"	$-c$
"	"	$180°$	"	$270°$	"	$-s$	"	$-c$
"	"	$270°$	"	$360°$	"	$-s$	"	$+c$

Checks.—In field work, $A_n = 360°$.

In computation of area, $s_1 + s_2 + s_3 + \text{etc.} = 0$,
$c_1 + c_2 + c_3 + \text{etc.} = 0$,

and area = area as found by two formulas.

Slight errors being unavoidable in practice, the traverses will usually require balancing in the manner of latitudes and departures.

Example.—Applying formula (*m*),

Dist.	Azimuths.	Traverses.		Coordinates.		Double Areas.	
		s	c	x	y	+	—
12.00	0° 00′	0.00	12.00	0.00	12.00		
8.30	120° 15′	7.17	—4.18	7.17	7.82	94.86	
10.20	152° 36′	4.68	—9.05	11.85	— 1 23	162.58	
6.60	224° 54′	—4.66	—4.67	7.19	— 5.90		8.84
9.30	309° 24′	—7.19	5.90	0.00	0.00		

Bearing of *AB*, N. 12° 30′ E.

$$\begin{array}{r} 257.44 \\ 8.84 \\ \hline 20)\ 248.60 \\ \hline 12.43\ \text{A.} \end{array}$$

SUGGESTION.—Let the student find the area by applying the formulas (*n*), (*o*) and (*p*).

Platting.—This is conveniently done as in Art. 84.

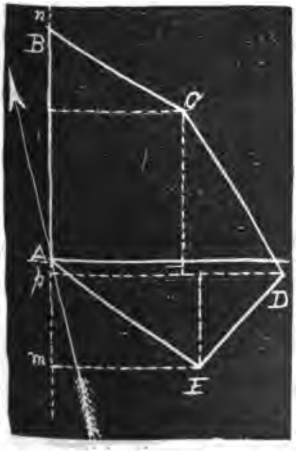


FIG. 107.

Example.—

Sta.	Coordinates.	
	x	y
A	0.00	0.00
B	0,00	12.00
C	7.17	7.82
D	11.85	— 1.23
E	7.19	— 5.90

As a check, the corners of the field should be seen to agree with the measurements of the sides.

The plat being drawn to a suitably large scale, the polygon may be reduced to a triangle, as *mDn*, and the area of the triangle as found from measurements on the plat may be employed to check the computation.

EXAMPLES.

129. Required contents and plats from the following notes:

(1)

Sta.	Azimuths.	Dist.
A	75° 32′	12.02
B	110° 25′	11.10
C	221° 03′	8 50
D	280° 20′	8.02
E	233° 30′	7.05
F	313° 20′	10.60

Bearing of *AB*, N, 70° E.

(2)

Sta.	Azimuths.	Dist.
A	33° 15′	18.80
B	105° 30′	30.95
C	133° 30′	20.00
D	234° 30′	13.80
E	270° 00′	21.25
F	301° 00′	27.85

Bearing of *AB*, N. 33° 15′ E.

3. 4 and 5. To be made by the student in the field.

130. Method of Radiation—It is sometimes a convenient method for area and plat to take the lengths and azimuths of lines from a single point, as *A*, to the corners of the field.

Thus, set the transit at *A*, turn on *B*, and note the reading; then on *C*, and note the reading, and so on around

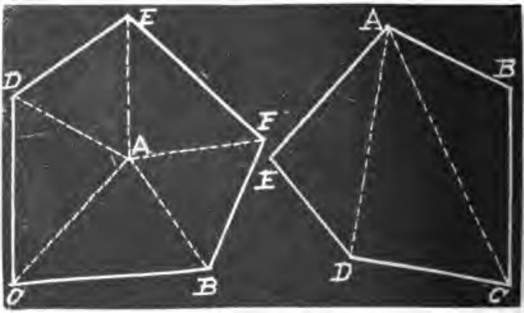


FIG. 108.

to *B* again. Measure *AB*, *AC*, *AD*, etc. Compute the areas of the triangles, (Art. 75, Prob. 1), and add them.

QUERY.—How would the solution be affected by the point *A* being outside of the field? How would the triangles outside be rendered, relatively, negative?

EXAMPLES.

131. Required area and plat from the following notes:

(1)

Lines.	Azimuths.	Dist.
AB	°0 00′	5.20
AC	35° 28′	7.34
AD	80° 43′	10.18
AE	112° 35′	8.50

Bearing of *AB*, N. 45° E.

(2)

Lines.	Azimuths.	Dist.
AB	0° 00′	6.14
AC	80° 14′	8.30
AD	142° 17′	7.10
AE	210° 25′	5.24
AF	325° 42′	9.06

Bearing of *AB*, N. 20° W.

3, 4 and 5. To be made by the student in the field.

STADIA AND GRADIENTER.

132. The Stadia, or **Micrometer,** is a compound cross-wire ring or diaphragm, shown in Fig. 109, having three horizontal wires, of which the middle one is cemented to the ring as usual, while the others, *bb* and *cc*, are fastened to small slides, held apart by a slender brass spring hoop, and actuated by independent screws, *dd*, by which the distance be- the two movable wires can be adjusted to include a given space, as one foot on a rod one hundred feet distant. These wires will in the same

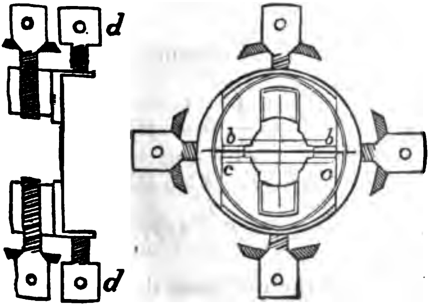


FIG. 109.

manner include two feet on a rod two hundred feet distant, or half a foot at a distance of fifty feet, and so on in the same proportion, thus furnishing a means of measuring distances, especially over broken ground, much more easily and even more accurately than with a tape or chain.

133. The principle of the stadia may be shown as follows:

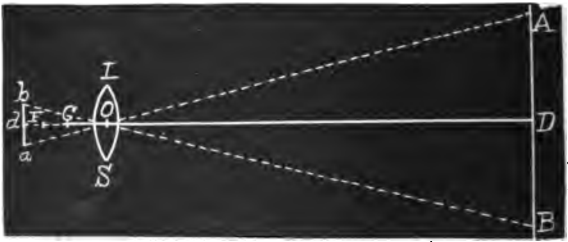


FIG. 110.

Let LS be the object-glass of the telescope, $AB = r$, a portion of a rod held vertically in front of the object glass, and $ab = s$, the image of AB as intercepted by the stadia wires bb and cc, (Fig. 109).

Let F be the focus of parallel rays, and C the center of the instrument, or the intersection of the plumb-line with the axis of the telescope.

Let f be the distance FO, f_1 the distance dO, and f_2 the distance OD.

By a principle of Optics $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ (1)

and from similar triangles, $f_1 : f_2 :: s : r$. (2)

Eliminating f_1, we obtain $f_2 = f\frac{r}{s} + f$. (3)

That is, the distance of the rod from the object-glass equals the focal length of the lens multiplied by the ratio of the intercepted space on the rod to the space between the stadia wires, increased by the focal length of the lens.

To find the distance d from the center of the instrument to the rod, the distance $CO = c$ must be added to f_2, giving

$$f_2 + c = d = f\frac{r}{s} + f + c. \quad (4)$$

The quantities f and c are constant for the same instrument. So also is s for any instrument carrying unadjustable wires.

Now, as the best avenue to the brain is by the way of the hands, the student will be best helped to understand the matter of the stadia by solving for himself the following problems:

134. Prob. 1 *To find f.*

SUGGESTIONS.—Focus the instrument upon some very distant object, for example the moon or a star, and measure the distance from the plane of the cross-wires to the plane of the object-glass.

135. Prob. 2. *To find c.*

SUGGESTIONS.—Focus the instrument upon an object at a distance of about 100 feet, and measure the distance from the center of the horizontal axis of the telescope to the object-glass.

136. Prob. 3. *To find s.*

SUGGESTIONS.—Set up and level the instrument over a point. Measure forward from the point a distance $f + c$, marking the point. From this point measure forward any convenient distance, as 400 or 500 feet, for a base. Level the telescope and direct it upon a leveling rod held vertically at the extremity of the base. Note carefully the space intercepted by the stadia wires.

Now, calling the base b, and the space on the rod r, we have, from formula (4),

$$d - (f + c) = b = f\frac{r}{s}. \quad (5)$$

$$\text{Whence, } s = f\frac{r}{b}. \quad (6)$$

Example.—Given $f = 6$ in., $b = 400$ ft., and $r = 4.8$ ft., to find s. Result, $s = 0.006$ ft.

SCH.—If the intercepted space on the rod should be required to be in a given ratio to the base, as 1 : 100, then must $s = 0.01 f$.

It is possible to set the wires so that any given reading of the rod shall correspond to any required length of base.

137. Prob. 4. *To find the base corresponding to a given setting of the wires.*

SUGGESTIONS.—Set up the instrument and measure from the plumb-line two distances, as 100 ft. and 1000 ft., as accurately as possible. Level the telescope and take carefully the wire interval at each distance. Do this four or five times, and take the mean of results as the value of r for that station. Substitute the values of d and the corresponding values of r in formula (4), forming, thus, two equations from which the values of the unknown quantities $\frac{f}{s}$ and $(f + c)$ may be found.

The first of these quantities is the constant number by which any reading of the rod is to be multiplied to produce the corresponding base, and the second is the number to be added to the base in finding the corresponding distance.

Example.—Given $d = 100$, $r = 1.25$ and $d = 1000$, $r = 12.61$, to find the base and distance for $r = 8.5$.

Results, $b = 673.20$; $d = 674.20$.

138. Prob. 5. *To make a stadia rod.*

SUGGESTIONS.—Procure a rod 1¼ in. square at the end, or a narrow strip of board 12 or 14 feet in length, in the latter case stiffening the back by a piece fastened along the middle. Give the rod two good coats of white paint. When ready for use, set up the transit, measure from the plumb-line the distance $f + c$, and set a point. From this point measure forward 100 feet to a second point, at which have an assistant hold the rod. The rod should be

provided with a plumb-line or level to insure a vertical position, and two movable targets to mark the space intercepted by the wires. Level the telescope and direct it upon the rod. Observe the intercepted space, and, if it should appear too large for the compass of the rod at distances of 1000 to 1500 feet, lessen it somewhat by means of the adjusting screws *dd* (Fig. 109). The space thus obtained is the unit in the graduation of the rod. This should be laid off from the bottom toward the top, with conspicuous numbering, and each divided into tenths and hundredths.

SCH. 1.—If the distances are required to be in chains and links, set the rod at 66 ft. instead of 100 ft. to obtain the unit. Graduate to this unit, and subdivide to hundredths as before.

SCH. 2.—Stadia rods are sometimes constructed with some style of diagrams of uniform scale upon their face, by means of which the reading of the rod is readily seen for any distance the rod is required to be used.

Having an instrument with the stadia wire attachment, the student should avail himself of considerable practice in its use.

139. In the foregoing consideration of stadia measurements, the line of sight has been regarded as horizontal. If, from the nature of the ground, the line of sight is considerably inclined to the horizon, the distance may be correctly taken on the rod as held inclined at an angle with the vertical equal to the angle of elevation or depression of the line of sight. The distance as thus obtained may then be reduced to a horizontal one by multiplying by the cosine of that angle.

It is more convenient, however, to hold the rod in a vertical position; and accordingly it is desirable to investigate the relation of the readings of the rod in the two positions.

To do this, let CO' (Fig. ~~109~~ 111) be the line of sight, at an angle e with the horizon.

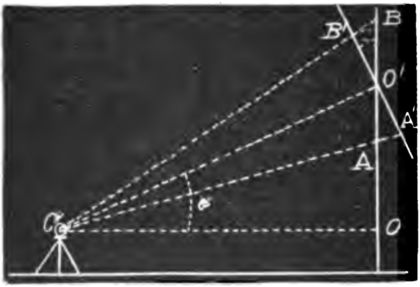


FIG. 111.

Let $A'B' = r'$ be the intercept on the rod as inclined at an angle e with the vertical; and let $b' = f\frac{r'}{s}$ formula (5) be the corresponding base.

Let $AB = r$ be the intercepted space on the rod as held vertically.

Let the angle $O'CB$ or $O'CA = v$.

We shall then have

Angle $OCB = e + v$, and angle $OCA = e - v$, whence angle $OBC = 90° - (e + v)$, and angle $OAC = 90° - (e - v)$. The angle $O'B'B = 90° + v$, and angle $O'A'A = 90° - v$.

In the triangle $O'B'B$ we have

$$\frac{O'B'}{O'B} = \frac{\sin [90° - (e + v)]}{\sin (90° + v)} \text{ or, } \frac{r'}{2O'B} = \frac{\cos (e + v)}{\cos v} \qquad (a)$$

In the triangle $O'A'A$ we have

$$\frac{O'A}{O'A} = \frac{\sin [90° - (e - v)]}{\sin (90° - v)} \text{ or } \frac{r'}{2O'A} = \frac{\cos (e - v)}{\cos v} \qquad (b)$$

Adding (a) and (b), we obtain

$$\frac{r'r}{2O'B \times O'A} = 2 \cos e \quad (c).$$

Multiplying (a) and (b) together, we obtain

$$\frac{r'r'}{4\,O'B \times O'A} = \frac{\cos^2 e \cos^2 v - \sin^2 e \sin^2 v}{\cos^2 v} \qquad (d)$$

Dividing (c) by (d), we have, after a little reduction,

$$\frac{r}{r'} = \frac{\cos e}{\cos^2 e - \sin^2 e \tan^2 v}, \quad (e)$$

which is an expression of the relation sought.

Cor.—With the wires adjusted to 1 foot on the rod for a base of 100 feet, we should have

$$\tan v = 0.005 \text{ ft. or } \tan^2 v = 0.000025 \text{ ft.}$$

Thus, $\tan^2 v = 0$, without material error.

Whence formula (e) becomes $r' = r \cos e$.

To find the distance CO' we have, from formula (4),

$$CO' = d' = f\frac{r'}{s} + f + c = b' + f + c.$$

Whence, $CO = d = (b' + f + c) \cos e$.

For vertical rod we have $b' = b \cos e$.

Whence, $d = b \cos^2 e + (f + c) \cos e$. (8)

The hight $OO' = h = \frac{1}{2} b \sin 2e + (f + c) \sin e$. (9)

Example.—Given $e = 10° \ 30'$, $r = 5.36$ ft., and $f + c = 1$ ft., to find d and h.

Solution.—Suppose the wires adjusted to give 1 ft. on the rod to the 100 ft., whence $b = 536$ ft.

$\text{Cos } e = 0.983$ and $\cos^2 e = 0.9668$.

Whence, $d = 536 \times 0.9668 + 0.98 = 519.18$ ft.

$\text{Sin } e = 0.182$, and $\frac{1}{2} \sin 2e = 0.1792$.

Whence, $h = 536 \times 0.1792 + 0.18 = 96.23$ ft.

Formula (8) may be put in the form

$$d = b \cos^2 e + (f + c) \cos^2 e + (f + c) \cos e\,(1 - \cos e).$$

Dropping the last term, we have

$$d = (b + f + c) \cos^2 e. \quad (10)$$

Assuming $f + c = 1$ ft., as a mean value in different instruments, the omission of the term $(f + c) \cos e$ $(1 - \cos e)$ introduces an error for ordinary elevations of less than 0.01 ft. in a base of 1000 ft.

Moreover, the use of formula (10) operates to diminish the very minute error introduced by use of formula (8).

For slight elevations, as from 1° to 2°, the reduction to horizontal may be omitted. For 5° 44′ the amount of the reduction is about one per cent. The correction for horizontal measurement is sometimes made by omitting to add $f + c$ to the base.

140. The Gradienter is an attachment to the transit for fixing grades and determining distances.

It consists mainly of a screw attached to the semicircular expanded arm of the ordinary clamp of the

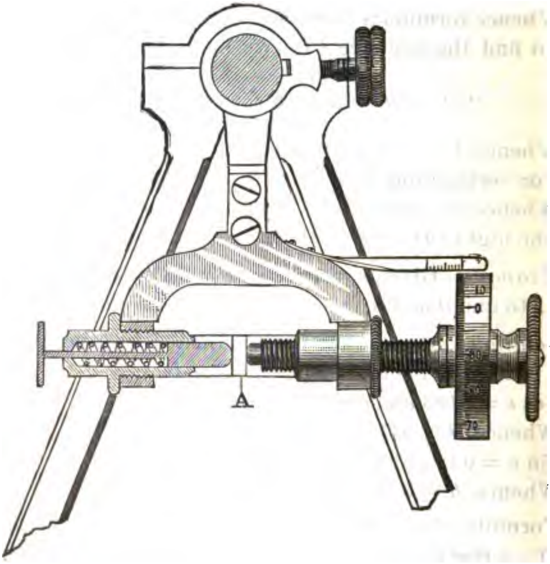


FIG. 112.

telescope axis; the screw is accurately cut to a given number of threads, and passing through a nut in one side of the arm, presses against a little stud, *A*, fixed to the inside surface of the right-hand standard.

In the other side of the semicircular arm is inserted a hollow cylinder containing a pin actuated by a strong spiral spring, the end of the pin pressing against the side of the stud opposite that in contact with the screw.

Near the other end of the screw, and turning with it, is a wheel, or micrometer, the rim of which is plated with silver, and divided into one hundred equal parts.

A small silver scale, attached to the arm and just above the micrometer wheel, is divided into spaces, each of

which is just equal to one revolution of the screw; so that by comparing the edge of the wheel with the divisions of the scale, the number of complete revolutions of the screw can be easily counted.

It will be seen that when the clamp is made fast to the axis of the clamp-screw, and the gradienter-screw turned, it will move the telescope vertically, precisely like the tangent-screw ordinarily used.

And as the value of a thread is such that a complete revolution of the screw will move the horizontal cross-wire of the telescope over a space of one foot on a rod at a distance of one hundred feet, it is clear that when the screw is turned through fifty spaces on the graduated head, the wire will pass over fifty one-hundredths, or one-half a foot on the rod, and so on in the same proportion.

In this way, the gradienter can be used in the measurement of distances, precisely like the stadia.

Grades can also be established, with great facility, as follows: Level the instrument; bring the telescope level to its centre by the clamp and gradienter screw; move the graduated head until its zero is brought to the edge of the scale; and then turn off as many spaces on the head as there are hundredths of feet to the hundred in the grade to be established.

Having a transit with gradienter attachment, let the student solve the following problems in the field.

141. **Prob. 1.** *To find the grade between two points.*

SUGGESTIONS.—Set the instrument over one of the points, level the plates and the telescope, and bring the zero of the screw to the edge of the scale.

Set the target of the leveling rod at hight of instrument.

With the rod held upon the other point, note the number of revolutions of the screw required in bringing the cross-wire upon the center of the target. That number, as so many feet, is the grade.

142. Prob. 2. *To find the distance between two points.*

SUGGESTIONS.—Set up and adjust the parts of the instrument as in Prob. 1. On a leveling rod held upon the other point, note the number of feet covered by one revolution of the screw, and multiply that number by 100.

If in order to cover r feet on a rod at a distance of d feet, n revolutions of the screw are required, then we should have $d : 100 :: r : n$; whence, $d = 100\, r \div n$.

Example.—Given $n = 2.30$, and $r = 5$ ft., to find d.

Result, $d = 217.39$ ft.

On inclined ground the horizontal sight line may be above or below the rod. In such cases, as in stadia measurement, a formula of reduction to a horizontal is employed, which may be deduced as follows:

Let $CO = d$ (Fig. 1) be a horizontal sight line;

Angle $OCO' = e$, the elevation of telescope to foot of rod;

Angle $O'CB = v$, the angle described by n revolutions of the screw;

$O'B' = r'$, the space on a rod perpendicular to CO', subtending angle v; and

$O'B = r$, the corresponding space on a vertical rod.

We shall then have, [Art. 139, Formula (a)],

$$\frac{r'}{r} = \frac{\sin\,[90^\circ - (e + v)]}{\sin\,(90^\circ + v)} = \frac{\cos e \cos v - \sin e \sin v}{\cos v}.$$

Whence, $r' = r\,(\cos e - \sin e \tan v)$.

Let $CO' = d'$. Then, $\tan v = \dfrac{r'}{d'} = \dfrac{n}{100}$.

$$\text{Whence, } d' = \frac{100\, r'}{n} = \frac{100\, r}{n}\left\{\cos e - \sin e \times \frac{n}{100}\right\}$$

$$\text{or}\quad d' = r\left\{\frac{100 \cos e}{n} - \sin e\right\}. \quad (1)$$

Now, $d = d' \cos e$.

$$\text{Whence, } d = r\left\{\frac{100}{n}\cos^2 e - \tfrac{1}{2}\sin 2e\right\}. \quad (2)$$

Cor.—If $n = 1$, we have

$$d' = r\,(100 \cos e - \sin e), \qquad (3)$$

$$\text{and } d = r\,(100 \cos^2 e - \tfrac{1}{2} \sin 2e), \qquad (4)$$

in which r is the space on a vertical rod, included by *one* revolution of the screw.

The numbers by which this value of r must be thus multiplied for various elevations are given in Table IX.

Examples.—1. Given $e = 15° 20'$, and $r = 5.42$ for one revolution of the screw, to find d' and d.

Solution.—We find in Table IX,

factor for inclined distance for $15° = 96.33$
" " " $15° 30' = 96.09$

Difference for $30' = 0.24$
whence, " " $20' = 0.16$

Whence, factor for inclined distance for $15° 20' = 96.17$.
Accordingly, $d = 5.42 \times 96.17 = 521.24$ ft.

Again, in Table IX we have

factor for horizontal distance for $15° = 93.05$
" " " $15° 30' = 92.59$

Difference for $30' = 0.46$
whence, " " $20' = 0.31$

Whence, factor for horizontal dist. for $15° 20' = 92.74$.

Hence, $d = 5.42 \times 92.74 = 502.65$ ft.

2. Given $e = 10.35$ rev. to foot of rod, and $r = 6.25$, to find d' and d.

Suggestion.—From Table X, find the angle e, and solve as above.

Sch.—When e is an angle of depression, the point O' is the upper end of the rod. The application of the formula is, however, the same in this case as in the one considered.

VI.

SECTION VII.

I. THE SOLAR COMPASS.

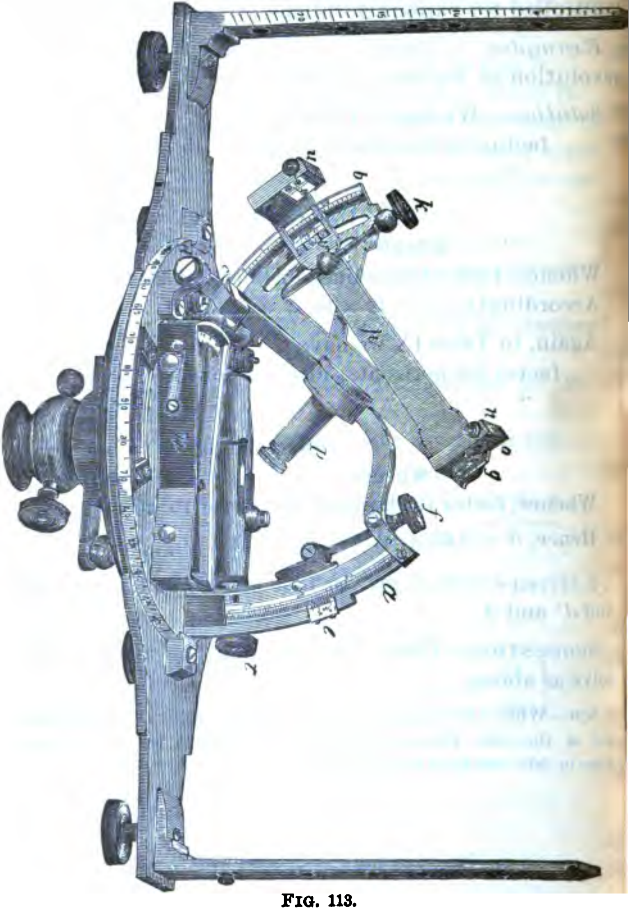


FIG. 113.

143. This instrument, so ingeniously contrived for readily determining a true meridian or north and south line, was invented by William A. Burt, of Michigan, and patented by him in 1836. It has since come into general use in the surveys of United States public lands, the principal lines of which are required to be run with reference to the true meridian.

The arrangement of its sockets and plates is similar to that of the surveyor's transit, except that the sight vanes are attached to the under plate or limb, and this revolves around the upper or vernier plate on which the solar apparatus is placed.

The limb is divided to half degrees, is figured in two rows, as usual, and reads by the two opposite verniers to single minutes.

144. The Solar Apparatus is seen in the place of the needle, and in fact operates as its substitute in the field.

It consists mainly of three arcs of circles, by which can be set off the latitude of a place, the declination of the sun, and the hour of the day.

These arcs, designated in the cut by the letters *a*, *b*, and *c*, are therefore termed the **latitude**, the **declination**, and the **hour** arcs, respectively.

145. The Latitude Arc, *a*, has its center of motion in two pivots, one of which is seen at *d*, the other is concealed in the cut.

It is moved either up or down within a hollow arc, seen in the cut, by a tangent-screw at *f*, and is securely fastened in any position by a clamp-screw.

The latitude arc is graduated to quarter degrees, and reads by its vernier, *e*, to single minutes; it has a range of about thirty-five degrees, so as to be adjustable to the latitude of any place in the United States.

146. The Declination Arc, *b*, is also graduated to quarter degrees, and has a range of about twenty-eight degrees.

Its vernier, *v*, reading to single minutes, is fixed to a movable arm, *h*, having its center of motion at the end of the declination arc, at *g*; the arm is moved over the surface of the declination arc, and its vernier set to any reading by turning the head of the tangent-screw, *k*. It is also securely clamped in any position by a screw, concealed in the engraving.

147. Solar Lenses and Lines.—At each end of the arm, *h*, is a rectangular block of brass, in which is set a small convex lens, having its focus on the surface of a little silver plate, *A*, (Fig. 114), fastened by screws to the inside of the opposite block.

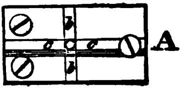


FIG. 114.

On the surface of the plate are marked two sets of lines intersecting each other at right angles; of these, *bb* are termed the hour lines, and *cc* the equatorial lines, as having reference respectively to the hour of the day and the position of the sun in relation to the equator.

In Fig. 114 the equatorial lines are those on the *lower* block, parallel to the surface of the hour arc, *c*; the hour lines are of course those at right angles to the first.

148. Equatorial Sights.—On the top of each of the rectangular blocks is seen a little sighting-piece, termed the equatorial sight, fastened to the block by a small milled head-screw, so as to be detached at pleasure.

They are used, as will be explained hereafter, in adjusting the different parts of the solar apparatus.

149. The Hour Arc, *c*, is supported by the two pivots of the latitude arc, already spoken of, and is also connected with that arc by a curved arm, as shown in the figure.

The hour arc has a range of about 120°, is divided to half degrees, and figured in two series, designating both the hours and the degrees, the middle division being marked 12 and 90 on either side of the graduated lines.

150. The Polar Axis.—Through the center of the hour arc passes a hollow socket, *p*, containing the spindle of

the declination arc, by means of which this arc can be moved from side to side over the surface of the hour arc, or turned completely round, as may be required.

The hour arc is read by the lower edge of the graduated side of the declination arc.

The axis of the declination arc, or indeed the whole socket *p*, is appropriately termed the **polar axis.**

151. The Adjuster.—Besides the parts shown in the cut, there is also an arm used in the adjustment of the instrument as described hereafter, but laid aside in the box when that is effected.

The parts above described constitute properly the solar apparatus.

Beside these, however, are seen the needle-box, *n*, with its arc and tangent-screw, *t*, and the spirit levels, for bringing the whole instrument to a horizontal position.

152. The Needle Box has an arc of about 36° in extent, divided to half degrees, and figured from the center or zero mark on either side.

The needle, which is made as in other instruments, except that the arms are of unequal lengths, is raised or lowered by a lever shown in the cut.

The needle-box is attached by a projecting arm to a tangent-screw, *t*, by which it is moved about its center, and its needle set to any variation.

This variation is also read off by the vernier on the end of the projecting arm, reading to three minutes a graduated arc, attached to the plate of the compass.

153. The Levels seen with the solar apparatus have ground glass vials, and are adjustable at their ends like those of other instruments.

The edge of the circular plate on which the solar work is placed, is divided and figured at intervals of ten degrees, and numbered, as shown, from 0 to 90 on each side of the line of sight.

These graduations are used in connection with a little brass pin, seen in the center of the plate, to obtain approximate bearings of lines, which are not important enough to require a close observation.

154. Lines of Refraction.—The inside faces of the sights are also graduated and figured, to indicate the amount of refraction to be allowed when the sun is near the horizon. These are not shown in the cut.

155. Principles of the Solar Compass.—The interval between two equatorial lines, *cc*, in Fig. 114, as well as between the hour lines, *bb*, is just sufficient to include the circular image of the sun as formed by the solar lens on the opposite end of the revolving arm, *h*, Fig. 113.

When, therefore, the instrument is made perfectly horizontal, the equatorial lines and the opposite lenses being accurately adjusted to each other by a previous operation, and the sun's image brought within the equatorial lines, his position in the heavens, with reference to the horizon, will be defined with precision.

Suppose the observation to be made at the time of one of the equinoxes; the arm *h*, set at zero on the declination arc *b*, and the polar axis *p*, placed exactly parallel to the axis of the earth.

Then the motion of the arm *h*, if revolved on the spindle of the declination arc around the hour circle *c*, will exactly correspond with the motion of the sun in the heavens, on the given day and at the place of observation; so that if the sun's image was brought between the lines *cc*, in the morning, it would continue in the same position, passing neither above nor below the lines, as the arm was made to revolve in imitation of the motion of the sun about the earth.

In the morning as the sun rises from the horizon, the arm *h* will be in a position nearly at right angles to that shown in the cut, the lens being turned toward the sun,

and the silver plate on which his image is thrown directly opposite.

As the sun ascends, the arm must be moved around, until when he has reached the meridian, the graduated side of the declination arc will indicate 12 on the hour circle, and the arm *h*, the declination arc *b*, and the latitude arc *a*, will be in the same plane.

As the sun declines from the meridian, the arm *h* must be moved in the same direction, until at sunset its position will be the exact reverse of that it occupied in the morning.

156. Allowance for Declination.—Let us now suppose the observation made when the sun has passed the equinoctial point, and when his position is affected by declination.

By referring to the Almanac, and setting off on the arc his declination for the given day and hour, we are still able to determine his position with the same certainty as if he remained on the equator.

When the sun's declination is south, that is, from the 22d of September to the 20th of March in each year, the arc *b* is turned toward the plates of the compass, as shown in the engraving, and the solar lens, *o*, with the silver plate opposite, are made use of in the surveys.

The remainder of the year, the arc is turned from the plates, and the other lens and plate employed.

When the solar compass is accurately adjusted, and its plates made perfectly horizontal, the latitude of the place, and the declination of the sun for the given day and hour, being also set off on the respective arcs, *the image of the sun cannot be brought between the equatorial lines until the polar axis is placed in the plane of the meridian of the place, or in a position parallel to the axis of the earth.* The slightest deviation from this position will cause the image to pass above or below the lines, and thus discover the error.

We thus, from the position of the sun in the solar system, obtain a certain direction absolutely unchangeable, from which to run our lines, and measure the horizontal angles required.

This simple principle is not only the basis of the construction of the solar compass, but the sole cause of its superiority to the ordinary or magnetic instrument. For in a needle instrument, the accuracy of the horizontal angles indicated, and therefore of all the observations made, depends upon the delicacy of the needle, and the constancy with which it assumes a certain direction, termed the magnetic meridian.

The principal causes of error in the needle, briefly stated, are the dulling of the pivot, the loss of polarity in the needle, the influence of local attraction, and the effect of the sun's rays, producing the diurnal variation. From all these imperfections the solar instrument is free.

The sights and the graduated limb being adjusted to the solar apparatus, and the latitude of the place and the declination of the sun also set off upon the respective arcs, we are able, not only to run the true meridian, or a due east and west course, but also to set off the horizontal angles with minuteness and accuracy from a direction which never changes, and is unaffected by attraction of any kind.

.57. **Adjustments.**—The adjustments of this instrument, with which the surveyor will have to do, are simple and few in number, and will now be given in order.

1st. **To Adjust the Levels.**—Proceed precisely as directed in the account of the other instruments we have described, by bringing the bubbles into the centre of the tubes by the leveling screws of the tripod, and then reversing the instrument upon its spindle, and raising or lowering the ends of the tubes, until the bubbles will remain in the centre during a complete revolution of the instrument.

2d. **To Adjust the Equatorial Lines and Solar Lenses.**—First detach the arm *h* from the declination arc, by withdrawing the screws shown in the cut from the ends of the posts of the tangent-screw *k*, and also the clamp-screw, and the conical pivot with its small screws by which the arm and declination arc are connected.

The arm *h*, being thus removed, attach the adjuster in its place by replacing the conical pivot and screws, and insert the clamp-screw so as to clamp the adjuster at any point on the declination arc.

Now level the instrument, place the arm *h* on the adjuster, with the same side resting against the surface of the declination arc as before it was detached. Turn the instrument on its spindle so as to bring the solar lens to be adjusted in the direction of the sun, and raise or lower the adjuster on the declination arc, until it can be clamped in such a position as to bring the sun's image as near as may be between the equatorial lines on the opposite silver plate, and bring the image precisely into position by the tangent of the latitude arc or the leveling-screws of the tripod. Then carefully turn the arm half way over, until it rests upon the adjuster by the opposite faces of the rectangular blocks, and again observe the position of the sun's image.

If it remains between the lines as before, the lens and plate are in adjustment; if not, loosen the three screws which confine the plate to the block, and move the plate under their heads, until one-half the error in the position of the sun's image is removed.

Again bring the image between the lines, and repeat the operation until it will remain in the same situation, in both positions of the arm, when the adjustment will be completed.

To adjust the other lens and plate, reverse the arm, end for end, on the adjuster, and proceed precisely as in the former case, until the same result is attained.

In tightening the screws over the silver plate, care must be taken not to move the plate.

This adjustment now being complete, the adjuster should be removed, and the arm *h*, with its attachments, replaced as before.

3d. To Adjust the Vernier of the Declination Arc.—Having leveled the instrument, and turned its lens in the direction of the sun, clamp to the spindle, and set the vernier *v*, of the declination arc, at zero, by means of the tangent-screw at *k*, and clamp to the arc.

See that the spindle moves easily and yet truly in the socket, or polar axis, and raise or lower the latitude arc by turning the tangent-screw *f*, until the sun's image is brought between the equatorial lines on one of the plates. Clamp the latitude arc by the screw, and bring the image precisely into position by the leveling-screws of the tripod or socket, and without disturbing the instrument, carefully revolve the arm *h*, until the opposite lens and plate are brought in the direction of the sun, and note if the sun's image comes between the lines as before.

If it does, there is no index error of the declination arc; if not, with the tangent-screw *k*, move the arm until the sun's image passes over half the error; again bring the image between the lines, and repeat the operation as before, until the image will occupy the same position on both plates.

We shall now find, however, that the zero marks on the arc and the vernier do not correspond, and to remedy this error, the little flat-head screws above the vernier must be loosened until it can be moved so as to make the zeros coincide, when the operation will be completed.

4th. To Adjust the Solar Apparatus to the Compass Sights.—First level the instrument, and with the clamp and tangent-screws set the main plate at 90° by the verniers and horizontal limb. Then remove the clamp-screw, and raise the latitude arc until the polar axis is by esti-

mation very nearly horizontal, and if necessary, tighten the screws on the pivots of the arc, so as to retain it in this position.

Fix the vernier of the declination arc at zero, and direct the equatorial sights to some distant and well marked object, and observe the same through the compass sights. If the same object is seen through both, and the verniers read to 90° on the limb, the adjustment is complete; if not, the correction must be made by moving the sights or changing the position of the verniers.

158. **To Use the Solar Compass.**—Before this instrument can be used at any given place, it is necessary to set off upon its arcs both the declination of the sun as affected by its refraction for the given day and hour, and the latitude of the place where the observation is made.

To Set off the Declination.—The declination of the sun, given in the ephemeris of the Nautical Almanac from year to year, is calculated for apparent noon at Greenwich, England.

To determine it for any other hour at a place in the United States, reference must be had, not only to the difference of time arising from the longitude, but also to the change of declination from day to day.

The longitude of the place, and therefore its difference in time, if not given directly in the tables of the Almanac, can be ascertained very nearly by reference to that of other places given, which are situated on, or very nearly on, the same meridian.

It is the practice of surveyors in the states east of the Mississippi, to allow a difference of *six* hours for the difference in the longitude, calling the declination given in the Almanac for 12 M., that of 6 A. M., at the place of observation.

Beyond the meridian of Santa Fe, the allowance would be about *seven* hours, and in California, Oregon, and Washington Territory about *eight* hours.

Having thus the difference of time, we very readily obtain the declination for a certain hour in the morning, which would be earlier or later as the longitude was greater or less, and the same as that of apparent noon at Greenwich on the given day. Thus, suppose the observation made at a place, say, five hours later than Greenwich, then the declination given in the Almanac for the given day at noon, affected by the refraction, would be the declination at the place of observation for 7 o'clock A. M.; this gives us the starting-point.

To obtain the declination for the other hours of the day, take from the Almanac the declination for apparent noon of the given day, and, as the declination is increasing or decreasing, add to or subtract from the declination of the first hour, the difference for one hour as given in the ephemeris, which will give, when affected by the refraction, the declination for the succeeding hour; and proceed thus in making a table of the declination for every hour of the day.

159. Refraction.—By reason of the increasing density of the atmosphere from its upper regions to the earth's surface, the rays of light from the sun are bent out of their course, so as to make his altitude appear greater than is actually the case.

The amount of refraction varies, according to the altitude of the body observed; being 0 when it is in the zenith, about one minute when midway from the horizon to the zenith, and almost 34′ when in the horizon.

160. Allowance for Refraction.—The proper allowance to be made for refraction in setting off the declination of the sun upon the Solar Compass has long been a source of perplexity to the surveyor. Accordingly, a table has been prepared, (Table XI), by which the amount of refraction for any hour of the day throughout the year may be readily obtained. The manner of using the table is shown in the solution of the following

Example.—1. To find the declination for the different hours of April 16, 1883, at Troy, N. Y.

Solution.—Latitude of Troy, about 42° 30′ N. Longitude, 4 hr., 54 min., 40 sec., practically 5 hr.

Apparent noon at Greenwich is 7 A. M. at Troy. Declination of sun at Greenwich at noon of April 16, 1883, as given by Nautical Almanac, N. 10° 6′ 2′′+, and hourly change, 53′′.

Refraction in Lat. 42° 30′, declination 10°, time 5 hr. before noon as given by table, 1′ 58′′.

Whence the following figures:

N. 10° 6′ 2′′ + Ref. 5 hrs. 1′ 58′′ = 10° 8′ 0′′ = Dec. at 7 A. M. Troy.
add hr. dif. 53′′

N. 10° 6′ 55′′ + " 4 " 1′ 11′′ = 10° 8′ 0′′.6 = " 8 "
add hr. dif. 53′′

N. 10° 7′ 48′′ + " 3 " 0′ 52′′ = 10° 8′ 40′′ = " 9 "
add hr. dif. 53′′

N. 10° 8′ 41′′ + " 2 " 0′ 39′′ = 10° 9′ 20′′ = " 10 "
add hr. dif. 53′′

N. 10° 9′ 34′′ + " 1 " 0′ 36′′ = 10° 10′ 10′′ = " 11 "
add hr. dif. 53′′

N. 10° 10′ 27′′ + " 0 " 0′ 36′′ = 10° 11′ 03′′ = " 12 M.
add hr dif. 53′′

N. 10° 11′ 20′′ + " 1 " 0′ 36′′ = 10° 11′ 56′′ " 1 P. M.
add hr. dif. 53′′

N. 10° 12′ 13′′ + " 2 " 0′ 39′′ = 10° 12′ 52′′ = " 2 "
add hr. dif. 53′′

N. 10° 13′ 06′′ + " 3 " 0′ 52′′ = 10° 13′ 58′′ = " 3 "
add hr. dif 53′

N. 10° 13′ 59′′ + " 4 " 1′ 11′′ = 10° 15′ 10′′ = " 4 "
add hr. dif. 53′

N. 10° 14′ 49′′ + " 5 " 1′ 58′′ = 10° 16′ 50′′ = " 5 "

Example.—2. To find the declination for the different hours of Oct. 16, 1883, at Troy, N. Y.

Solution.—Declination of sun at Greenwich at noon of Oct. 16, 1883, as given by Nautical Almanac S. 8°51′47′′.7. hourly change 55′′.

Refraction 5 hr. before noon, Lat. 42° 30′, Dec. — 9°, is very nearly 9′ 24″, and operates *to diminish* the declination.

Whence the following:

S. 8° 51′ 47″.7 add hr. dif. 55″	—Ref. 5 hr.	9′ 24″	= 8° 42′ 23″ =	Dec. at 7 A. M.	at Troy,
S. 8° 52′ 42″ add hr. dif 55″	— " 4 "	2′ 49″	= 8° 49′ 53″ =	" 8	"
S. 8° 53. 37″ add hr. dif. 55″	— " 3 "	1′ 49″	= 8° 51′ 48″ =	" 9	"
S. 8° 54′ 32″ add hr. dif 55″	— " 2 "	1′ 26″	= 8° 53′ 06″ =	" 10	"
S. 8° 55′ 27″ add hr. dif. 55″	— " 1 "	1′ 14″	= 8° 54′ 13″ =	" 11	"
S. 8° 56′ 22″ add hr. dif. 55″	— " 0 "	1′ 14″	= 8° 55′ 08″ =	" 12 M.	
S. 8° 57′ 17″ add hr. dif. 55″	— " 1 "	1′ 14″	= 8° 56′ 03″ =	" 1 P. M.	
S. 8° 58′ 12″ add hr. dif. 55″	— " 2 "	1′ 26″	= 8° 56′ 46″ =	" 2 "	
	etc.	etc.	etc.		

161. To Set Off the Latitude.—Find the declination of the sun for the given day at noon, at the place of observation, as just described, and with the tangent-screw set it off upon the declination arc, and clamp the arm firmly to the arc.

Observe in the Almanac the equation of time for the given day, in order to know about the time the sun will reach the meridian.

Then, about fifteen or twenty minutes before this time, set up the instrument, level it carefully, fix the divided surface of the declination arc at 12 on the hour circle, and turn the instrument upon its spindle until the solar lens is brought into the direction of the sun.

Loosen the clamp-screw of the latitude arc, and with the tangent-screw raise or lower this arc until the image of the sun is brought precisely between the equatorial lines, and turn the instrument from time to time so as to keep the image also between the hour lines on the plate.

As the sun ascends, its image will move below the lines, and the arc must be moved to follow it. Continue thus, keeping it between the two sets of lines until its image begins to pass above the equatorial lines, which is also the moment of its passing the meridian.

Now read off the vernier of the arc, and we have the latitude of the place, which is always to be set off on the arc when the compass is used at the given place.

It is the practice of surveyors using the solar compass to set off, in the manner just described, the latitude of the point where the survey begins, and to repeat the observation and correction of the latitude arc every day when the weather is favorable, there being also nearly an hour at mid-day when the sun is so near the meridian as not to give the direction of lines with the certainty required.

162. To Run Lines with the Solar Compass.—Having set off in the manner just given, the latitude and declination upon their respective arcs, the instrument being also in adjustment, the surveyor is ready to run lines by the sun.

To do this, the instrument is set over the station and carefully leveled, the plates clamped at zero on the horizontal limb, and the sights directed north and south, the direction being given, when unknown, approximately by the needle.

The solar lens is then turned to the sun, and with one hand on the instrument, and the other on the revolving arm, both are moved from side to side, until the sun's image is made to appear on the silver plate; when by carefully continuing the operation, it may be brought precisely between the equatorial lines.

Allowance being now made for refraction, the line of sights will indicate the true meridian; the observation may now be made, and the flag-man put in position.

When a due east and west line is to be run, the verniers of the horizontal limb are set at 90°, and the sun's image kept between the lines as before.

The solar compass being so constructed that when the sun's image is in position the limb must be clamped at 0 in order to run a true meridian line, it will be evident that the bearing of any line from the meridian may be read by the verniers of the limb precisely as in the ordinary magnetic compass, the bearings of lines are read from the ends of the needle.

163. Use of the Needle.—In running lines, the magnetic needle is always kept with the sun; that is, the point of the needle is made to indicate 0 on the arc of the compass box, by turning the tangent-screw connected with its arm on the opposite side of the plate. By this means, the lines can be run by the needle alone in case of the temporary disappearance of the sun; but, of course, in such cases the surveyor must be sure that no local attraction is exerted.

The variation of the needle, which is noted at every station, is read off in degrees and minutes on the arc, by the edge of which the vernier of the needle-box moves.

164. Allowance for the Earth's Curvature—When long lines are run by the solar compass, either by the true meridian, or due east and west, allowance must be made for the curvature of the earth.

Thus, in running north or south, the latitude changes about one minute for every distance of 92 chains 30 links, and the side of a township requires a change on the latitude arc of 5′ 12″, the township, of course, being six miles square.

This allowance is of constant use where the surveyor fails to get an observation on the sun at noon, and is a very close approximation to the truth.

In running due east and west, as in tracing the stand-

ard parallels of latitude, the sights are set at 90° on the limb, and the line is run at right angles to the meridian.

If no allowance were made for the earth's curvature, these lines would, if sufficiently produced, reach the equator, to which they are constantly tending.

Of course, in running short lines either east or west, the variation from the parallel would be so small as to be of no practical importance; but when long sights are taken, the correction should be made by taking fore and back sights at every station, noting the error on the back sight, and setting off one-half of it on the fore sight on the side toward the pole.

165. Time of Day by the Sun.—The time of day is best ascertained by the solar compass when the sun is on the meridian, as at the time of making the observation for latitude.

The time thus given is that of apparent noon, and can be reduced to mean time by merely applying the equation of time as directed in the Almanac, and adding or subtracting as the sun is slow or fast.

The time, of course, can also be taken before or after noon, by bringing the sun's image between the hour lines, and noticing the position of the divided edge of the revolving arm, with reference to the graduations of the hour circle, allowing four minutes of time for each degree of the arc, and thus obtaining apparent time, which must be corrected by the equation of time as just dedescribed.

166. Caution as to the False Image.—In using the compass upon the sun, if the revolving arm be turned a little one side of its proper position, a false or reflected image of the sun will appear on the silver plate in nearly the same place as that occupied by the true one. It is caused by the reflection of the true image from the surface of the arm, and is a fruitful source of error to the

inexperienced surveyor. It can, however, be readily distinguished from the real image by being much less bright, and not so clearly defined.

167. Approximate Bearings.—When the bearings of lines, such as the course of a stream, or the boundaries of a forest, are not desired with the certainty given by the verniers and horizontal limb, a rough approximation of the angle they make with the true meridian is obtained by the divisions on the outside of the circular plate.

In this operation, a pencil, or thin straight edge of any sort, is held perpendicularly against the circular edge of the plate, and moved around until it is in range with the eye, the brass center-pin, and the object observed.

The bearing of the line is then read off at the point where the pencil is placed.

Time for Using the Solar Compass.—The solar compass, like the ordinary instrument, can be used at all seasons of the year, the most favorable time being, of course, in the summer, when the declination is north, and the days are long, and more generally fair.

It is best not to take the sun at morning and evening, when it is within half an hour of the horizon, nor, for about the same interval, before and after it passes the meridian.

II. THE SOLAR ATTACHMENT.

168. The Solar Attachment is essentially the solar apparatus of Burt placed upon the cross-bar of the ordinary transit, the polar axis only being directed above instead of below, as in the solar compass. A little circular disk of an inch and a half diameter, and having a short round pivot projecting above its upper surface, is first screwed firmly to the axis ofthe telescope.

Upon this pivot rests the enlarged base of the polar axis, which is also firmly connected with the disk by four

capstan-head screws passing from the under side of the disk into the base already named.

These screws serve to adjust the polar axis, as will be explained hereafter.

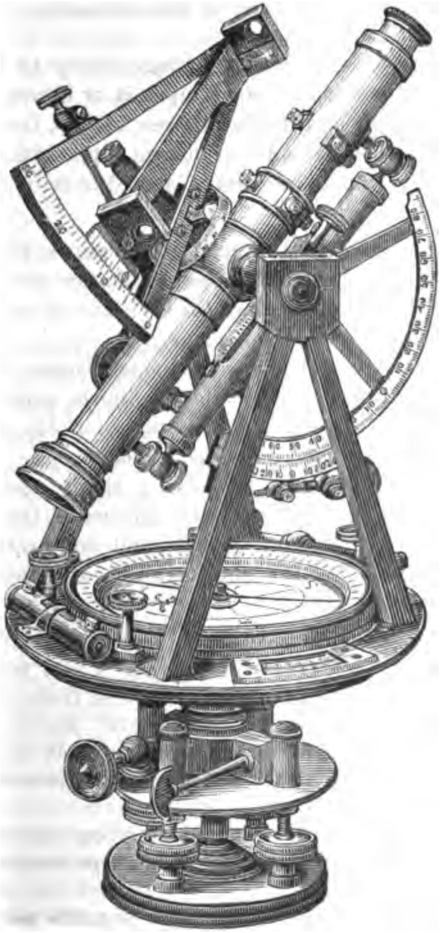


FIG 114..

169. **The hour circle** surrounding the base of the polar axis is easily movable about it, and can be fastened at any point desired by two flat-head screws above. It is divided to five minutes of time; is figured from I. to XII., and is read by a small index fixed to the declination circle, and moving with it.

A hollow cone, or socket, fitting closely to the polar axis and made to move snugly upon it, or clamped at any point desired by a milled-head screw on top, furnishes by its two expanded arms below, a firm support for the declination arc, which is securely fastened to it by two large screws.

170. **The declination arc** is of about five inches radius, is divided to quarter degrees, and reads by its vernier to single minutes of arc, the divisions of both vernier and limb being in the same plane.

The declination arm has the usual lenses and silver plates on the two opposite blocks, made precisely like those of the ordinary solar compass, but its vernier is outside the block, and more easily read.

The declination arm has also a clamp and tangent movement, as shown in the cut. The arc of the declination limb is turned on its axis and one or the other solar lens used, as the sun is north or south of the equator.

171. **The latitude** is set off by means of a large vertical limb having a radius of two and a half inches; the arc is divided to thirty minutes, is figured from the centre, each way, in two rows, viz. from 0 to 80°, and from 90° to 10°, the first series being intended for reading vertical angles; the last series for setting off the latitude, and is read by its vernier to single minutes.

It has also a clamp-screw inserted near its centre, by which it can be set fast to the telescope axis in any desired position.

The vernier of the vertical limb is made movable by the tangent-screw attached, so that its zero and that of

the limb are readily made to coincide when, in adjusting the limb to the level of the telescope, the arc is clamped to the axis.

The usual tangent movement to the telescope axis serves, of course, to bring the vertical limb to the proper elevation, as hereafter described.

A level on the under side of the telescope, with ground vial and scale, is indispensable in the use of the Solar attachment.

The divided arcs, vernier, and hour circle are all on silver plate, and are thus easily read and preserved from tarnishing.

172. Adjustments.—These pertain to the solar lenses and lines, the declination arc, the polar axis and hour arc, as follows:

(1) **The solar lenses and lines** are adjusted precisely like those of the ordinary Solar, the declination arm being first detached by removing the clamp and tangent-screws, and the conical centre with its two small screws, by which the arm is attached to the arc.

The adjuster, which is a short bar furnished with every instrument, is then substituted for the declination arm, the conical centre screwed into its place, at one end, and the clamp-screw into the other, being inserted through the hole left by the removal of the tangent-screw, thus securing the adjuster firmly to the arc.

The arm is then turned to the sun, as described in the article on the Solar Compass, and reversed by the opposite faces of the blocks upon the adjuster, until the image will remain in the centre of the equatorial lines.

(2) **The vernier of the declination arc** is adjusted by setting the vernier at zero, and then raising or lowering the telescope by the tangent-screw until the sun's image appears exactly between the equatorial lines.

Having the telescope axis clamped firmly, carefully revolve the arm until the image appears on the other plate.

If precisely between the lines, the adjustment is complete; if not, move the declination arm by its tangent-screw, until the image will come precisely between the lines on the two opposite plates; clamp the arm and remove the index error by loosening two screws that fasten the vernier; place the zeros of the vernier and limb in exact coincidence, tighten the screws, and the adjustment is finished.

(3) **To Adjust the Polar Axis.**—First level the instrument carefully by the long level of the telescope, using in the operation the tangent movement of the telescope axis in connection with the leveling screws of the parallel plates until the bubble will remain in the centre during a complete revolution of the instrument upon its axis.

Place the equatorial sights on the top of the blocks as closely as is practicable with the distinct view of a distant object; and having previously set the declination arm at zero, sight through the interval between the equatorial sights and the blocks at some definite point or object, the declination arm being placed over either pair of the capstan-head screws on the under side of the disk.

Keeping the declination arm upon the object with one hand, with the other turn the instrument half around on its axis, and sight upon the same object as before. If the sight strikes either above or below, move the two capstan-head screws immediately under the arm, loosening one and tightening the other as may be needed until half the error is removed.

Sight again and repeat the operation, if needed, until the sight will strike the same object in both positions of the instrument, when the adjustment of the axis in one direction will be complete.

Now turn the instrument at right angles, keeping the sight still upon the same object as before; if it strikes the same point when sighted through, the axis will be truly vertical in the second position of the instrument.

If not, bring the sight upon the same point by the other pair of capstan-head screws now under the declination arc, reverse as before, and continue the operation until the same object will keep in the sight in all positions, when the polar axis will be made precisely at right angles to the level and to the line of collimation.

It should here be noted that, as this is by far the most delicate and important adjustment of the solar attachment, it should be made with the greatest care, the bubble kept perfectly in the center and frequently inspected in the course of the operation.

(4) **To Adjust the Hour Arc.**—Whenever the instrument is set in the meridian, as will be hereafter described, the index of the hour arc should read apparent time.

If not, loosen the two flat-head screws on the top of the hour circle, and with the hand turn the circle around until it does, fasten the screws again, and the adjustment will be complete.

To obtain mean time, of course the correction of the equation for the given day, as given in the Nautical Almanac, must always be applied.

173. To Find the Latitude.—First level the instrument very carefully, using, as before, the level of the telescope until the bubble will remain in the center during a complete revolution of the instrument, the tangent movement of the telescope being used in connection with the leveling screws of the parallel plates, and the axis of the telescope firmly clamped.

Next clamp the vertical arc, so that its zero and that of its vernier coincide as near as may be, and then bring them into exact line by the tangent screw of the vernier.

Then, having the declination of the sun for 12 o'clock of the given day as affected by the meridianal refraction carefully set off upon the declination arc, note also the equation of time, and fifteen or twenty minutes before noon, the telescope being directed to the north, and the

object-end lowered until, by moving the instrument upon its spindle and the declination arc from side to side, the sun's image is brought nearly into position between the equatorial lines. Now bring the declination arc directly in line with the telescope, clamp the axis firmly, and with the tangent screw bring the image precisely between the lines and keep it there with the tangent screw, raising it as long as it runs below the lower equatorial line, or in other words, as long as the sun continues to rise in the heavens.

When the sun reaches the meridian, the image will remain stationary for an instant and then begin to rise on the plate.

The moment the image ceases to run below is of course apparent noon, when the index of the hour arc should indicate XII, and the latitude be determined by the reading of the vertical arc.

It must be remembered, however, that the angle through which the polar axis has moved in the operation just described is measured from the zenith instead of the horizon as in the ordinary solar, so that the angle read on the vertical limb is the complement of the latitude.

The latitude itself is readily found by subtracting this angle from 90°; thus, at Troy, the reading of the limb being found as above directed to be 47° 16′, the latitude will be 90° — 47° 16′ = 42° 44′.

It will be noticed that with this apparatus the latitude of any place can be most easily ascertained without any index error, as in the usual solar compass.

174. To Run Lines with the Solar Attachment.—Having set off the complement of the latitude of the place on the vertical arc, and the declination for the given day and hour, as in the solar, the instrument being also carefully leveled by the telescope bubble, set the horizontal limb at zero and clamp the plates together, loosen the lower clamp so that the transit moves easily

upon its lower socket, set the instrument approximately north and south, the object end of the telescope pointing to the north, turn the proper solar lens to the sun, and with one hand on the plates and the other on the revolving arm, move them from side to side until the sun's image is brought between the equatorial lines on the silver plate.

The lower clamp of the instrument should now be fastened and any further lateral movement be made by the tangent screw of the tripod. The necessary allowance being made for refraction, the telescope will be in the true meridian, and being unclamped, may be used like the sights of the ordinary solar compass, but with far greater accuracy and satisfaction in establishing meridian lines. Of course when the upper or vernier plate is unclamped from the limb, any angle read by the verniers is an angle from the meridian, and thus parallels of latitude or any other angles from the true meridian may be established as with the solar compass.

The bearing of the needle, when the telescope is on the meridian, will also give the declination of the needle at the point of observation.

The declination of the needle being set off, the needle kept then at zero, or "with the sun," lines may be run by the needle alone, when the sun is obscured.

The sun, however, must ever be regarded as the most reliable guide, and should, if possible, be taken at every station.

SECTION ~~VIII.~~ VII.

I. LEVELING.

175. The surface of water at rest is an example of a level surface. It is a surface whose points are all equidistant from the center of gravity of the earth.

The intersection of a vertical plane with a level surface is a **line of true level.** A line of true level is therefore a curve line whose center is at or near the center of the earth.

A line tangent to a line of true level is a **line of apparent level**

A line of apparnet level is a horizontal one passing through the point of observation.

A point is *above* or *below* another accordingly as it is at a greater or a less distance from the center of the earth.

The difference of the distances of two points from the center of the earth is called the **Difference of Level** of the points.

The operation of measuring the difference of level of points is called **Leveling.**

In leveling, two principal instruments are required: (1) an instrument capable of adjustment to a horizontal line; and (2) an instrument adapted to measuring vertical distances.

These instruments are called, respectively, a **Level** and a **Leveling Rod.**

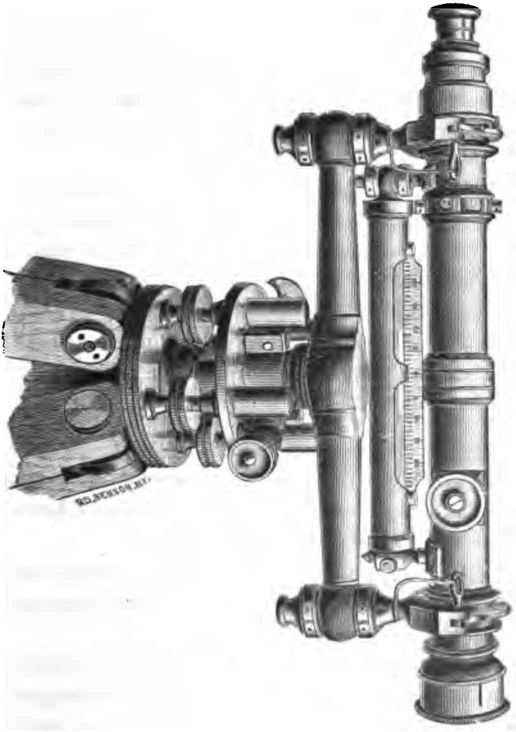


FIG. 116.

176. The Telescope has at each end a ring of bell-metal, turned very truly, and both of exactly the same diameter; by these it revolves in the wyes, or can be at pleasure clamped in any position when the clips of the wyes are brought down upon the rings, by pushing in the tapering-pins.

177. The Level or ground bubble tube is attached to the under side of the telescope, and furnished at the op-

posite ends with the usual movements, in both horizontal and vertical directions.

The aperture of the tube, through which the glass vial appears, is about five and one-fourth inches long, being crossed at the center by a small rib or bridge, which greatly strengthens the tube.

The level scale which extends over the whole length, is graduated into tenths of an inch, and figured at every fifth division, counting from zero at the center of the bridge; the scale is set close to the glass.

The level vial is made of thick glass tube, selected so as to have an even bore from end to end, and finely ground on its upper interior surface, that the run of the air-bubble may be uniform throughout its whole range.

178. The Wyes of the level are made large and strong, of the best bell-metal, and each has two nuts, both being adjustable with the ordinary steel pin.

The clips are brought down on the rings of the *tele*scope-tube by the Y pins, which are made tapering, so as to clamp the rings very firmly.

The clip of one of the wyes has a little pin projecting from it, which, entering a recess filed in the edge of the ring, ensures the vertical position of the level and cross-wire.

179. The Level-Bar is made round, of the best bell-metal, and shaped so as to possess the greatest strength in the parts most subject to sudden strains.

Connected with the level-bar is the head of the tripod-socket.

180. The Tripod-Socket is compound: the interior spindle upon which the whole instrument is supported, is made of steel, and nicely ground, so as to turn evenly and firmly in a hollow cylinder of bell-metal; this again

has its exterior surface fitted and ground to the main socket of the tripod-head.

The bronze cylinder is held upon the spindle by a washer and screw, the head of the last having a hole in its center, through which the string of the plump-bob is passed.

Adjustments.—The adjustments which require attention by the engineer in the use of the level, are the following:

181. To Adjust the Line of Collimation.—Set the tripod firmly; remove the Y pins from the clips, so as to allow the telescope to turn freely, clamp the instrument to the tripod-head, and, by the leveling and tangent-screws, bring either of the wires upon a clearly marked edge of some object, distant from one hundred to five hundred feet.

Then with the hand carefully turn the telescope half-way around, so that the same wire is compared with the object assumed.

Should it be found above or below, bring it half-way back by moving the capstan-head screws at right angles to it, remembering always the inverting property of the eye-piece; now bring the wire again upon the object, and repeat the first operation until it will reverse correctly.

Proceed in the same manner with the other wire until the adjustment is completed.

Should both wires be much out, it will be well to bring them nearly correct before either is entirely adjusted.

When this is effected, unscrew the covering of the eye-piece centering screws, and move each pair in succession with a small screw-driver, until the wires are brought into the center of the field of view.

The inverting property of the eye-piece does not affect this operation, and the screws are moved direct.

To test the correctness of the centering, revolve the telescope, and observe whether it appears to shift the position of an object.

Should any movement be perceived, the centering is not perfectly effected.

It may here be repeated, that in all telescopes the position and adjustment of the line of collimation depends upon that of the object-glass; and, therefore, that the movement of the eye-piece does not affect the adjustment of the wires in any respect.

When the centering has been once effected, it remains permanent, the cover being screwed on again to conceal and protect it from derangement at the hands of the curious or inexperienced operator.

182. To Adjust the Level-Bubble.—Clamp the instrument over either pair of leveling-screws, and bring the bubble into the center of the tube.

Now turn the telescope in the wyes, so as to bring the level-tube on either side of the center of the bar. Should the bubble run to the end, it would show that the vertical plane, passing through the center of the bubble, was not parallel to that drawn through the axis of the telescope rings.

To correct the error, bring the bubble entirely back, with the capstan-head screws, which are set in either side of the level-holder, placed usually at the object end of the tube.

Again bring the level-tube over the center of the bar, and the bubble to the center, turn the level to either side, and, if necessary, repeat the correction until the bubble will keep its position, when the tube is turned half an inch or more, to either side of the center of the bar.

The necessity for this operation arises from the fact that when the telescope is reversed end for end in the wyes in the other and principal adjustment of the bubble,

we are not certain of placing the level-tube in the same vertical plane; and therefore it would be almost impossible to effect the adjustment without a lateral correction.

Having now, in great measure, removed the preparatory difficulties, we proceed to make the level-tube parallel with the bearings of the Y rings.

To do this, bring the bubble into the center with the leveling-screws, and then, without jarring the instrument, take the telescope out of the wyes and reverse it end for end. Should the bubble run to either end, lower that end, or what is equivalent, raise the other by turning the small adjusting nuts, on one end of the level, until by estimation half the correction is made; again bring the bubble into the center and repeat the whole operation, until the reversion can be made without causing any change in the bubble.

It would be well to test the lateral adjustment, and make such correction as may be necessary in that, before the horizontal adjustment is entirely completed.

183. To Adjust the Wyes.—Having effected the previous adjustments, it remains now to describe that of the wyes, or, more precisely, that which brings the level into position at right angles to the vertical axis, so that the bubble will remain in the center during an entire revolution of the instrument.

To do this, bring the level-tube directly over the center of the bar, and clamp the telescope firmly in the wyes, placing it as before, over two of the leveling-screws, unclamp the socket, level the bubble, and turn the instrument half-way around, so that the level-bar may occupy the same position with respect to the leveling-screws beneath.

Should the bubble run to either end, bring it half-way back by the Y nuts on either end of the bar; now move the telescope over the other set of leveling-screws, bring the bubble again into the center, and proceed precisely as

above described, changing to each pair of screws, successively, until the adjustment is very nearly perfected, when it may be completed over a single pair.

The object of this approximate adjustment is to bring the upper parallel plate of the tripod-head into a position as nearly horizontal as possible, in order that no essential error may arise, in case the level. when reversed, is not brought precisely to its former situation. When the level has been thus completely adjusted, if the instrument is properly made, and the sockets well fitted to each other and to the tripod-head, the bubble will reverse over each pair of screws in any position.

Should the engineer be unable to make it perform correctly, he should examine the outside socket carefully to see that it sets securely in the main socket, and also notice that the clamp does not bear upon the ring which it encircles.

When these are correct, and the error is still manifested, it will, probably, be in the imperfection of the interior spindle.

After the adjustments of the level have been effected, and the bubble remains in the center, in any position of the socket, the engineer should turn the telescope in the wyes until the pin on the clip of the wye will enter the little recess in the ring to which it is fitted, and by which is ensured the vertical position of the spirit-level and cross-wire.

When the pin is in its place, the vertical wire may be applied to the edge of a building, and in case it should not be parallel with it, two of the cross-wire screws that are at right angles with each other may be loosened, and by the screws outside, the cross-wire ring turned until the wire is vertical. The line of collimation must then be corrected again, and the adjustments of the level will be complete.

184. To Use the Level.—Set the legs firmly into the ground. The bubble should then be brought over each pair of leveling-screws in succession and leveled in each position until the bubble will remain in the center of the tube as the telescope is turned upon its axis in any direction.

Bring the wires precisely in focus and the object distinctly in view, so that all errors of parallax may be avoided.

This error is seen when the eye is moved to either side of the center of the eye-piece of a telescope in which the foci of the object and eye-glasses are not brought precisely upon the cross-wires and object; in such a case the wires will appear to move over the surface, and the observation will be liable to inaccuracy.

In all instances, the wires and object should be brought into view so perfectly, that the cross-wires will appear to be fastened to the surface, and will remain in that position however the eye is moved.

185. Leveling Rods.—The various leveling rods used by American engineers are made in two or more parts, which slide from each other as they are extended in use.

They are of different patterns, distinguished as the New York, Philadelphia, and Boston or Yankee rods. The New York and Boston rods are divided into feet, tenths and hundredths, and read by verniers to thousandths.

186. The New York Rod shown in the engraving as cut in two, so that the ends may be exhibited, is made of maple, in two pieces like the former, but sliding one from the other, the same end being always held on the ground, and the graduations starting from that point.

The graduations are made to tenths and hundredths of a foot, the tenth figures being black, and the feet marked with a large red figure.

The front surface, on which the target moves, reads to about six and a half feet; when a greater hight is required,

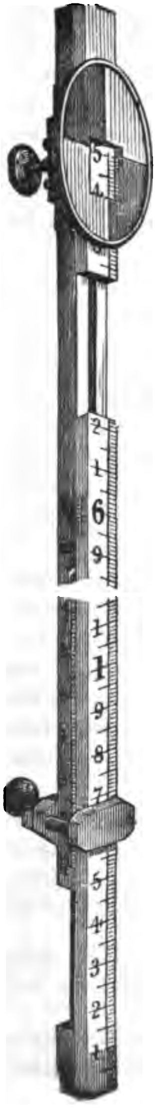


FIG. 117.

the horizontal line of the target is fixed at the highest graduation, and the upper half of the rod, carrying the target, is moved out of the lower, the reading being now obtained by a vernier on the graduated side, up to an elevation of twelve feet.

The target is round, made of thick sheet brass, having, to strengthen it still more, a raised rim, which also protects the paint from being defaced.

The target moves easily on the rod, being kept in any position by the friction of the two flat plates of brass which are pressed against two alternate sides, by small spiral springs, working in little thimbles attached to the band which surrounds the rod.

There is also a clamp-screw on the back, by which it may be securely fastened to any part of the rod.

The face of the target is divided into quadrants, by horizontal and vertical diameters, which are also the boundaries of the alternate colors with which it is painted.

The colors usually preferred are white and red; sometimes white and black.

The opening in the face of the target is a little more than a tenth of a foot long, so that in any position a tenth, or a foot figure, can be seen on the surface of the rod.

The right edge of the opening is chamfered, and divided into ten equal spaces, corresponding with nine-hundredths on the rod; the divisions start from the horizontal line which separates the colors of the face.

187. Problem. *To find the deviation of a line of apparent, from one of true level.*

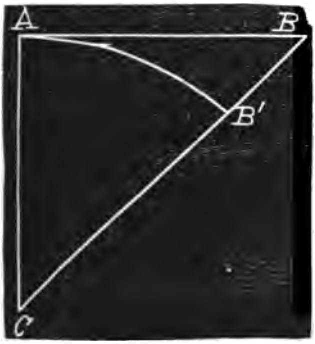


FIG. 118.

Solution.—Let C be the center of the earth. Let AB be a line of apparent level, and AB' a line of true level.

The distance $B'B$ for any measured distance AB' is the deviation sought.

From Geometry,

$$B'A^2 = B'B\,(B'B + 2B'C),$$

whence, $B'B = \dfrac{B'A^2}{B'B + 2B'C}$.

For ordinary distances, the length of the arc $B'A$ will not differ sensibly from that of the tangent AB, nor will the diameter $2B'C$ differ appreciably from the secant $B'B + 2B'C$.

Making these substitutions in the above formula, we have

$$B'B = \frac{B'A^2}{2B'C}.$$

That is, *The deviation of the apparent from the true level, between two points, is equal to the square of the distance between the points, divided by the diameter of the earth.*

Cor.—*The deviations for different distances are proportional to the squares of the distances.*

Calling the diameter of the earth 7920 miles, and supposing AB' to be one mile, we find

$$B'B = 0.000126 \text{ miles} = 0.665 \text{ feet} = 7.98 \text{ inches}$$

or 8 inches, nearly.

For m miles, we have $B'B = 8m^2$ inches.

For n chains, $B'B = \dfrac{1}{800} n^2$ inches $= 0.00125\, n^2$ inches.

SCH. 1.—The effect of refraction of light is apparently to increase the difference between true and apparent level. Accordingly, for considerable distances, the correction for curvature, as above found, is sometimes diminished by about one-sixth of itself.

SCH. 2.—If in leveling, the instrument is placed midway between the points whose difference of level is required, the difference of true level of the points is the same as their difference of apparent level.

The student having a level and a rod for use in practice may now solve the following problems in the field.

188. Prob. 1. *To find the difference of level of two points.*

CASE 1.—*When the difference of level may be found by one setting of the instrument.*

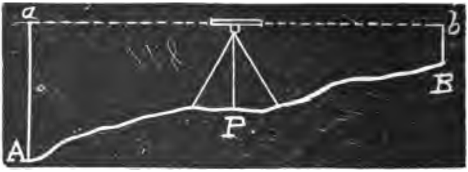


FIG. 119.

SUGGESTIONS.—Suppose A and B to be the points.

Set up the instrument, as at P, about equidistant from A and B, though not necessarily on line between them.

Have an assistant hold the leveling rod at A. Read the hight Aa. In the same manner take the hight Bb.

The difference between the hights Aa and Bb is the difference of level sought.

CASE 2.—*When the difference of level cannot be found by one setting of the instrument.*

SUGGESTIONS.—Suppose A and D to be the two points.

Set up the level at a convenient point, as P. Take the readings Aa' and Bb as in Case 1. Remove to a point, as Q, and take the readings Bb' and Cc, and so on to D.

The difference between the sum

$$Aa' + Bb' + Cc', \text{ etc.,}$$

and the sum $Bb + Cc + Dd$, etc.,

is the difference of level sought.

A reading taken in the direction in which the work is proceeding is called a **Fore-Sight**, and in the opposite direction, a **Back-Sight**.

A convenient form of Field Notes in cases like the above consists of three columns, (1) for stations, (2) for back-sights, and (3) for foresights, as shown in the following

Example.—Required the difference of level between the points A and E, from the accompanying notes:

Sta.	Back-Sights	Fore-Sights.
A	3.28	5.3
B	2.14	7.15
C	3.25	8.5
D	4.7	3.45
E	5.83	2.75

Which point is the higher?

FIG. 120.

It is common in operations of leveling to refer the hights of points to an imaginary level plane called a **Datum Plane**. This plane may be assumed to pass through any point at pleasure. It is, however, customarily referred to as a plane at some convenient distance below a fixed point called a **Bench Mark** or simply a **Bench**, and indicated in the notes by the letters B. M.

In order to avoid negative hights, the datum plane is usually assumed at such a distance below the bench, as to pass beneath the lowest station likely to occur in any part of the survey for which it is used.

It is customary to establish bench marks at convenient points along a line of levels, by which the work may be reviewed as occasion requires, or at which it may be resumed after temporary cessation.

The stations are usually designated by numbers beginning with zero.

The distances between stations are measured or not, as the nature of the work may require.

Distances in leveling are usually given in feet.

189. Prob. 2. *To find the hights above a datum plane, of several stations on a given line.*

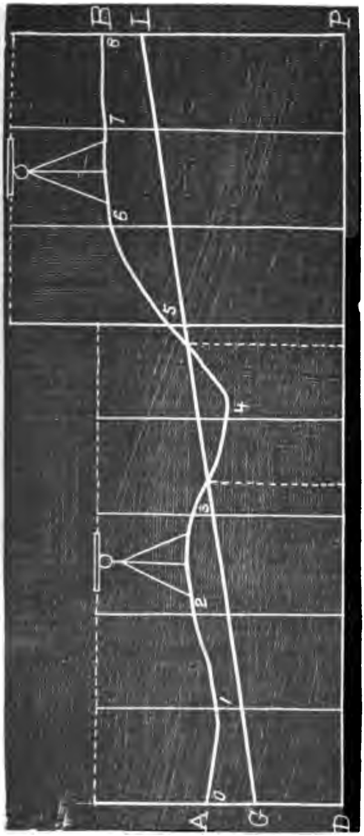


FIG. 121.

SUGGESTIONS. — Let AB be the given line and DP the datum plane assumed at any convenient distance, say 10 ft., below a bench near A.

Set up the level at some convenient point, for example between stations 2 and 3.

Take the reading of the rod upon the bench and add it to the assumed hight of the bench above the datum. The sum is the hight of the instrument.

Take the readings upon stations 0, 1, 2, 3, 4 and 5 in succession, and subtract each from the hight of the instrument. The remainders are the hights, respectively, of those stations above the datum.

Carry the instrument forward to another position, as between stations 6 and 7.

Take the reading of the rod a second time on station 5 and add it to the hight of station 5 as before found. The

sum is the new hight of instrument, with which proceed as before.

A point used as station 5, as above indicated, is called a **Turning Point.** In practice, a bench is often adopted as a turning point.

The reading of the rod upon a turning point or bench-mark is usually taken with somewhat greater precision than upon other points.

A reading upon a bench or turning point is added to the hight of the point above the datum in finding hight of instrument; and a reading upon any point is subtracted from the hight of instrument in finding the hight of the point.

Accordingly, an observation for the former is called a **Plus Sight,** denoted by + S, and for the latter, a **Minus Sight,** denoted by — S.

The hight of instrument is denoted by H. In., and the hight of any point above the datum, by H.

The following is an example of the notes made in solving the above problem:

Sta.	+ S.	H. In.	— S.	H.	Remarks.
B. M.	3.426	13.426		10.000	A stone 20 ft. S. E. of 0.
0			5.45	7.976	
1			7.30	6.126	
2			5.35	8.08	
3			5.40	8.03	
4			6.23	6.20	
5	8.274		3.76	9.666	
6		17.940	5.25	12.69	
7			5.10	12.84	
8			5.00	12.94	

190. Prob. 3. *To find the cut or fill, to grade, at points between two given points.*

SUGGESTIONS.—Let A and B, (Fig. 121), denote the given points. Beginning at A, for example, measure the

distance AB, at the same time marking it off into convenient divisions of equal length, as 33 ft., 50 ft., 66 ft., or 100 ft., for example, by driving pegs down to the surface of the ground. The last division will usually be fractional. Number the divisions, 0, 1, 2, 3, etc., beginning at A. A point of any fractional division is conveniently denoted by the number of the last preceding full division with the number of feet in the fraction written at the right. Thus, with stations 100 ft. apart, a point 720 feet from the beginning would be denoted by 7^{20}.

Find now, (Prob. 2), the hights of the points, 0, 1, 2, 3, etc., above some convenient datum.

For illustration, suppose the hights to be as given in the above Table (Prob. 2). Also suppose the hight of the grade line at A to be 5 ft.; and at B, 9 ft.

The distance from A to B consisting of 8 equal parts, say of 50 ft., we should then have

$$(9 \text{ ft.} - 5 \text{ ft.}) \div 8 = 0.5 \text{ ft.} = \text{rise per station.}$$

Beginning at A or station 0, we have

$$\begin{array}{rcrl}
7.98 - 5. & = & 2.98 & = \text{cut at } 0 \\
6.13 - 5.50 & = & 0.63 & = \text{ “ “ } 1 \\
8.08 - 6.00 & = & 2.08 & = \text{ “ “ } 2 \\
8.03 - 6.50 & = & 1.53 & = \text{ “ “ } 3 \\
6.20 - 7.00 & = & -0.98 & = \text{fill “ } 4 \\
9.67 - 7.50 & = & 2.17 & = \text{cut “ } 5 \\
\text{etc.} & & \text{etc.} & \quad \text{etc.}
\end{array}$$

Observe that we subtract the hight of the grade line at each station, from the hight of the station; and since we have here proceeded from lower to higher points of the grade, we have *added* the rise of the grade per station to the hight of the grade at the last preceding station.

Let the student find the cut at each station, beginning at B, all other things being as above.

Again, supposing the hights of the stations to be as above, let the student find the depths of cut and fill under

the supposition that the hight of the grade at A is 6 ft., and at B, 8.4 ft.

191. **Drawing Profile.**—Fig. 121 represents a section formed by a vertical plane passing through the points A and B, and meeting the datum plane in the line DP. The irregular line AB represents the intersection of the vertical plane with the surface of the ground, and is called the **Profile.**

The manner of drawing the profile is as follows:

Draw a horizontal line to represent the datum line, on which lay off to a convenient scale the distance between the stations. In Fig. 121 the scale employed for the horizontal measurements is 100 ft. to the inch.

At the points of division of the datum line, erect perpendiculars, on which lay off the surface hights of the several stations, in their order, but to a scale usually ten times greater than that used for the horizontal distances. Thus, in Fig. 121 the vertical measurements are laid down to the scale of 10 ft. to the inch.

A line drawn through the points thus located forms the profile.

The use of a larger scale in drawing the vertical distances serves to render the irregularities of the surface more apparent to the eye than they would be if drawn to the same scale with the horizontal measurements.

The grade line is drawn through any two points at the proper distances from the datum line. The position and inclination of the grade line depend upon certain conditions required to be fulfilled by the work, such as the flowage of water, ease of travel, economy of construction, etc.

In road work the grade is often adopted with reference to an equalization of "cut" and "fill", so that the material furnished by excavations shall make the embankments. The required position of the grade line, in order to fulfill this condition most advantageously, is conven-

iently got by stretching a thread across the profile, varying the position of the thread until the areas intercepted by it and the profile on opposite sides appear to be equal.

EXERCISES.

192. 1. Find depths of cut or fill, and draw profile and grade line from the following notes:

Sta.	+ S.	H. In.	— S.	H.	H. Gr.	Cut.	Fill.
0		14.26	4.26	10.00	8.00		
2			6.30				
2			8.45				
3	4.12		3.23				
3^{40}		15.15	8.20				
4			4.63				
5			5.53				
5^{25}			5.75		9.575		

Distance between stations, 100 ft.

2—5. Examples made by the student in the "Field."

II. DRAINAGE SURVEYING.

193. Of the many applications of leveling, the most common, perhaps, in the province of the ordinary surveyor, is that relating to drainage. Almost every neighborhood offers occasions for work of this kind. Vast tracts of swamp and marsh covered with stagnant water occur in nearly every county and township of the State; and statutory provisions are made for drainage "whenever the same shall be conducive to the public health, convenience or welfare." Officers called, in Michigan, **Drain Commissioners** are provided to have charge of the work.

In addition to public provisions of this general kind, account is also to be taken of the fact, almost universally

accepted among intelligent men, that a very large proportion of land ordinarily classed as dry is improved by drainage.

The Committee on Draining in their Report to the State Agricultural Society of New York, said: "There is not one farm out of every seventy-five in this State but needs draining—yes, much draining—to bring it into high cultivation."

194. Drains are of two forms: the **Open Drain** or **Ditch**, and the **Under Drain.**

The former is adapted to the case of water lying upon the surface of the ground, and the latter to water underlying the surface. Under drains are usually discharged into open drains, which are thus rendered an essential auxiliary to thorough drainage.

195. **Locating Drains.**—The Michigan Drain Law provides that "before the Commissioner takes any action toward locating or establishing any drain, there shall be filed with him an application signed by not less than five freeholders of the township or townships in which such drain or the lands to be drained thereby may be situated, giving a general description of the beginning, the route and the terminus thereof."

"Upon the filing of such application, the Commissioner authorized to act thereon shall, as soon as practicable, proceed personally to examine the route of the proposed drain, and if, in his opinion, it is necessary and conducive to the public health, convenience or welfare that the application should be granted, he shall, as a means of determining the practicability thereof, make, or cause to be made by a competent surveyor, a survey and measurement of the line of the proposed drain."

196. **Making the Survey.**—This will be, in the first place, a careful reconnoissance of the locality respecting the general "lay of the land," natural water courses, etc. In this will be determined the proper commencement,

the engineer or principal surveyor, two men are required —a rod-man, and an ax-man to make and drive pegs.

The pegs should be driven down even with the surface of the ground and at such a distance from the stakes marking the stations that they may be used without disturbance in excavating. Some practice driving them, say six inches, in front of the stakes; others set them opposite and at such a uniform distance from the record stakes as not to be disturbed by the digging.

Bench marks should be made at convenient distances, for example at every tenth station, and far enough from the line not to be disturbed.

198. Platting.—The field work having been completed, the next thing is to make a plat of the line and also of the sections or tracts of land which will be affected by the drain, writing the owner's name and number of acres on each. On some convenient part of the plat, the courses and their corresponding distances should be noted, also the number of linear feet of drain on each separate tract.

Next comes the drawing of the profile. This is most conveniently done by use of paper, called **Profile** paper, prepared specially for the purpose. Taking a piece of the proper width and of sufficient length to contain also the title and necessary explanatory notes, at the left hand, we begin on the edge next to us and write the numbers of all the ~~sections~~ in their order toward the right, upon the vertical lines. We then mark with the point of a sharp pencil the point of elevation of each station as taken from the column of elevations in the level notes. Connecting the points thus marked, by an ink line, we have the profile of the surface of the ground on the line of the drain. We then take a black thread and stretch it on the profile between the points assumed *as grade*, at the first and the last station. From this inspection, it will be seen whether it is necessary or desirable to introduce one

or more changes of grade between the extreme points in order to avoid objectionable cuts.

Having determined the situation of the grade lines, we then draw them in their places, preferably with red ink.

Under the grade lines and upon the vertical lines of the several stations should be written in red ink the elevations of the grade and below that, in black ink, the elevations of the surface. In a similar manner, above the profile may be written first, in red ink, the depths of the cuts, and, second, the widths of the ditch at bottom and top.

The names of the land owners through whose land the ditch passes with the number of linear feet on each may be conveniently written upon the datum line.

199. Depth and Width.—The depth of a drain obviously depends upon the situation of the grade line with respect to the surface. In adjusting the grade line it is more important to guard against the drain being too shallow rather than too deep; most open drains are too shallow.

Again, it should be taken into account, if the drain is to run through soft marshes and hard ridges, that the soft ground, on the withdrawal of the water, will settle; and so the drain may need to be dug deeper in some places than would otherwise be necessary.

The necessary width of a drain of given depth and grade depends upon the quantity of water it is required to discharge in a given time.

The width at the top is determined from the width at the bottom and the slope or inclination given the sides, which is usually from one to one and one-half feet on the horizontal to each foot in the depth.

200. Quantity of Discharge.—The amount of water which a drain may discharge in a given time obviously depends upon the area of the water-way or cross-section of the drain and the velocity of the stream.

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Thus, denoting by Q the quantity of discharge, by a the area of the water-way, and by v the mean velocity of discharge, we should have

$$Q = av \quad (1).$$

As an approximate formula for computing the mean velocity of water flowing in an open canal of uniform cross-section and fall, Trautwine gives the formula

$$V = \left\{ \frac{af \times 8975}{p} \right\}^{\frac{1}{2}} - .1089 \quad (2)$$

in which V = mean velocity in feet per second, a = area of water-way in square feet, f = fall in feet per foot, and p = wet perimeter or the water border of the channel.

REMARK.—In applying the above formula, it is customary to use 9000 for 8975 and .11 for .1089.

Example.—Required the velocity and the capacity of a drain 5 ft. wide at the bottom, the sides having a slope of 1 to 1, depth of water 3 ft., and the fall 2 ft. to 1000 ft.

Solution.—Width at top = 5 ft. + 2 × 3 ft. = 11 ft.
Area of water-way = $1\frac{1}{2}$ (11 ft. + 5 ft.) = 24 sq. ft.
Wet perimeter = 5 ft. + $6\sqrt{2}$ ft. = 13.5 ft.
Fall per foot = 0.002 ft.

Substituting in (2), $V = \left\{ \frac{24 \times 0.002 \times 9000}{13.5} \right\}^{\frac{1}{2}} - 0.11 = 5.55.$

Substituting in (1), $Q = 24 \times 5.55 = 133.2$ cu. ft. per second, or 11,508,480 cu. ft. per day.

Trautwine gives also the following formula, with the remark that it is applicable also to sewers:

$$V = \left\{ \frac{a}{p} \times 2F \right\}^{\frac{1}{2}} \quad (3)$$

in which a and p are as above described, and F is the fall in feet per mile.

REMARK.—In connection with the above formulas, as well as with others of similar import, Trautwine repeats again and again the caution that they are to be regarded only as approximately true.

By the kindness of Mr. B. F, Welles, C.E., of Marshall, Mich., we are permitted to give Table XII, showing approximately the number of acres served by drains having bottom widths of 1 to 10 ft., with side slopes of 1 to 1, and various rates of fall per station, on the supposition of 1 inch rain-fall in 24 hours, one-half of which reaches the drain.

201. Amount of Rainfall.—All calculations of requisite capacity of drains must be based upon the probable amount or number of inches of rainfall in a given time. The soil, however, acts as a reservoir up to the point of saturation, depending upon its texture, keeping from the drains altogether a portion of the rainfall, which passes off by evaporation or is absorbed by plants.

The average annual rainfall in Michigan, Indiana, Illinois and Missouri is about 35 inches. In Ohio, for a period of ten years, it was reported to be 37.86 inches.

In the matter of rainfall in Michigan, we are indebted to Prof. Carpenter for the following data:

"By consulting the State Agricultural College meteorological reports for the five years of '72-3-4-5-6, we find that during these five years, in the months of March, April, May, June, July, August, September, October, and November, there have been a total number of 304 rainfalls, in nineteen of which more than one inch of water fell during twenty-four hours. In nine of these rainfalls more than one and one-half inches fell during twenty-four hours, and in four rainfalls the amount exceeded two inches in depth in twenty-four hours. Expressing this in percentage, six and three-tenths per cent. of the number of showers gave a rainfall exceeding one inch in depth, while but three per cent. gave a rainfall exceeding one and one-half inches.

"By a further consultation of the meteorological records we learn that, although these large showers in which the rainfall exceeds one inch occur comparatively seldom (on the average only four times a year), yet they bring with them twenty-eight per cent. of our total rainfall during that period, and consequently they must be fully provided for in any works for thorough drainage. The following table is compiled from the meteorological records kept at the college, and shows the comparative depth and number of showers from the months of March to December for five years. The last column shows the total percentage of rainfall in all the showers of a given depth. The last

column but one shows the total percentage of the number of showers compared with the whole number. Although this table is not extended sufficiently far back to give very accurate results, it is thought (since one year's rainfall does not differ greatly from that of another year) to be sufficiently reliable to produce data for any ordinary case of farm drainage in this part of the United States. In cases where greater accuracy is needed it will probably be best to conduct a series of independent observations for a longer time.

TABLE OF SHOWERS FROM MARCH TO DECEMBER.

Depth of Rainfall in Inches.	Number of Showers.						Percentage of Total.	
	1872.	1873.	1874.	1875	1876	Total	No. of Showers.	Am't of Rainfall.
.00 to .25	19	40	28	35	43	165	54.2	17
.25 to .50	20	14	13	9	11	67	22.0	21
.50 to .75	6	8	6	10	5	35	11.5	21
.75 to 1 00	2	6	5	2	3	18	06.0	13
1.00 to 1.25	......	......	......	......	2	2	00.7	2
1.25 to 1.50	3	2	......	......	3	8	02 6	9
1.50 to 1.75	1	......	1	1	......	3	01.0	4
1.75 to 2.00	......	1	......	1	......	2	00.7	3
2.00 to 2.25	......	......	......	......	......	......	......	......
2.25 to 2 50	......	1	......	......	......	1	00.3	2
2.50 to 2.75	1	......	......	......	......	1	00.3	2
2.75 to 3.00	......	......	......	......	......	......	......	......
3.00 to 3.25	......	......	......	1	1	2	00.7	6
Totals	......	......	......	......	......	304	100.00	100

The amount of discharge of drains as compared with the rainfall is usually estimated at about 50 per cent. So that in order to produce thorough drainage it is necessary to assume that the capacity of the drains shall be sufficient to carry off during twenty-four hours one-half the water that fell the previous twenty-four hours. The probability of the rainfall in any day exceeding one inch is so slight that we shall be safe in assuming as the necessary carrying capacity of drains one-half of 3,630 cu. ft., or 1,815 cu. ft. of water for each acre drained."

202. Under Drains are formed in various ways; sometimes of brush, rails or loose stone trenched in, sometimes of tubes made of logs or of iron, sometimes of plank or of brick or stone laid in cement, and again of earthern tubes of which there are various forms, called **Tiles**.

The prevailing method of underdrainage for agricultural purposes consists in the use of cylindrical tiles

which are made of different sizes and usually about a foot in length.

It is of this form of under drain, only, that we propose to write briefly.

203. Surveying for Under Drains.—Very much of what has been said upon surveying for the ditch or open drain applies also to the tile drain. The same preliminary inspection is required to determine the best location of the outlet and the proper directions of trunk and branch lines. Indications as to source of water, whether from springs on the premises or on lands situated above, whether from rain fall, merely, upon the particular tract or also as flowing off from neighboring areas; the directions of slopes, whether of surface or of underlying strata; the character of the soil, etc., all have to be carefully observed and their bearing duly considered.

204. Location of Drains.—As above intimated, any well conducted survey for under drains contemplates the execution of a *system* of drains working together and depending upon each other, This will include usually a principal drain, called a **Main** and lateral drains, called **Minors**, which discharge into the main. In an extended system, auxiliary mains called **Sub Mains** are also introduced.

Since it is the direct office of the minors to remove the surplus water from the ground, it is of the first importance that they be so located as successfully to perform their functions. To do this requires the exercise of careful judgment on the part of the engineer, respecting the proper directions of the minors and also their distances from each other. Equal care is requisite also in regard to the location of the main, so as properly to receive the water from the minors and discharge it at the principal outlet.

As a rule, the main should be located at the foot of the regular slopes or, either in straight or curved lines, along

the valleys of the field; and, in general, the minors should run directly down the slopes, discharging themselves obliquely into the main.

Cases, however, will sometimes occur that require departure from the above rules, but these are to be regarded as "exceptions which prove the rule."

The distances of the minors from each other will be governed largely by the character of the soil as to permeability, and to some extent by the depth of the drains. In a porous soil, as a general rule, the deeper the drain the further it will draw.

Through the kindness of Prof. Carpenter, I am enabled to illustrate the location of drains by referring the reader to Fig. 122, which is a sketch map not drawn to scale, of the drains constructed in the west part of the grounds of the Agricultural College at Lansing, Mich.

Describing the system, Prof. Carpenter says "the outlet is protected by a stone wall at *O*, the main drain of four-inch tile passes up the valley, changing direction with it to *E*, a distance of 33 rods, having an average fall of two inches per rod. At this point the contour of the ground required an abrupt bend, and a silt basin, (see Fig. 124), was sunk at this point, principally to lessen friction from the abrupt bend, also to allow the workings of the drain to be examined, and retain silt from drains 12 and 13. From the silt basin a sub-main, marked 12 in the cut, was run, which received a number of laterals making an angle of forty-five degrees with the main, each terminating at the summit of the ridge.

From the main, *OD*, a series of laterals, marked 1, 2, 3, 4, etc., were run directly up the slope; the angles made with the main drain being likely to make back currents, they were connected with the main by curves, as shown in the plan. Between drains 5 and the ends of 1, 2, 3 and 4, was a terrace, which it was impossible to pass through, and drains 5 and 6 were put in running diagonally down

the slope. The sizes of the tile used were, for main drain, four-inch; for drain No. 12, three-inch; for lower part of 13, three-inch; for the remainder of the drains, two-inch;

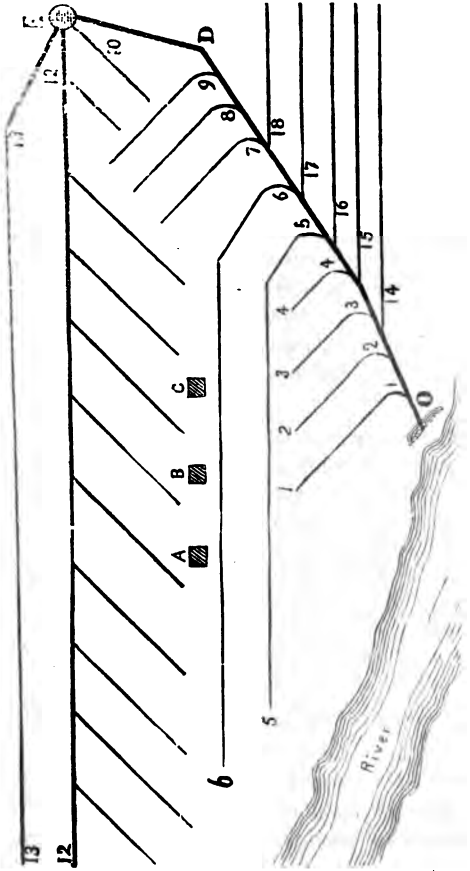

FIG. 122.

area drained about twelve acres. The fall of the drain marked 12 was two inches per rod. The fall of the short laterals was often as much as four to six inches per rod.

The foregoing example was chosen, not for its perfectness of detail, but as one which, although on a small scale, would show the advantages and disadvantages of applying our principle of location.

Drains should run directly down the slope. Thus in Fig. 122, drains marked 1, 2, 3 and 4 run directly down the slope; each has then the maximum fall to be obtained on that ground; that is, they strike the main drain at a point near the outlet and are short as compared with drains marked 5 and 6.

Again, drain 5, being on lower ground than 6, receives water principally from one side, viz.: toward 6; on the other hand, each of the drains marked 1, 2, 3 and 4 receives water equally from both sides, consequently the water has to travel through the ground nearly twice as far, on the average, to reach drains 5 and 6 as to reach drains 1, 2, 3 and 4."

The above example illustrates the general principles relating to the development of a system of under drains. Of course, another case precisely like it could not be expected to occur. Circumstances are infinitely varied. Every situation is a new one and must be treated on its own merits. None but the most general instruction can be given in any treatise on this point. About as practital a suggestion as may be afforded the student is, *Go into the field and there mix plenty of brains with your work.*

205. Running the Lines.—Having settled the question of the proper system of drains to be adopted, the next thing to be done is to lay out and measure the lines. This is perhaps most conveniently done in the case of under drains, by beginning at the outlet measuring and staking out, first, the main lines of the system and then the branches.

A distance of 50 ft. between stations is a convenient one in tile draining In some instances, as where the fall is very slight, a less distance may be desirable; in

others a greater one may give equally good results. In addition to the stakes driven at the uniform distances of the stations, a stake should mark the entrance of each minor, and the distance to it should be entered in the notes, in the usual manner. Such stakes mark the points of beginning in running out the minors.

To facilitate examinations for "faults," the points of entrance of the branches in the main drain should be established by witnesses.

206. **Taking the Levels.**—This is done in the same manner as in the case of open drains, but, perhaps, with a somewhat greater degree of care and precision. The point assumed for the outlet must, of course, be sufficiently low to receive all the water of the field; and at the same time the outlet ought to be high enough to be at all times above the back water of the stream into which the drain empties. A drain is of little more use under a violation of the latter condition than under a disregard of the former.

In assuming the grade, due consideration must be had for proper depth consistently with required fall.

The depth of an under drain should be, at the least, two feet. All the better if three or four feet in most soils.

Henry F. French, author of Farm Drainage, says: "We cannot, however, against the overwhelming weight of authority, and against the reasons for deeper drainage, which to us seem so satisfactory, conclude that even three feet is, in general, deep enough for under drains. Three-foot drains will produce striking results on almost any wet lands, but four-foot drains will be more secure and durable, will give wider feeding-ground to the roots, better filter percolating water, warm and dry the land earlier in Spring, furnish a larger reservoir for heavy rains, and, indeed, more effectually perform every office of drains."

Accordingly, the rule should be to approximate as closely as possible to what are thus regarded as desirable

depths, admitting depths very much below the standard, only when we *must*, in order to have any drains at all.

Upon the question of necessary amount of fall, with which the surveyor is so often confronted in connection with the requirement of desirable depths, it is to be observed in the first place that large, deep streams require less fall than small ones; and, again, the form and the condition of the channel have much to do with the movement of water.

"It has been found in practice that a water-course thirty feet wide and six feet deep will flow at the rate of one mile per hour, with a fall of no more than *six inches per mile.*"*

Examples are cited of successful operation of drains with three inches or even two and one-half inches fall to one hundred feet.

These, however, are to be regarded, probably, as exceptional cases or as presenting, perhaps, the lowest limit that, even under the most favorable conditions of ordinary drainage, ought to be attempted.

A very excellent authority says, "as to the fall necessary in tile draining, I consider one foot in one hundred yards the least fall to work upon with safety."*

The above considerations will be perceived to bear upon the situation of the grade line, in order, on the one hand, to avoid too shallow drains, and on the other, to secure the requisite fall for the proper movement of the water.

Changes of grade, though undesirable, are admissible when not easily avoided. If possible, the heaviest grades should be in the direction of the outlet. When this cannot be, it may be desirable to introduce silt-wells at points of any considerable change of grade.

The hights of the outlets of minor drains into the main are usually the hights of grade in the main drain for the same points.

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* Farm Drainage.

207. Constructing the Drain.—The principal point is the method of opening the trench and laying the tiles on the grade line.

To do this systematically requires a measuring rod six or eight feet in length, divided into feet, tenths and hundredths of feet, the larger divisions being numbered upward, as in the ordinary leveling rod. A cord or wire, also, is needed, which is to be stretched above the line of the drain and adjusted to a position parallel to the grade line. This is done by inverting the measuring rod on the grade peg and bringing the cord or wire to the division of the rod indicating the cut at that point. The cord is thus placed at the full length of the measuring rod from the grade line or intended bottom of the trench.

The cord may be held each fifty or one hundred feet by two slats, each about seven feet long, and movable about a bolt passing through a little distance from the upper end. These are called **Shears**. The cord or wire is prevented from slipping by a couple of turns, and is tied to a stake eight or ten feet from the shears.

Another device consists in the use of stakes or posts driven on opposite sides of the ditch, and connected with a cross-bar arranged so that either end may be raised or lowered to a level, and fastened to the posts by a clamp and thumb-screw. The cross-bars being adjusted to the proper hight as above described, the cord or wire is drawn tightly across them, directly over the center line of the drain.

Again, single stakes or posts driven on one side of the ditch, each having attached at right angles an arm which may be raised or lowered, and secured in place by a clamp and screw, are sometimes employed.

By such means as the above, the ditch is readily dug to just the proper depth, and the tile laid to grade with exceeding accuracy and with great rapidity. The proper distance from the top of the tile to the cord may be indicated by an arm attached to the measuring rod.

208. Size of Tile.—The size of tile required in a given case will depend upon the quantity of water to be removed and the fall available to remove it. Formulas are given in works upon Hydraulics, to express the velocity and discharge of water flowing in pipes, but the conditions are so different in case of tiles that such formulas, at best, give only the most roughly approximate results.

Thus, for example, the following, which is Poncelet's formula:

$$V = 48 \left\{ \frac{D \times H}{L + 54\,D} \right\}^{\frac{1}{2}}$$

in which, $V =$ approximate velocity in feet per second, $D =$ diameter of pipe in feet, $H =$ total head in feet, and $L =$ total length of pipe in feet.

Having found the velocity, we have

Discharge in cu. ft. = vel. × cross-section of pipe.

Tables XII and XIII are used for the above purpose, the latter quite extensively by drainage engineers in this State, and has been found to give good results.

As regards size of tile for main and sub-main drains a good authority says, "*that* can be regulated only by the person in charge of the drainage at any particular place, after seeing the land opened up and the minor drains discharging. As a rule a circular pipe of three inches internal diameter will discharge the *ordinary* drainage of six statute acres, and give sufficient space for the circulation of the air."

This estimate is based upon an amount of annual rainfall of from twenty-six to thirty inches, which differs but slightly from that of Michigan and adjoining States.

In addition to the above, it may be remarked that if the fall in the main is slight, a larger size of tile would be required than if the fall was considerable.

And, again, in order to provide suitably for the accumulation of water which occurs toward the outlet, a

larger size may be there required than that used in the upper part of the main.

209. Protection at Outlets.—The outlets of underdrains should be protected by some construction, as shown in the annexed cut, to prevent the earth from falling down in front of the drain. A retaining wall of masonry laid in hydraulic cement is the best provision for the purpose. The outlets should be protected also by a coarse grating of some sort in front of the tile to prevent muskrats and other creatures from getting in.

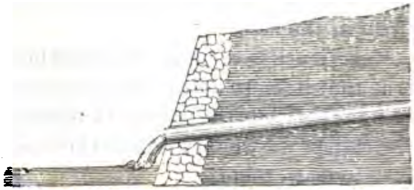


FIG. 123.

A common practice is to introduce at the outlet a box made of plank a few feet in length, into which the tile is made to discharge.

210. Silt Well.—This is a contrivance, shown in the cut, for catching the silt gathered by the drains above it. It serves also the purpose of affording a means of inspecting the working of the drains. Silt-wells may be constructed with a view, chiefly, to facilitating the movement of the water at an abrupt bend in the drain. And again, they may be constructed somewhat with reference to convenience of obtaining a pail of water for any purpose, in the field.

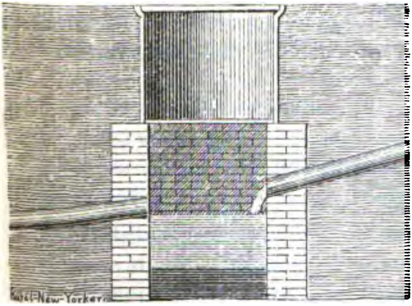


FIG. 124.

Conclusion.—The foregoing treatment of the matter of drainage embraces only the most general considerations pertaining to the subject. A thousand questions of detail which arise in the work have not been touched, but have been left to be answered, as probably it is best they should be, by the student's actual experience with them in the field.

Experience is the best of all teachers. Nor is it strictly true that it keeps a dear school for such only as can learn in no other; but the real truth is that direct contact with the things we would know is the cheapest and the wisest way to learn them.

SECTION VIII.

ORIGINAL SURVEYS.

211. In land surveying, the surveyor has two distinct classes of problems to deal with. In the first class, he is called upon

(*a*) To lay down upon the ground the corners and boundary lines of tracts of land of specified dimensions; and

(*b*) To find the areas of tracts which are already defined by natural or artificial boundaries.

In this class is included the original, marking out upon the ground of the boundaries of every tract of land however great or small. Hence we call surveys of this nature **Original Surveys.**

212. When the boundaries have once been laid down upon the ground and marked by persons having authority to do so, then the surveyor, who is afterward called upon, has a different class of problems to deal with. He then has

(*a*) To find the corner posts and monuments;

(*b*) To re-locate them when lost; and

(*c*) To retrace old boundary lines.

Surveys of this nature we shall call **Resurveys.**

213. Original Surveys include: *First.* The rectangular surveys of the United States, known as the government survey; similar surveys in Canada and other countries by government authority, and the subdivision of sections. *Second.* Surveys made by the proprietors in those regions where the government surveys do not extend, including in the United States the surveys of all

land not granted by the original states of the Union to the general government; and surveys for town plats, highways and like purposes.

214. United States Survey.—The territory embraced within the present States of Ohio, Indiana, Illinois, Michigan, Wisconsin, and Tennessee, that part of Minnesota lying east of the Mississippi River, and all of Alabama and Mississippi lying north of the thirty-first parallel, was held by Massachusetts, Connecticut, New York, Virginia, North Carolina, South Carolina, and Georgia, under grants from Great Britain, during their colonial condition. These territorial interests were surrendered to the General Government of the Union by the last named States at different times hereinafter set forth, and constituted the nucleus of our public domain with some reservations as to former grants, and was the remainder of the territory conceded to the United States under the definitive treaty of 1783, and consisted of 404,955.91 square miles, or 259,171,787 acres. This was the public domain of the United States on April 30, 1803, the date of the Louisiana purchase, and for which the original survey and disposition laws were made.

The United States were recognized by the Crown in the definitive treaty of peace with Great Britain as "free sovereign and independent States, and that he treats with them as such, and for himself, his heirs, and successors relinquishes all claims to the government, proprietary and territorial rights of the same, and every part thereof."

The Government of the United States acquired as custodian for the Nation, lands known as the public domain as follows:

From States (colonies prior to July 4, 1776) ceded under the Confederation and under the Constitution.

This was in pursuance of a resolution of the Congress of the Confederation passed Tuesday, October 10, 1780, providing for the reception and care of such unappropri-

ated lands as might be ceded by States to the United States, and for the disposition of the same for the common benefit of the United States.

The dates of cession of these lands to the United States were as follows:

Colony.	State.	Date of Cession.
New Hampshire	New Hampshire.	No cession.
New York	New York.	March 1, 1781.
Rhode Island and Providence Plantations	Rhode Island.	No cession.
New Jersey	New Jersey.	Do.
New Castle, Kent and Sussex, on Delaware	Delaware.	Do.
Pennsylvania	Pennsylvania.	Do.
Virginia	Virginia.	March 1, 1784, and December 30, 1788.*
Maryland	Maryland.	No cession.
Massachusetts Bay	Massachusetts.	April 19, 1785.
Connecticut	Connecticut.	September 13, 1786; confirmed May 30, 1800.
South Carolina	South Carolina.	August 9, 1787.
North Carolina	North Carolina.	February 25, 1790.
Georgia	Georgia.	April 24, 1802.

*An act to change the conditions of the cession of March 1, 1784, only so far as to ratify the fifth article of the compact of the ordinance of 1787.

AREA OF CESSIONS.

	Sq. miles.	Acres.
Massachusetts (disputed) claimed (estimated)*	54,000.00	34,560,000
Connecticut (disputed) and Western Reserve and Fire-lands (estimated)*	40,000.00	25,600,000
From New York and Massachusetts cession, actual	315.91	202,187
From Virginia (disputed and undisputed) to the United States exclusive of Kentucky and including area of Western Reserve and the Fire-lands)†	265,562.00	169,959,680
South Carolina cession	4,900.00	3,136,000
North Carolina cession, nominal, because the area of Tennessee was almost covered with reservations	45,600.00	29,184,000
Georgia cession	88,578.00	56,689,920
Total actual State cessions to the United States for public domain	404,955.91	259,171,787

*The area above was also claimed by Virginia and included in her cession.

†Connecticut's jurisdictional cession of the Western Reserve and Fire-lands, containing about 3,800,000, included under Virginia cession.

AREA OF PURCHASES—PUBLIC AND NATIONAL DOMAIN.

	Sq. miles.	Acres.
Louisiana purchase, April 30, 1803	1,182,752	756,961,280
East and West Florida Feb 22, 1819	59,268	37,931,520
Gaudalupe Hidalgo, February 2, 1848	522,568	334,443,520
State of Texas, November 25, 1850	96,707	61,892,480
Gadsden purchase, December 30, 1853	45,535	29,142,400
Alaska purchase, March 30, 1867	577,390	369,529,600
	2,484,220	1,589,900,800

At a total cost of $88,157,389.98.

The Texas annexation of 1845 added to the national domain the area of the present State of Texas, viz., 274,356 square miles, or 175,587,840 acres, included in the national domain, besides the purchase of 1850 from the State, now public domain.

The total area of purchased and annexed territory, included in the national and public domain since 1803, is 2,758,576 square miles, or 1,765,488,640 acres, at a total cost of $88,157,389.98 for the purchase, and including the Georgia cession of 1802, $8,200,000.

215. The present system of survey of the public lands was inaugurated by a committee appointed by the Continental Congress, and consisting of the following delegates: Hon. Thomas Jefferson, chairman, Virginia; Hon. Hugh Williamson, North Carolina; Hon. David Howell, Rhode Island; Hon. Elbridge Gerry, Massachusetts; Hon. Jacob Read, South Carolina.

On the 7th of May, 1784, this committee reported "An ordinance for ascertaining the mode of locating and disposing of lands in the western territory, and for other purposes therein mentioned." This ordinance required the public lands to be divided into "hundreds" of ten geographical miles square, and those again to be subdivided into lots of one mile square each, to be numbered from 1 to 100, commencing in the *north-western* corner, and continuing from west to east and from east to west

consecutively. This ordinance was considered, debated, and amended, and reported to Congress April 26, 1785, and required the surveyors "to divide the said territory into townships of 7 miles square, by lines running due north and south, and others crossing these at right angles. * * * The plats of the townships, respectively, shall be marked by subdivisions into sections of 1 mile square, or 640 acres, in the same direction as the external lines, and numbered from 1 to 49. * * * And these sections shall be subdivided into lots of 320 acres."

This is the first record of the use of the terms "township" and "section."

May 3, 1785, on motion of Hon. William Grayson, of Virginia, seconded by Hon. James Monroe, of Virginia, the section respecting the extent of townships was amended by striking out the words "seven miles square" and substituting the words "six miles square." The records of these early sessions of Congress are not very full or complete; but it does not seem to have occurred to the members until the 6th of May, 1785, that a township six miles square could not contain 49 sections of 1 mile square. At that date a motion to amend was made, which provided, among other changes, that a township should contain 36 sections; and the amendment was *lost*. The ordinance as finally passed, however, on the 20th of May, 1785, provided for townships 6 miles square, containing 36 sections of 1 mile square. The first pnblic surveys were made under this ordinance. The townships, 6 miles square, were laid out in ranges, extending northward from the Ohio River, the townships being numbered from south to north, and the ranges from east to west. The region embraced by the surveys under this law forms a part of the present State of Ohio, and is usually styled "The Seven Ranges." In these initial surveys only the *exterior lines* of the townships were surveyed, but the plats were marked by subdivisions into sections of 1 mile square, and mile corners were established on the town-

ship lines. The sections were numbered from 1 to 36, commencing with No. 1 in the *southeast* corner of the township, and running from *south* to *north* in each tier to No. 36 in the *northwest* corner of the township, as shown in the following diagram:

36	30	24	18	12	6
35	29	23	17	11	5
34	28	22	16	10	4
33	27	21	15	9	3
32	26	20	14	8	2
31	25	19	13	7	1

The surveys were made under the direction of the Geographer of the United States.

The act of Congress approved May 18, 1796, provided for the appointment of a surveyor-general, and directed the survey of the lands northwest of the Ohio River, and above the mouth of the Kentucky River, "in which the titles of the Indian tribes have been extinguished." Under this law *one-half* of the townships surveyed were subdivided into sections "by running through the same, each way, parallel lines at the end of every two miles, and by making a corner on each of said lines at the end of every mile," and it further provided that "the sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately, through the township, with progressive numbers till the thirty-sixth be completed." This method

of numbering sections, as shown by the following diagram, is still in use:

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

The act of Congress approved May 10, 1800, required the "townships west of the Muskingum, which * * * are directed to be sold in quarter townships, to be subdivided into half sections of three hundred and twenty acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each mile on those running from south to north. * * * And the interior lines of townships intersected by the Muskingum, and of all the townships lying east of that river, which have not been heretofore actually subdivided into sections, shall also be run and marked. * * * And in all cases where the exterior lines of the townships thus to be subdivided into sections or half sections shall exceed, or shall not extend, six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half sections in such township, according as the error may be in running the lines from east to west or from south to north."

[illegible] The acts of Congress defining the system of public land surveys, and the principles to be employed in carrying [illegible] are to be found in the United States Statutes as follows:

[illegible]	[illegible]	Volume	1,	Chap.	29.
"	[illegible]	"	2,	"	55.
"	[illegible]	"	2,	"	14.
"	[illegible]	"	3,	"	51.
"	[illegible]	"	4,	"	55.
"	[illegible]	"	12,	"	86.
"	[illegible]	"	18,	"	130.
"	[illegible]	"	19,	"	105.

[illegible] acts as are now in force [illegible] a volume entitled [illegible] Those Sections which refer di- [illegible] are as follows:

[illegible] to Surveys and [illegible] There shall be appointed by the [illegible] and consent of the [illegible] States and Territories [illegible] the surveying [illegible] Minnesota, Kansas, [illegible] and Iowa, Dakota, [illegible] Washington, Montana, [illegible]

[illegible] 11 id. 212: [illegible] 17 [illegible] R. S. 2207.

[illegible] shall, before entering [illegible] and deliver to the Sec- [illegible] with good and sufficient [illegible] thousand dollars, [illegible] according to [illegible] and for the [illegible] his office; and the [illegible] require a new

bond and additional security, under the direction of the Secretary of the Interior, for the lawful disbursement of public moneys.

3 Stat. 697; R. S. 2215, 2216. U. S. *v.* Vanzandt, 11 Wheat, 184; U. S. *v.* Tingey, 5 Pet. 115; Farrar and Brown *v.* U. S., 5 *id.* 373; U. S. *v.* Bradley, 10 *id.* 343; U. S. *vs.* Linn, 15 *id.* 290; U. S. *v.* Prescott, 3 How. 578; U. S. *v.* Boyd, 5 *id.* 29; Bryan *v.* U. S., 1 Black, 140; Boyden *v.* United States, 13 Wall. 17; Bevans *v.* U. S, 13 *id.* 56; U. S. *v.* Thomas, 15 *id.* 337; U. S. *v.* Stephenson, 1 McClean, C. C. 462; U. S. *v.* Linn, 2 *id.* 501; U. S. *v.* Ward, 3 *id.* 179. 8 Op. Att. Gen. 7. Cir. G. L. O., July 1, 1871; *id.* May 14, 1879. Treasury Cir., July 13, 1871 (Copp's L. L. 783; 1 Lester's L. L. 312, 314).

SEC. 85. The commission of each surveyor-general shall cease and expire in four years from the date thereof, unless sooner vacated by death, resignation, or removal from office.

3 Stat. 697; R. S. 2217. Best *v.* Polk, 18 Wall. 112. Decision Com. G. L. O., Feb 20, 1858 (1 Lester's L. L. 340).

SEC. 86. Every surveyor-general, except where the President sees cause otherwise to determine, is authorized to continue in the uninterrupted discharge of his regular official duties after the day of expiration of his commission and until a new commission is issued to him for the same office, or until the day when a successor enters upon the duties of such office; and the existing official bond of any officer so acting shall be deemed good and sufficient and in force until the date of the approval of a new bond to be given by him, if recommissioned, or otherwise, for the additional time he may so continue officially to act, pursuant to the authority of this section.

10 Stat. 247; 18 *id*, 62; R. S. 2222.

SEC. 87. Whenever the surveys and records of any surveying district are completed, the surveyor-general thereof shall be required to deliver over to the Secretary of State of the respective states, including such surveys, or to such other officer as may be authorized to receive them, all the field-notes, maps, records, and other papers appertaining to land titles within the same; and the office of

surveyor-general in every such district shall thereafter cease and be discontinued.

5 Stat. 384; 19 *id.* 121; R. S. 2218.

SEC. 88. In all cases of discontinuance, as provided in the preceding section, the authority, powers, and duties of the surveyor-general in relation to the survey, resurvey, or subdivision of the lands therein, and all matters and things connected therewith, shall be vested in and devolved upon the Commissioner of the General Land Office.

10 Stat. 152; R. S. 2219.

SEC. 89. Under the authority and direction of the Commissioner of the General Land Office, any deputy surveyor or other agent of the United States shall have free access to any such field-notes, maps, records, and other papers for the purpose of taking extracts therefrom or making copies thereof without charge of any kind; but no transfer of such public records shall be made to the authorities of any State until such State has provided by law for the reception and safe-keeping of such public records and for the allowance of free access thereto by the authorities of the United States.

10 Stat. 152; 18 *id.* 62; R. S. 2220, 2221.

SEC. 90. Every surveyor-general shall engage a sufficient number of skillful surveyors as his deputies, to whom he is authorized to administer the necessary oaths upon their appointments. He shall have authority to frame regulations for their direction, not inconsistent with law or the instructions of the General Land Office, and to remove them for negligence or misconduct in office.

Taylor and Quarlls *v.* Brown, 5 Cranch, 234; Craig *et al.* *v.* Braxford, 3 Wheat. 594; Ellicott *et al.* *v.* Pearl, 10 Pet. 412; Brown's Lessee *v.* Clements, 3 How. 650. Reed *v.* Conway 20 Mo. 22; same case, 26 *id* 13; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth. 2 Ind. 274; McClintock *v.* Rodgers, 11 Ills. 279. Cir. G. L. O., June 26, 1880.-

Second. He shall cause to be snrveyed, measured, and marked, without delay, all base and meridian lines through

such points and perpetuated by such monuments, and such other correction parallels and meridians as may be prescribed by law or by instructions from the General Land Office in respect to the public lands within his surveying district, to which the Indian title has been or may be hereafter extinguished.

Gazzen *v.* Phillips' Lessee, 20 How. 372. 3 Op. Att. Gen., 281, 284. Atshire *v.* Hulse, 1 Ohio, 170; Hastings *v.* Stevenson, 2 *d.* 9; McKinney *v.* McKinney, 8 *id.* 423; Hamil *v.* Carr, 21 Ohio St. 258; Hendrick *v*, Eno, 42 Iowa, 411; Saint Louis *v.* Walker, 40 Mo. 383: Jordan *v.* Barrett, 13 La. 24; Fowler *v.* Duval, 11 *id.* 561; Cox *v.* Jones, 47 Cal 412. Cir. G. L. O., June 26, 1880.

Third. He shall cause to be surveyed all private land claims within his district after they have been confirmed by authority of Congress, so far as may be necessary to complete the survey of the public lands.

Menard's Heirs *v.* Massey, 8 How. 293; Kissell *v.* St. Louis Public Schools, 18 *id.* 19; Stanford *v* Taylor. 18 *id.* 409; Ballance *v.* Forsyth, 24 *id*, 183; U. S. *v.* Fossat, 25 *id.* 445; Carondelet *v.* St. Louis, 1 Black. 179; U. S. *v.* Sepulveda, 1 Wall. 104; U. S. *v.* Halleck, 1 *id.* 439; U. S *v.* Billings, 2 *id.* 444; Sutter's case, 2 *id.* 562; U. S. *v.* Pacheco, 2 *id.* 587; Fossat case, 2 *id.* 649; Dehon *v.* Bernal, 2 *id.* 774; U. S. *v.* Armijo, 5 *id.* 444; Higueras *v.* U. S., 5 *id.* 827: Maguire *v.* Tyler, 8 *id.* 650; Lynch *v.* Bernal, 9 *id.* 315; Henshaw *a.* Bissell, 18 *id.* 255; Shepley *et al. v.* Cowan *et al.*, 1 Otto, 330; Miller *et al. v.* Dale *et al.*, 2 *id.* 473; Van Reynegand *v.* Bolton, 5 *id.* 33; U. S. *v.* Throckmorton, 8 *id* 61; Snyder *v.* Sickles, 8 *id* 203; Scull *v.* U. S., 8 *id.* 410. Bissell *v.* Henshaw, 1 Saw. C. C. 553; Leroy *v.* Jamison, 3 *id.* 369. Gibson *v.* Chouteau, 39 Mo. 536; Milburn *v.* Hardy, 28 *id.* 514; Funkhouser *v.* Hantz, 29 *id.* 540; Dent *v.* Legesson, 29 *id.* 489; Carondelet *v.* St. Louis, 29 *id.* 527; Maguire *v*, Tyler, 30 *id.* 202; Robins *v.* Eckler, 36 *id.* 494; Clark *v.* Heammerle, 36 *id.* 620; Gibson *v.* Chouteau, 39 *id.* 536; Vasquez *v.* Ewing, 42 *id.* 247; Glasgow *v.* Lindell 50 *id.* 60; Rector *v.* Gaines, 19 Ark. 70; Ashley *v.* Rector, 20 *id.* 359; Meaux *v.* Breaux, 10 Martin (La.) 364; Moon *v.* Wilkinson, 13 Cal. 478; Boggs *v.* Mining Co., 14 *id.* 279; Mott *v.* Smith, 16 *id.* 534; Johnson *v.* Van Dyke, 20 *id.* 225; McGarrahan *v.* Maxwell, 27 *id.* 75; Treadway *v.* Semple, 28 *id.* 652; Searle *v.* Ford, 29 *id.* 104; Mahoney *v.* Van Winkle, 33 *id.* 448; Morrill *v.* Chapman, 35 *id.* 85; Yates *v.* Smith, 38 *id.* 60; San Diego *v.* Allison, 46 *id.* 163. Decisions Sec. Int., July 16, 1872; Aug. 8, 1876; Aug. 17, 1876; arch 16, 1877. Decisions Com. G. L. O., Aug. 18, 1860; Sept. 18, 1874; Nov. 3, 1874; Sept. 18, 1875; Oct. 28, 1875; June 26, 1879. Cir. G. L. O., June 26, 1880.

Fourth. He shall transmit to the register of the respective land offices within his district general and particular plats of all lands surveyed by him for each land district; and he shall forward copies of such plats to the Commissioner of the General Land Office.

Barnard *v.* Ashley, 18 How. 43; Water and Mining Co. *v.* Bugbee, 6 Otto. 165; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth, 2 Ind. 274; Pope *v.* Athearn, 42 Cal. 606; Com. G. L. O. Instructions to Surveyor-General, April 17, 1879.

Fifth. He shall, so far as is compatible with the desk duties of his office, occasionally inspect the surveying operations while in progress in the field, sufficiently to satisfy himself of the fidelity of the execution of the work according to contract, and the actual and necessary expenses incurred by him while so engaged shall be allowed; and where it is incompatible with his other duties for a surveyor-general to devote the time necessary to make a personal inspection of the work in progress, then he is authorized to depute a confidential agent to make such examination, and the actual and necessary expenses of such person shall be allowed and paid for that service, and five dollars a day during the examination in the field; but such examination shall not be protracted beyond thirty days, and in no case longer than is actually necessary; and when a surveyor-general, or any person employed in his office at a regular salary, is engaged in such special service he shall receive only his necessary expenses in addition to his regular salary.

1 Stat. 464; 13 *id.* 325; 4 *id.* 492; 10 *id.* 245, 247; 18 *id.* 34; 19 *id.* 126; R. S. 2223. Sec. Int. Instructions, July 1, 1874; Sept. 21, 1874. Cir. G. L. O., June 26, 1880.

SEC. 91. Every deputy surveyor shall enter into a bond, with sufficient security, for the faithful performance of all surveying contracts confided to him: and the penalty of the bond, in each case, shall be double the estimated amount of money accruing under such contracts, at the rate per mile stipulated to be paid therein. The suffici-

ency of the sureties to all such bonds shall be approved and certified by the proper surveyor-general.

4 Stat. 493; 10 *id.* 247; R. S. 2230. U. S. *v.* Vanzandt, 11 Wheat. 184; U. S. *v.* Tingey, 5 Pet. 115; Farrar *et al.* *v.* U. S., 5 *id.* 373; U. S. *v.* Bradley, 10 *id.* 343; U. S. *v.* Linn, 15 *id.* 290. U. S. *v.* Stephenson, 1 McLean, C C. 462.

SEC. 92. The surveyor-general, in addition to the oath now authorized by law to be administered to deputies on their appointment to office, shall require each of their deputies, on the return of his surveys, to take and subscribe an oath that those surveys have been faithfully and correctly executed according to law and the instructions of the surveyor-general.

9 Stat. 79; R. S. 2231. Ellicott and Meredith *v.* Pearle, 10 Pet. 412; U. S. *v* Hanson, 16 *id.* 196; Bollard *et al.* *v.* Dwight *et al.*, 4 Cranch, 421; Taylor *et al.* *v.* Brown, 5 *id.* 234. Cir. G. L. O., June 26, 1880.

SEC. 93. The district attorney of the United States, in whose district any false, erroneous, or fraudulent surveys have been executed, shall, upon the application of the proper surveyor-general, immediately institute suit upon the bond of such deputy, and the institution of such suit shall act as a lien upon any property owned or held by such deputy or his sureties at the time such suit was instituted.

9 Stat. 79; R. S. 2232.

SEC. 99. The public lands shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of six miles square, unless where the line of an Indian reservation, or of tracts of land heretofore surveyed or patented, or the course of navigable rivers, may render this impracticable; and in that case this rule must be departed from no further than such particular circumstances require.

McKinney *v.* McKinney, 8 Ohio, 423; Hamil *v.* Carr, 21 Ohio St. 258. Decision Sec. Int., Jan. 24, 1880. Cir. G. L. O , June 26, 1880.

Second. The corners of the townships must be marked with progressive numbers from the beginning, each dis-

tance of a mile between such corners must be also distinctly marked with marks different from those of the corners.

Third. The township shall be subdivided into sections, containing, as nearly as may be, six hundred and forty acres each, by running through the same, each way, parallel lines at the end of every two miles; and by making a corner on each of such lines, at the end of every mile. The sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately through the township with progressive numbers till the thirty-six be completed.

Grogan *v.* Knight, 27 Cal 516. Decision Sec. Int., April 14, 1879. Cir. G. L. O., June 26, 1880.

Fourth. The deputy surveyors, respectively, shall cause to be marked on a tree near each corner established in the manner described, and within the section, the number of such section, and over it the number of the township within which such section may be; and the deputy surveyors shall carefully note, in their respective field-books, the names of the corner-trees marked and the numbers so made.

Cir. G. L O., June 26, 1880.

Fifth. Where the exterior lines of the townships which may be subdivided into sections or half-sections exceed, or do not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half-sections in such townships, according as the error may be in running the lines from east to west, or from north to south; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats respectively, and all others as containing the complete legal quantity.

Knight *v.* Elliott, 57 Mo. 317; Vaughn *v.* Tate, 64 *id.* 491; Walters *v.* Commons, 2 Port. (Ala.) 38; Lewen *v.* Smith, 7 *id.* 428. Decision Sec. Int., April 14, 1879. Cir. G. L. O., June 26, 1880.

Sixth. All lines shall be plainly marked upon trees, and measured with chains, containing two perches of sixteen and one-half feet each, subdivided into twenty-five equal links; and the chain shall be adjusted to a standard to be kept for that purpose.

Bradley *v.* Taylor, 5 Cranch, 191; McIvers *v.* Walker. 9 *id.* 173; Shipp *v.* Miller's Heirs, 2 Wheat. 316; Holmes *v.* Trout, 7 Pet. 171; Brown *v.* Huger, 21 How. 305; Meron *v.* Whitney, 5 Otto, 551; Robinson *v.* Moon, 4 McLean, C. C. 279. Oakley *v.* Stuart, 52 Cal. 521. Cir. G L O., June 26, 1880.

Seventh. Every surveyor shall note in his field-book the true situations of all mines, salt licks, salt springs, and and mill-seats which come to his knowledge; all water courses over which the line he runs may pass; and also the quality of the lands.

Newsom *v.* Pryor's Lessee, 7 Wheat. 7; Preston *v.* Bowman, 6 *id.* 580; Patterson *v* Jenks, 2 Pet. 216.

Eighth. These field books shall be returned to the surveyor-general, who shall cause therefrom a description of the whole lands surveyed to be made out and transmitted to the officers who may superintend the sales. He shall also cause a fair plat to be made of the townships and fractional parts of townships contained in the lands, describing the subdivisions thereof and the marks of the corners. This plat shall be recorded in books to be kept for that purpose; and a copy thereof shall be kept open at the surveyor-general's office for public information, and other copies shall be sent to the places of the sale and to the General Land Office.

1 Stat. 465; 2 *id.* 73; 19 *id.* 348; R. S. 2395. Taylor *et al.* *v.* Brown, 5 Cranch, 234; Barnard *v.* Ashley, 18 How. 43; Water and Mining Co. *v.* Bugbee, 6 Otto, 165. Rector *v.* Gaines, 19 Ark. 70; Lewen *v.* Smith, 5 Port. (Ala.) 428; Mott *v.* Smith, 16 Cal. 534; Hamil *v.* Carr, 21 Ohio St. 258; Doe *v.* Hildreth, 2 Ind. 274; McClintock *v.* Rodgers, 11 Ills. 279. Decision Sec. Int., Jan. 15, 1878. Decision Com. G. L. O., April 17, 1879.

SEC. 100. The boundaries and contents of the several sections, half-sections, and quarter-sections of the public

lands shall be ascertained in conformity with the following principles:

First. All the corners marked in the surveys, returned by the surveyor-general, shall be established as the proper corners of sections, or subdivisions of sections, which they were intended to designate; and the corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from two corners which stand on the same line.

Second. The boundary lines, actually run and marked in the surveys returned by the surveyor-general, shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines, as returned, shall be held and considered as the true length thereof. And the boundary lines which have not been actually run and marked shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships where no such opposite corresponding corners have been or can be fixed, the boundary lines shall be ascertained by running from the established corners due north and south or east and west lines, as the case may be, to the water-course, Indian boundary line, or other external boundary of such fractional township.

Mott *v.* Smith, 16 Cal. 534; Guin *v.* Brandon, 29 Ohio St. 656; McClintock *v.* Rodgers, 11 Ills. 279; Goodman *v*, Myrick, 5 Oreg. 65. Cir. G. L. O., June 26, 1880.

Third. Each section or subdivision of section, the contents whereof have been returned by the surveyor-general, shall be held and considered as containing the exact quantity expressed in such return; and the half-sections and quarter-sections, the contents whereof shall not have been thus returned, shall be held and considered as containing the one-half or the one-fourth part, respectively,

of the returned contents of the section of which they make part.

2 Stat. 313; R. S. 2396. Lindsey *v.* Hawes, 2 Black, 554; U. S. *v.* Pacheco, 2 Wall. 587; Railway Co. *v.* Schurmier, 7 *id.* 272; County of Saint Clair *v.* Livingston, 23 *id.* 46; Heidekoper *v.* Brooms, 1 Wash C. C. 109; Coon *v.* Pen, 1 Pet. C. C. 496. 2 Op. Att. Gen. 578. Knight *v.* Elliott, 57 Mo. 317; Vaughn *v.* Tate, 64 *id.* 491; Waters *v.* Commons, 2 Port. (Ala.) 38; Lewen *v.* Smith, 7 *id.* 428; Billingsly *v.* Bates, 30 Ala. 376; Doe *v.* Hildreth, 2 Ind. 274; Grogan *v.* Knight, 27 Cal. 516. Decision Com. G. L. O., May 17, 1875. Cir. G. L. O., June 26, 1880.

SEC. 101. In every case of the division of a quarter-section the line for the division thereof shall run north and south, and the corners and contents of half quarter-sections which may thereafter be sold shall be ascertained in the manner and on the principles directed and prescribed by the section preceding, and fractional sections containing one hundred and sixty acres or upwards shall in like manner, as nearly as practicable, be subdivided into half quarter-sections, under such rules and regulations as may be prescribed by the Secretary of the Interior, and in every case of a division of a half quarter-section, the line for the division thereof shall run east and west, and the corners and contents of quarter quarter-section, which may thereafter be sold, shall be ascertained, as nearly as may be, in the manner and on the principles directed and prescribed by the section preceding; and fractional sections containing fewer or more than one hundred and sixty acres shall in like manner, as nearly as may be practicable, be subdivided into quarter quarter-sections, under such rules and regulations as may be prescribed by the Secretary of the Interior.

3 Stat. 566; 4 *id.* 503; R. S. 2397. Gazzam *v.* Phillips' Lessee, 20 How. 372; Railway Co. *v.* Schurmier, 7 Wall. 272. Buel *v.* Tuley, 4 McLean, C. C. 268. Wharton *v.* Littlefield, 30 Ala. 245. 3 Op. Att. Gen. 281, 284. Decision Sec. Int., April 14, 1879. Decision Com. G. L. O., May 17, 1875. Cir. G. L. O., June 26, 1880.

SEC. 102. Whenever, in the opinion of the President, a departure from the ordinary method of surveying land

on any river, lake, bayou, or water-course would promote the public interest, he may direct the surveyor-general, in whose district such land is situated, and where the change is intended to be made, to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water-course, and running back the depth of forty acres; which tracts of land so surveyed shall be offered for sale entire, instead of in half quarter-sections, and in the usual manner, and on the same terms in all respects as the other public lands of the United States.

4 Stat. 34; R. S. 2407.

SEC. 103. In extending the surveys of the public lands in the State of Nevada, the Secretary of the Interior may vary the lines of the subdivisions from a rectangular form, to suit the circumstances of the country.

14 Stat. 86; R. S. 2408. Heydenfeldt *v.* Mining Co., 3 Otto, 634.

SEC. 104. The Secretary of the Interior, if he deems it advisable, is authorized to continue the surveys in Oregon and California, to be made after what is known as the geodetic method, under such regulations and upon such terms as have been or may hereafter be prescribed by the Commissioner of the General Land Office; but none other than township lines shall be run where the land is unfit for cultivation; nor shall any deputy surveyor charge for any line except such as may be actually run and marked or for any line not necessary to be run.

9 Stat. 496; 10 *id.* 245; R. S. 2409.

SEC. 105. Whenever, in the opinion of the Secretary of the Interior, a departure from the rectangular mode of surveying and subdividing the public lands in California would promote the public interests, he may direct such change to be made in the mode of surveying and designating such lands as he deems proper, with reference to the existence of mountains, mineral deposits, and the advantages derived from timber and water privileges; but such lands shall not be surveyed into less than one hun-

dred and sixty acres or subdivided into less than forty acres.

10 Stat. 245: R. S. 2410. Cir. G. L. O., June 26, 1880.

SEC. 106. The public surveys shall extend over all mineral lands, and all subdividing of surveyed lands into lots less than one hundred and sixty acres may be done by county and local surveyors at the expense of claimants; but nothing contained in this section shall require the survey of waste or useless lands.

10 Stat. 15, 21; 16 *id.* 218; R. S. 2406.

SEC. 107. The printed manual of instructions relating to the public surveys, prepared at the General Land Office, and bearing date February twenty-second, eighteen hundred and fifty-five, the instructions of the Commissioner of the General Land Office, and the special instructions of the surveyor-general, when not in conflict with such printed manual or the instructions of the Commissioner, shall be taken and deemed to be a part of every contract for surveying the public lands.

12 Stat. 409; R. S. 2399. Cir. G. L. O., June 26, 1880.

SEC. 108. Legal subdivisions of forty acres of placer lands may be subdivided into ten-acre lots.

16 Stat. 213; R. S. 2330.

SEC. 2320. Mining claims upon veins or lodes of quartz or other rock in place bearing gold, silver, cinnabar, lead, tin, copper, or other valuable deposits, heretofore located, shall be governed as to length along the vein or lode by the customs, regulations, and laws in force at the date of their location. A mining-claim located after the tenth day of May, eighteen hundred and seventy-two, whether located by one or more persons, may equal, but shall not exceed, one thousand five hundred feet in length along the vein or lode; but no location of a mining-claim shall be made until the discovery of the vein or lode within the limits of the claim located. No claim shall extend more than three hundred feet on each side of the middle

of the vein at the surface, nor shall any claim be limited by any mining regulation to less than twenty-five feet on each side of the middle of the vein at the surface, except where adverse rights existing on the tenth day of May, eighteen hundred and seventy-two, render such limitation necessary. The end-lines of each claim shall be parallel to each other.

10 May, 1872, c. 152, s. 2, v. 17, p. 91.

SEC. 2322. The locators of all mining locations heretofore made or which shall hereafter be made, on any mineral vein, lode, or ledge, situated on the public domain, their heirs and assigns, where no adverse claim exists on the tenth day of May, eighteen hundred and seventy-two, so long as they comply with the laws of the United States, and with State, Territorial, and local regulations not in conflict with the laws of the United States governing their possessory title, shall have the exclusive right of possession and enjoyment of all the surface included within the lines of their locations, and of all veins, lodes, and ledges throughout their entire depth, the top or apex of which lies inside of such surface-lines extended downward vertically, although such veins, lodes, or ledges may so far depart from a perpendicular in their course downward as to extend outside the vertical side-lines of such surface locations. But their right of possession to such outside parts of such veins or ledges shall be confined to such portions thereof as as lie between vertical planes drawn downward as above described, through the end-lines of their locations, so continued in their own direction that such planes will intersect such exterior parts of such veins or ledges. And nothing in this section shall authorize the locator or possessor of a vein or lode which extends in its downward course beyond the vertical lines of his claim to enter upon the surface of a claim owned or possessed by another.

10 May, 1872, c. 152, s. 3, v. 17, p. 91.

SEC. 2323. Where a tunnel is run for the development of a vein or lode, or for the discovery of mines, the own-

ers of such tunnel shall have the right of possession of all veins or lodes within three thousand feet from the face of such tunnel on the line thereof, not previously known to exist, discovered in such tunnel, to the same extent as if discovered from the surface; and locations on the line of such tunnel of veins or lodes not appearing on the surface, made by other parties after the commencement of the tunnel, and while the same is being prosecuted with reasonable diligence, shall be invalid; but failure to prosecute the work on the tunnel for six months shall be considered as an abandonment of the right to all undiscovered veins on the line of such tunnel.

10 May, 1872, c. 152, s. 4, v. 17, p. 92.

SEC. 2324. The miners of each mining-district may make regulations not in conflict with the laws of the United States, or with the laws of the State or Territory in which the district is situated, governing the location, manner of recording, amount of work necessary to hold possession of a mining-claim, subject to the following requirements: The location must be distinctly marked on the ground so that its boundaries can be readily traced. All records of mining-claims hereafter made shall contain the name or names of the locators, the date of the location, and such a description of the claim or claims located by reference to some natural object or permanent monument as will identify the claim.

10 May, 1872, c. 152, s. 5, v. 17, p. 92.

SEC. 109. The surveyor-general of the United States may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointment to survey mining claims. The expenses of the survey of vein or lode claims, and the survey and subdivision of placer claims into smaller quantities than one hundred and sixty acres, shall be paid by the applicants, and they shall be at liberty to obtain the same at the most reasonable rates, and they shall also be at liberty to employ any United States deputy surveyor to make the

survey. The Commissioner of the General Land Office shall have power to establish the maximum charges for such surveys; and to the end that he may be fully informed on the subject, each applicant shall file with the register a sworn statement of all charges and fees paid by such applicant for surveys, which statement shall be transmitted to the Commissioner of the General Land Office.

17 Stat. 95; 19 *id.* 52; R. S. 2334. Decision Com. G. L. O., April 20, 1877.

SEC. 110. The surveyor-general of the United States shall prepare or cause to be prepared a plat and field-notes of all mining surveys made by authority of law, which shall show accurately the boundaries of such claims; and, when warranted by the facts, he shall give to the claimant his certificate that five hundred dollars' worth of labor has been expended or improvements made upon the claim by the claimant or his grantors, and that the plat is correct, with such further description by such reference to natural objects or permanent monuments as shall identify the claim, and furnish an accurate description, to be incorporated in the patent.

17 Stat. 92; R. S. 2325

SEC. 111. Contracts for the survey of the public lands shall not become binding upon the United States until approved by the Commissioner of the General Land Office, except in such cases as the Commissioner may otherwise specially order

12 Stat. 409; R. S 2398. Maguire *v.* Tyler, 1 Black, 201; Parks *v.* Ross, 11 How. 362; Spencer *v.* Lapsley, 20 *id* 264. Reed *v.* Conway, 26 Mo. 13. Decision Sec. Int., Feb. 27, 1878.

SEC. 112. The Commissioner of the General Land Office has power, and it shall be his duty, to fix the prices per mile for public surveys, which shall in no case exceed the maximum established by law; and, under instructions to be prepared by the Commissioner, an accurate account shall be kept by each surveyor-general of the cost of sur-

veying and platting private land claims, to be reported to the General Land Office, with the map of such claim; and patents shall not issue for any such private claim, nor shall any copy of such survey be furnished, until the cost of survey and platting has been paid into the Treasury by the claimant or other party; and before any land granted to any railroad company by the United States shall be conveyed to such company or any persons entitled thereto, under any of the acts incorporating or relating to said company, unless such company is exempted by law from the payment of such cost, there shall first be paid into the Treasury of the United States the cost of surveying, selecting, and conveying the same by the said company or persons in interest.

12 Stat. 409; 18 *id.* 384; 19 *id.* 122; R. S. 2400. Railway Co. *v.* Prescott, 16 Wall. 603; Railway Co. *v.* McShane, 22 *id.* 444; Hannewell *v.* Cass Co., 22 *id.* 464; Colorado Co. *v.* Commissioners, 5 Otto, 259. Decisions Sec. Int., Dec. 17, 1874; Feb. 27, 1878; Feb. 20, 1879; March 5, 1879; April 2, 1879. Decisions Com. G. L. O., April 18, 1867; August 18, 1867; Feb. 17, 1869; March 26, 1870. Cir. G. L. O., June 26, 1880.

SEC. 113. The Commissioner of the General Land Office may authorize, in his discretion, public lands in Oregon, densely covered with forests or thick undergrowth, to be surveyed at augmented rates, not exceeding eighteen dollars per mile for standard parallels, fifteen dollars for townships, and twelve dollars for section lines; and under like conditions he may allow augmented rates in California, and in Washington Territory, not exceeding eighteen dollars per linear mile for standard parallels, sixteen dollars for township, and fourteen dollars for section lines.

16 Stat. 304, 305; 17 *id.* 358; R. S. 2404, 2405. Decision Sec. Int., June 16, 1879. Cir. G. L. O., June 26, 1880.

SEC. 114. Whenever the public surveys, or any portion of them, in the States of Oregon and California, are so required to be made as to render it expedient to make compensation for the surveying thereof by the day instead

of by the mile, it shall be lawful for the Commissioner of the General Land Office, under the direction of the Secretary of the Interior, to make such fair and reasonable allowance, as, in his judgment, may be necessary to insure the accurate and faithful execution of the work.

10 Stat. 247; R. S. 2411. Decision Sec. Int., June 16, 1879. Cir. G. L. O., June 26, 1880.

SEC. 118. Each surveyor-general, when thereunto duly authorized by law, shall cause all confirmed private land claims within his district to be accurately surveyed, and shall transmit plats and field-notes thereof to the Commissioner of the General Land Office for his approval. When publication of such surveys is authorized by law, the proof thereof, together with any objections properly filed and all evidence submitted either in support of or in opposition to the approval of any such survey, shall also be transmitted to said Commissioner.

2 Stat. 326, 352; 3 *id.* 325; 5 *id.* 740; 9 *id.* 242, 633; 10 *id.* 244, 308, 599; 11 *id.* 294; 12 *id.* 172, 209, 369, 409; 13 *id.* 332, 344; 14 *id.* 218; 16 *id.* 64, 304; 18 *id.* 305; 19 *id.* 121, 202: R. S. 2447. Bissell *v.* Penrose, 8 How. 317; Villalobus *v.* U. S., 10 *id.* 541; Ledoux *v.* Black, 18 *id.* 473; U. S. *v.* Fossat, 20 *id.* 413; Brown *v.* Huger, 21 *id.* 305; U. S *v.* Fossat, 21 *id.* 445; Castro *v.* Hendricks, 23 *id.* 438; Ballance *v.* Forsyth, 24 *id.* 183; U. S. *v.* Sepulveda, 1 Wall. 104; U. S. *v.* Halleck, 1 *id* 439; U. S. *v.* Vallejo, 1 *id.* 658; Sutter's case 2 *id.* 562; Fossat case, 2 *id.* 649; Higueras *v.* U. S, 5 *id.* 827; Alviso *v.* U. S., 8 *id.* 337. 12 Op. Att. Gen. 116, 250; 14 *id*, 74, 601. U. S, *v.* Garcia, 1 Saw. C.C. 383; Russell *v.* Henshaw, 1 *id.* 553; Leroy *v.* Jamison, 3 *id.* 369; U. S. *v.* Flint, 4 *id.* 42. Dent *v.* Sergerson, 29 Mo. 480; Fowler *v.* Duvall, 11 La. Ann. 561; Waterman *v.* Smith, 13 Cal. 373; Moore *v* Wilkerson, 13 *id.* 478; Merrit *v.* Judd, 14 *id.* 60; Mott *v.* Smith, 16 *id.* 534; Johnson *v.* Van Dyke, 20 *id.* 225; McGarraghan *v.* Maxwell, 27 *id*, 75; Sealc *v.* Ford, 29 *id.* 104. Cir. G. L. O., June 26, 1880.

SEC. 120. Every person who in any manner, by threat or force, interrupts, hinders, or prevents the surveying of the public lands, or of any private land claim which has been or may be confirmed by the United States, by the persons authorized to survey the same, in conformity with the instructions of the Commissioner of the General

Land Office, shall be fined not less than fifty dollars nor more than three thousand dollars, and be imprisoned not less than one nor more than three years.

4 Stat. 417; R S. 2412.

SEC. 121. Whenever the President is satisfied that forcible opposition has been offered, or is likely to be offered, to any surveyor or deputy surveyor in the discharge of his duties in surveying the public lands, it may be lawful for the President to order the marshal of the State or district, by himself or deputy, to attend such surveyor or deputy surveyor with sufficient force to protect such officer in the execution of his duty, and to remove force should any be offered.

4 Stat. 417; R. S. 2413.

SEC. 122. The President is authorized to appoint surveyors of public lands, who shall explore such vacant and unappropriated lands of the United States as produce the live-oak and red cedar timbers, and shall select such tracts or portions thereof, where the principal growth is of either of such timbers, as in the judgment of the Secretary of the Navy may be necessary to furnish for the Navy a sufficient supply of the same. Such surveyors shall report to the President the tracts by them selected, with the boundaries ascertained and accurately designated by actual survey or water-courses.

3 Stat. 347; R. S. 2459. U. S. v. Briggs, 9 How. 351.

SEC. 123. The director of the geological survey shall, under the Interior Department, have the direction of the geological survey and the classification of the public lands and examination of the geological structure, mineral resources, and products of the national domain.

20 Stat. 394.

218. Manner of Field Work and Changes that have been Made.—In accordance with these laws, instructions have been issued from time to time, by the

Commissioners of the General Land Office, directing the manner in which the field work should be performed.

In the earlier surveys under the act of 1796 (Sec. 2395 R. S. See p. 244, Sec. 99, Third,) the township was subdivided by parallel lines two miles apart. The mile posts were planted on these lines, but no half mile (or quarter-section) corners set.

The act of 1800 provided that the townships west of the Muskingum River should be subdivided into half sections of 320 acres each, as near as may be, by parallel lines run through them from east to west and from north to south at distances of a mile apart. Half-mile posts were to be set on the east and west lines, but not on the lines running north and south.

The act of 1805 (Sec. 2396 R. S. P. 245, Sec. 100) covers in its provisions the two classes of surveys above noted, as well as the principles governing all subsequent surveys of the public lands.

Since that time, few changes have been made in the manner of carrying on the surveys.

The principal change has been in the manner of closing the subdivision lines on the exterior line of the township.

In some of the earlier surveys, three sets of corners were marked in the range lines. The first set was marked when the range line was run, and were not really corners of the subdivisions.

The other two sets were marked at the points where the subdivision lines of the townships, both east and west, intersected the range line—those lines not being required to close on the corners previously set on the range line.

Later the surveyors were required to close their subdivision lines upon the corners previously set on the east line of the township, but not on the north or west. Double corners were thus produced on all the exterior lines of the township.

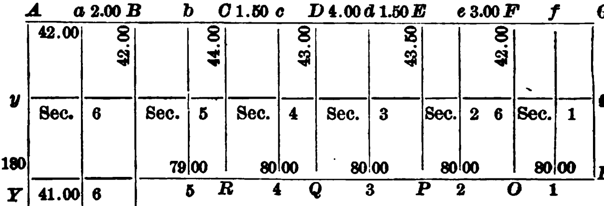
Most of the surveys before 1846 were made under this system, which is thus laid down in the Instructions of 1815:

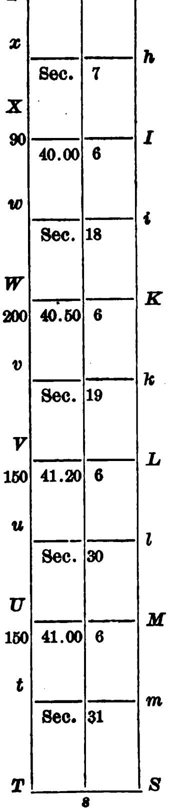
"Each side of a section must be made one mile in measure by the chain, and quarter-section corners are to be established at every half mile, except when in the closing of a section, if the measure of the closing side should vary from 80 chains or one mile, you are in that case to place the quarter-section corners equidistant, or at an average distance from the corners of the section; but in running out the sectional lines on the west or north side of the township, you will establish your quarter-section posts or corners at the distance of half a mile from the last corner, and leave the remaining excess or defect on the west or north tier of quarter-sections, which balance or remainder you will carefully measure and put down in your field-notes in order to calculate the remaining or fractional quarter-section on the north and west side of the township: also in running to the western or northern boundary, unless your sectional lines fall in with the posts established there for the corners of sections in the adjacent townships, you must set post and mark bearing trees at the points of intersection of your lines with the town boundaries, and take the distance of your corners from the corners of the sections of the adjacent townships, and note that and the side on which it varies in chains or links, or both.

The sections must be made to close by running a random line from one corner to another, except on the north and west ranges of sections, and the true line between them is to be established by means of offsets."

The following, from the Instructions of 1815, will explain what has been to many surveyors an incomprehensible matter, viz., how the fractional areas laid down in the plats of the early United States surveys were arrived at:

219. Method by which to Calculate the Northern





and Western Tier of Fractional Quarter-Sections.—"You will commence, say at the northeast corner of the township, the length of the line from *G* to *g* being 40 chains, as established in running the exterior boundary of the township; you will proceed by adding the length of the line from 6 to *F* on the line from *O* to *F*, which is 42 chains, to the length *G g*, 40 chains, and divide it by two, which will give you the length of the line from the center of the section to *f*, on the town boundary, which, being added to the length of the line *G g* and divided by two, will give you the length of one of the lines required for calculating the N. E. quarter of section No. 1; then the length of the line from *G* to *f* being 40 chains, and the south boundary of the section being 80 chains, the length of the line *H* to 1 is 40 chains—the length of the line from *G* to *f*—therefore there is no necessity for additions or divisions, as the line from 1 to *f* is parallel to the line *G H*; then by multiplying those two sides together and cutting off as many decimals as there are in the sums multiplied, and dividing by 10, you have the contents of the N. E. quarter of section 1 in acres and decimal parts of an acre. You will

then proceed to calculate the N. W. quarter of the same section by taking the length of the line from the center of the section to *f*, as found in your former calculation, to which add 42 chains, the length of the line from 6 to *F*, and divide by two, which gives you the length of one of the lines required. Then as the line from *O* to *F* intersected the town boundary 3 chains east of the section corner, the length of the line from *F* to *f* is only 37 chains, which, added to 40 chains (the length of the south line of the southwest quarter of section 1), and divided by two, will give you the length of the line from 6 to the center of the section, which being added to 37 and divided by two, will give you the length of the other line required, which you will calculate in the same manner as above.

As the length of the line from *F* to *f* is only 37 chains, the length of the line from *e* to *F* must be 43 chains; the length of the line *E* to *e* is 38.50 chains; the length of the line from *d* to *E*, 41.50 chains, etc., the quarter-section corners not being placed at the average distance between the section corners, except when you strike the corners of the sections established in running the exterior lines of the township."

220. Instructions of 1881.—The leading points of interest to the surveyor in the Instructions of 1881, to United States deputy surveyors, are as follows:

1. The public lands of the United States are ordinarily surveyed into rectangular tracts, bounded by lines conforming to the cardinal points.

2. The public lands shall be laid off, in the first place, into bodies of land of 24 miles square, as near as may be. This shall be done by the extension of standard lines from the principal meridian every 24 miles, and by the extension, from the base and standard lines, of auxiliary meridians every 24 miles. Thereafter they shall be laid off into bodies of land of 6 miles square, as near as may be, called **Townships**, containing as near as may be 23,040

acres. The townships shall be subdivided into 36 tracts, called **Sections**, each containing as near as may be 640 acres. Any number or series of contiguous townships, situate north or south of each other, constitute a **Range**.

The law requires that the lines of the public surveys shall be governed by the true meridian, and that the townships shall be *six miles square*—two things involving in connection a mathematical impossibility—for, strictly to conform to the meridian, necessarily throws the township out of square, by reason of the convergency of meridians, and hence, by adhering to the true meridian, results the necessity of departing from the strict requirements of law, as respects the precise area of townships and the subdivisional parts thereof, the township assuming something of a trapezoidal form, which inequality develops itself more and more as such, the higher the latitude of the surveys. It is doubtless in view of these circumstances that the law provides (see section 2 of the act of May 18, 1796,) that the sections of a *mile* square shall contain the quantity of 640 acres, *as nearly as may be;* and, moreover, provides (see section 3 of the act of May 10, 1800,) in the following words: "And in all cases where the exterior lines of the townships, thus to be subdivided into sections or half-sections, shall exceed, or shall not extend 6 miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west, or from south to north; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity."

The section lines are surveyed from *south* to north on true meridians, and from *east* to west, in order to throw the excesses or deficiencies in measurements on the north

and west sides of the township, as required by law. In case where a township has been partially surveyed, and it is necessary to complete the survey of the same, or where the character of the land is such that only the north or west portions of the township can be surveyed, this rule can not be strictly adhered to, but, in such cases, must be departed from only so far as is absolutely necessary. It will also be necessary to depart from this rule where surveys close upon State or Territorial boundaries, or upon surveys extending from different meridians.

3. The townships are to bear numbers in respect to the base line, either north or south of it; and the tiers of townships called "ranges" will bear numbers in respect to the meridian line according to their relative position to it, either on the west or east.

4. The thirty-six sections into which a township is subdivided are numbered, commencing with number *one* at the *northeast* angle of the township, and proceeding west to number six, and thence proceeding east to number twelve, and so on, alternately, until the number thirty-six in the southeast angle. In all cases of surveys of fractional townships, the sections should bear the same numbers as they would if the township was full.

5. Standard parallels shall be established at intervals of every 24 miles, north and south of the base line, and auxiliary meridians at intervals of every 24 miles, east and west of the principal meridian; the object being to confine the errors resulting from convergence of meridians, and inaccuracies of measurements, within the tracts of lands bounded by the lines so established.

6. The survey of all principal base and meridian, standard parallels, and auxiliary meridian, and township lines, must be made with an instrument operating independently of the magnetic needle. Burt's *improved solar compass*, or other instrument of equal utility, must be used of necessity in such cases; and it is deemed best that such instrument should be used under all circumstances.

Where the needle can be relied on, however, the ordinary compass may be used in subdividing and meandering. Whenever deputies use instruments with magnetic apparatus only, they must test the accuracy of their work and the condition of their instruments by at least three observations upon a circumpolar star, upon different days, between the commencement and the close of surveying operations in any given township. Deputies using instruments with solar apparatus are not required to make observations of the star Polaris, but they must test their instruments by taking the latitude daily, the weather permitting, in running base, standard, meridian, and range lines, and upon three different days during the execution of subdivisional surveys in each township. They must make complete records in their field-notes, under proper dates, of the making of all observations in compliance with these instructions, showing the style and condition of the instrument in use, and the angle formed, by comparing the line run with the meridian as determined by observation.

7. The construction and adjustments of all surveying instruments used in the surveying of the public lands of the United States must be tested at least once a year, and oftener if necessary, by comparison with the true meridian, established under the direction of the surveyor-general of the district; and the instruments must be so modified in construction, or corrected in such a way, as may be necessary to produce the closest possible approximation to accuracy and uniformity in the operation of all such instruments. A record will be made of such examinations, showing the number and style of the instrument, name of the maker, the quantity of instrumental error discovered by comparison, in either solar or magnetic apparatus, or both, and means taken for correction. The surveyor-general will allow no surveys to be made until the instruments to be used therein have been approved by him.

8. The township lines and also the subdivision lines will usually be measured by a two-pole chain of 33.03 feet in length, consisting of 50 links, and each link being 7 inches and ninety-two hundredths of an inch long. On uniform and level ground, however, the four-pole chain may be used. The measurements will, however, always be represented according to the four-pole chain of 100 links. The four-pole chains must be adjusted to lengths of 66.06 feet. The object in adding six-hundredths of a foot to the 66 feet of a four-pole chain is to assure thereby that 66 feet will be set off upon the earth's surface without the application of a greater strain than about 20 pounds by the chainmen, thus providing for loss by vertical curvature of the chain, and at the same time avoiding the uncertain results attending the application of strains taxing its elasticity. The deputy surveyor must provide himself with a measure of the standard chain kept at the office of the surveyor-general, to be used by him as a field standard. The chain in use must be compared and adjusted with this field standard each working day, and such field standard must be returned to the surveyor-general's office for examination when his work is completed.

9. **Of Tally Pins.**—You will use eleven tally pins made of steel, not exceeding 14 inches in length, weighty enough toward the point to make them drop perpendicularly, and having a ring at the top, in which is to be fixed a piece of red cloth, or something else of conspicuous color, to make them readily seen when stuck in the ground.

10. **Process of Chaining.**—In measuring lines with a two-pole chain, every *five* chains are called a **Tally**; and in measuring lines with a four-pole chain, every *ten* chains are called a **Tally**, because at that distance the last of the ten tally pins with which the forward chainman set out will have been stuck. He then cries "tally"; which cry is repeated by the other chainman, and each

registers the distance by slipping a thimble, button, or ring of leather, or something of the kind, on a belt worn for that purpose, or by some other convenient method. The hind chainman then comes up, and having counted in the presence of his fellow the tally pins which he has taken up, so that both may be assured that none of the pins have been lost, he then takes the forward end of the chain, and proceeds to set the pins. Thus the chainmen alternately change places, each setting the pins that he has taken up, so that one is forward in all the odd, and the other in all the even tallies. Such procedure, it is believed, tends to insure accuracy in measurement, facilitates the recollection of the distances to objects on the line, and renders a mis-tally almost impossible.

11. **Leveling the Chain and Plumbing the Pins.**—The length of every line you run is to be ascertained by precise horizontal measurement, as nearly approximating to an air line as is possible in practice on the earth's surface. This all-important object can only be attained by a rigid adherence to the three following observances:

(1) Ever keeping the chain *stretched* to its utmost degree of tension on even ground.

(2) On uneven ground, keeping the chain not only stretched as aforesaid, but horizontally *leveled*. And when ascending and descending steep ground, hills, or mountains, the chain will have to be *shortened* to one-half its length (and sometimes more), in order accurately to obtain the true horizontal measurement.

(3) The careful plumbing of the tally pins, so as to attain precisely *the spot* where they should be stuck. The more uneven the surface, the greater the caution needed to set the pins.

12. **Marking Lines.**—All lines on which are to be established the legal corner boundaries are to be marked after this method, viz.: Any trees which may intercept the line must have two chops or notches cut on each

side of them, without any other marks whatever. These are called **Sight Trees** or **Line Trees**. A sufficient number of other trees standing within 50 links of the line, on either side of it, are to be *blazed* on two sides diagonally, or quartering toward the line, in order to render the line conspicuous, and readily to be traced, the blazes to be opposite each other, coinciding in direction with the line where the trees stand very near it, and to approach nearer each other the farther the line passes from the blazed trees. Due care must ever be taken to have the lines so well marked as to be readily followed, and to cut the blazes deep enough to leave recognizable scars as long as the trees stand.

Where trees 2 inches or more in diameter are found, the required blazes must not be omitted.

Bushes on or near the line should be bent at right angles therewith, and receive a blow of the ax at about the usual height of blazes from the ground sufficient to leave them in a bent position, but not to prevent their growth.

On trial, or random lines, the trees are not to be blazed, unless occasionally, from indispensable necessity, and then it must be done so guardedly as to prevent the possibility of confounding the marks of the trial line with the *true*. But bushes and limbs of trees may be lopped, and *stakes set* on the trial or random line, at every *ten* chains, to enable the surveyor on his return to follow and correct the trial line and establish therefrom the *true line*. To prevent confusion, the temporary stakes set on the trial or random line must be *pulled up* when the surveyor returns to establish the true line.

13. **Insuperable Objects on Line—Witness Points.**—Under circumstances where your course is obstructed by impassable obstacles, such as ponds, swamps, marshes, lakes, rivers, creeks, etc., you will prolong the line across such obstacles by means of right angle offsets; or, if

such be inconvenient, by a traverse or trigonometrical operation, until you regain the line on the opposite side. And in case a north and south, or a true east and west, line is regained in advance of any such obstacle, you will prolong and mark the line back to the obstacle so passed, and state all the particulars in relation thereto in your field-book. And, at the intersection of lines with both margins of impassable obstacles, you will establish a *witness point*, for the purpose of perpetuating the intersections therewith, by setting a post, and giving in your field-book the course and distance therefrom to two trees on opposite sides of the line, each of which trees you will mark with a blaze and notch facing the post; but on the margins of navigable water-courses, or navigable lakes, you will mark the trees with the proper number of the fractional section, township and range.

The best marking tools adapted to the purpose must be provided for marking neatly and *distinctly* all the letters and figures required to be made at corners, *Arabic* figures being used exclusively; and the deputy is always to have at hand the necessary implements for keeping his marking tools in order.

Establishing Corners.—To procure the faithful execution of this portion of a surveyor's duty is a matter of the utmost importance. After a true coursing and most exact measurements, the establishment of corners is the consummation of the work. If, therefore, the corner be not perpetuated in a permanent and workmanlike manner the *great aim* of the surveying service will not have been attained.

The following are the different points for perpetuating corners, viz.:

1. For township boundaries, at intervals of every 6 miles.

2. For section boundaries, at intervals of every mile, or 80 chains.

3. For quarter-section boundaries, at intervals of every half mile, or 40 chains. Exceptions, however, occur as fully set forth hereafter in that portion of the manual showing the manner of running township lines, and method of subdividing.

4. Meander corners are established at all those points where the lines of the public surveys intersect the banks of such rivers, bayous, lakes, or islands as are by law directed to be meandered.

Standard quarter-section corners on standard lines must be established in all respects like other quarter-section corners, with the addition of the letters S. C., and if bearing trees are established for such corners, each tree must be marked S. C. ¼ S. B. T.

SEC. 9. When a pit is dug at a meander corner, it must be 8 lks. from the corner, on the side opposite the river or lake meandered.

SEC. 10. The letters M. C., for meander corner, must be marked on the side facing the river or lake meandered.

Witness Corners.—A witness corner must bear the same marks that would be placed upon the corner for which it is a witness, with the addition of the letters W. C., and be established in all respects like such corner.

If bearing trees are established for a witness corner, each tree must be marked W. C., in addition to the usual marks.

Miscellaneous.—SEC. 1. When a rock in place is established for a corner, its dimensions above ground must be given, and a cross (×) marked at exact corner point. In other respects, form for stone corners will be used.

SEC. 2. Where mounds of earth are raised "alongside" of corners, on N. and S. lines, they must be placed on the W. and on E. and W. lines on the N. side of corner. In case the character of the land is such that this cannot be done, the deputy will state in his notes instead of "alongside," "S." (or E.)

SEC. 3. In case where pits are practicable, the deputy prefers raising a mound of stone, or stone covered with earth, as more likely to perpetuate the corner, he will use the form given for mound of stone, omitting the words "pits impracticable," and adding "covered with earth," when so established.

SEC. 4. Where the requisite number of trees can be found within 300 links of the corner point, three (3) bearing trees should be established for every *standard* or *closing corner*, four (4) for every *corner* common to 4 *townships* or *sections*, and two (2) for every *quarter section corner* or *meander corner*. In case the requisite number cannot be found within limits, the deputy must state in his field-notes after describing those established, "no other trees within limits," and "dug pits in Secs. — & —," or "raised a mound of stone alongside."

SEC. 5. Stones 18 in. and less long must be set two thirds, and over 18 in. long, three-fourths of their length in the ground. No stones containing less than 504 cubic inches must be used for corners.

SEC. 6. Particular attention is called to the "Summary of objects and data required to be noted," on pages — and — of these instructions, and it is expected that the deputy will thoroughly comply with same in his work and field-notes.

SEC. 7. No mountains, swamp lands, or lands not classed as surveyable, are to be meandered, and all lines approaching such lands must be discontinued at the section or quarter section corner.

SEC. 8. Where by reason of impassable objects, the south boundary of a township cannot be established, an east and west line should be run through the township, first random, then corrected, from one range line to the other, and as far south as possible, and from such line the section lines will be extended in the usual manner, except over any fraction south of said line, which may be sur-

veyed in the opposite direction from the section corners on the auxiliary base thus established.

SEC. 9. When no part of the east or west boundaries can be run, both the north and south boundaries will be established as true lines.

SEC. 10. Allowance for the convergency of meridians must be made whenever necessary.

SEC. 11. All letters and figures cut in posts or trees must be marked over with red chalk to make them still more plain and durable.

SEC. 12. Township corners common to four townships, and section corners common to four sections, are to be set diagonally in the earth, with the angles in the direction of the lines. All other corners are to be set square, with the sides facing the direction of the lines.

SEC. 13. The sizes of wooden posts, mounds, and pits noted in foregoing descriptions of corners, are to be regarded as *minimum*, and whenever practicable to increase their dimensions it is desirable to do so.

SEC. 14. In establishing corners, stones should be used wherever practicable; then posts; and lastly, mounds, with stake in pit.

SEC. 15. It is expected that deputy surveyors will carefully read and familiarize themselves with these instructions, and all others contained in this volume, and will instruct their assistants as to their duties before commencing work. Extra copies will be furnished the deputies for the use of their assistants.

Meandering.—SEC. 1. Proceeding *down* stream, the bank on the *left* hand is termed the **Left Bank**, and that on the *right* hand the **Right Bank**. These terms are to be universally used to distinguish the two banks of a river or stream.

SEC. 2. Both banks of *navigable* rivers are to be meandered by taking the general courses and distances of their sinuosities, and the same are to be entered in the field-book.

At those points where either the township or section lines intersect the banks of a navigable stream, corners are to be established at the time of running these lines. These are called **Meander Corners**; and in meandering, you are to commence at one of those corners, coursing the banks, and measuring the distance of each course from your commencing corner to the next meander corner. By the same method you are to meander the opposite bank of the same river.

The crossing distance *between the meander corners on same line* is to be ascertained by triangulation, in order that the river may be protracted with entire accuracy. The particulars to be given in the field notes.

Rivers not embraced in the class denominated "navigable" under the statute, but which are well-defined natural arteries of internal communication, will only be meandered *on one bank*. For the sake of uniformity, the surveyor will traverse the *right bank* when not impracticable; but where serious obstacles are met with, rendering it difficult to course along the right bank, he may cross to the left bank and continue the meanders as far as necessary; but all changes from one bank to the other will be made at the point of intersection of some line of the public surveys with the stream being meandered.

The subdividing deputies will be required to establish meander corners on both banks of such meanderable streams at the intersection of all section lines, and the distances across the river will be noted in the field-book.

In meandering water-courses, where a distance is more than *ten chains* between stations, even chains only should be taken; but if the distance is *less* than ten chains, and it is found convenient to employ chains and links, the number of links should be a multiple of ten, thereby saving time and labor in testing the closings both in the field and in the surveyor-general's office.

SEC. 3. You are also to meander, in manner aforesaid, all lakes, bayous, and deep ponds, which may serve as

public highways of commerce. Shallow lakes or ponds, readily to be drained or likely to dry up, are not to be meandered. Lakes, bayous, and ponds lying entirely within a section, are not to be meandered.

In meandering lakes, bayous, or ponds you are to commence at a meander corner, and proceed as above directed for meandering the banks of navigable streams; and from said corner take the courses and distances of the entire margin of the same, noting the intersections with all meander corners established thereon.

You will notice all streams of water falling into the river, lake, or bayou you are surveying, stating the width of the same at their mouth; also all springs, noting the size thereof and depth, and whether the water be pure or mineral; also the head and mouth of all bayous; and all islands, rapids, and bars are to be noticed, with intersections to their upper and lower points to establish their exact situation. You will also note the elevation of the banks of rivers and streams, the hights of falls and cascades, and the length of rapids.

SEC. 4. Meander lines should not be established at the segregation line between dry and swamp or overflowed land but at the ordinary low-water mark of the actual margin of the rivers or lakes on which such swamp or overflowed lands border. In cases where such meander lines were formerly established at the segregation line between dry and swamp or overflowed lands, new and proper meander lines may be established under the direction of the surveyor-general, and the township and section-lines extended over such swamp or overflowed lands and the corners established, as hereinbefore provided, in order that the plats and field-notes of surveys may show the actual facts in the case.

SEC. 5. The precise relative position of islands, in a township made fractional by the river in which the same are situated, is to be determined trigonometrically; sighting to a flag or other fixed object on the island, from a

special and carefully measured base line, connected with the surveyed lines, on or near the river bank, you are to form connection between the meander corners on the river to points corresponding thereto, in direct line, on the bank of the island, and there establish the proper meander corners, and calculate the distance across.

SEC. 6. In taking the connection of an island with the mainland, when there is no meander corner in line, opposite thereto, to sight from, you will measure a special base from the meander corner nearest to such island, and from such base you will triangulate to some fixed point on the shore of the island, ascertain the distance across, and there establish a *special* meander corner, wherefrom you will commence to meander the island.

SEC. 7. The field-notes of meanders will be set forth in the field-books showing the dates when the work is performed, as illustrated in the specimen notes annexed. They are to state and describe particularly the meander corner at which they commenced, and each one upon which they may close, and are to exhibit the meanders of each fractional section separately; following, and composing a part of such notes, will be given a description of the land, timber, depth of inundation to which the bottom is subject, and the banks, current, and bottom of the stream or body of water you are meandering. The utmost care must be taken to pass no object of topography, *or change therein*, without giving a particular description thereof in its proper place in your meander notes.

Surveying.—Initial points from which the lines of the public surveys are to be extended must be established whenever necessary under such special instructions as may be prescribed in each case by the Commissioner of the General Land Office. The locus of such initial points must be selected with great care and due consideration for their prominence and easy identification, and must be established astronomically.

The initial point having been established, the lines of the public surveys are to be extended therefrom as follows:

Base Line.—The base line shall be extended east and west from the initial point by the use of solar instruments or transits, as may be directed by the surveyor-general, in his special written instructions. Where solar instruments are used, the deputy must test said instruments in every 12 miles of line run, by taking the latitude, or by observation on the polar star; and in all cases where he has reason to suppose that said instrument is in error, he must take an observation on the polar star, and if error be found, must make the necessary corrections before proceeding with his survey. The proper corners shall be established at each 40 and 80 chains, and at the intersection of the line with rivers, lakes, or bayous that should be meandered, in accordance with the instructions for the establishment of corners. In order to check errors in measurement, two sets of chainmen, operating independently of each other, must be employed.

Where transits are used, the line will be run by setting off at the point of departure on the principal meridian a tangent to the parallel of latitude, which will be a line falling at right angles to the said meridian. The survey will be continued on this line for twelve (12) miles, but the corners will be established at the proper points by offsets northerly from said line, at the end of each half mile. In order to offset correctly from the tangent to the parallel, the deputy will be guided by the table of offsets and azimuths contained in this volume. As the azimuth of the tangent is shown, the angle thence to the true meridian at each mile is readily found, thus indicating the direction of the offset line. The computations are made for a distance of 12 miles, at the end of which observations on the polar star must be taken for the projection of a new tangent. The computations are also upon even degrees of latitude; offsets for intervening

parallels can be readily determined by interpolation. Where offset distances to quarter-section corners exceed 50 links, their direction to the parallel can be determined in like manner by interpolation for azimuth.

Principal Meridian.—The principal meridian shall be extended north and south from the initial point, by the use of solar instruments or transits, as may be directed by the surveyor-general in his special written instructions. Where solar instruments are used, the line will be run in the same manner as prescribed for running the base line by solar instruments. Where transits are used, observations upon the polar star must be taken within each 12 miles of line run. In addition to the above general instructions, it is required that in all cases where the establishment of a new principal meridian seems to be necessary to the surveyor-general, he shall submit the matter, together with his reasons therefor, to the Commissioner of the General Land Office, and the survey of such principal meridian shall not be commenced until written authority, together with such special instructions as he may deem necessary, shall have been received from the Commissioner.

Standard Parallels.—Standard parallels, which are also called correction lines, shall be extended east and west from the principal meridian, at intervals of every 24 miles north and south of the base line, in the same manner as prescribed for running the base line.

Auxiliary Meridians.—Auxiliary meridians shall be extended north and south from the base line, at intervals of every 24 miles east and west from the principal meridian, in the same manner as prescribed for running the principal meridian.

It is contemplated that these base, principal meridian, standard, and auxiliary meridian lines shall first be extended over the territory to be surveyed, and that afterwards township and section lines shall be run, where

needed, within these tracts of 24 miles square, formed by the extension of these principal lines; and each surveyor-general will therefore cause said principal lines to be extended as rapidly as practicable.

Exteriors or Township Lines.—The east and west boundaries of townships are always to be run from south to north on a true meridian line; and the north and south boundaries are to be run from east to west, or from west to east (according to the location of the township to be surveyed with reference to prior surveys), on a *random* or trial line and corrected back on a true line. The distance north or south of the township corner to be closed upon, from the point of intersection of these random lines with the east or west boundary of the township, must be carefully measured and noted. Should it happen, however, that such random line should fall short, or overrun in length, or intersect the east or west boundary more than *three chains'* distance from the township corner thereon, as compared with the corresponding boundary on the south (due allowance being made for convergency), the line, and if necessary the entire exterior boundaries of the township, must be retraced, so as to discover and correct the error. In running random lines, temporary corners are to be set at each 40 and 80 chains, and permanent corners established upon the true line as corrected back, in accordance with instructions, throwing the excess or deficiency on the west half mile, as prescribed by law. Permanent corners are to be established in accordance with instructions, on the east and west township boundaries at the time they are run. Whenever practicable, the township lines within these tracts of 24 miles square must be surveyed in regular order from *south* to *north; i. e.*, the exterior boundaries of the township in any one range lying immediately north of the south boundary of such tract of 24 miles square must first be surveyed, and the exteriors of the other three townships in said range extended therefrom, in regular

order, from *south* to *north;* and it is preferable to survey, first, the entire range of townships in such tract adjoining the east boundary, or adjoining the west boundary, and the other three ranges in regular sequence. In cases, however, where the character of the land is such that this rule cannot be complied with, the following will be observed:

In extending the *south* or *north* boundaries of a township to the *west*, where the *southwest* or *northwest* corners cannot be established in the regular way by running a north and south line, such boundaries will be run *west* on a *true* line, allowing for convergency on the west half mile; and from the township corner established at the end of such boundary, the west boundary will be run *north* or *south*, as the case may be. In extending *south* or *north* of a township to the *east*, where the *southeast* or *northeast* corner cannot be established in the regular way, the same rule will be observed, except that such boundaries will be run *east* on a *true line*, and the *east* boundary run *north* or *south*, as the case may be. One set of chainmen only is required in running township lines.

Method of Subdividing.—1. The first mile, both of the south and east boundaries of each township you are required to subdivide, is to be carefully traced and measured before you enter upon the subdivision thereof. This will enable you to observe any change that may have taken place in the magnetic variation, as it existed at the time of running the township lines, and will also enable you to compare your chaining with that upon the township lines.

2. Any discrepancy arising either from a change in the magnetic variation or a difference in measurement, is to be carefully noted in the field-notes.

3. After adjusting your compass to a variation which you have thus found will retrace the eastern boundary of the township, you will commence at the corner of sections

35 and 36. on the south boundary, and run a line parallel to the range line, forty chains, to the quarter-section corner, which you are to establish between sections 35 and 36; continuing on said course forty chains farther, you will establish the corner to sections 25, 26, 35, and 36.

4. From the section corner last named, run a *random* line, without blazing, *due east*, for the corner of sections 25 and 36, on east boundary, and at forty chains from the starting point set a post for *temporary* quarter-section corner. If you intersect exactly at the corner, you will blaze your random line back, and establish it as the *true* line; but if your random line intersects the said east boundary either north or south of said corner, you will measure the distance of such intersection, from which you will calculate a course that will run a *true* line back to the corner from which your random started. You will establish the *permanent* quarter-section corner at a point equidistant from the two terminations of the *true* line.

5. From the corner of sections 25, 26, 35, and 36, run due north between sections 25 and 26, setting the quarter-section post as before, at forty chains, and at eighty chains establishing the corner of sections 23, 24, 25, and 26. Then run a random *due east* for the corner of sections 24 and 25 on east boundary; setting temporary quarter-section post at forty chains; correcting back, and establishing *permanent* quarter-section corner at the equidistant point on the *true* line, in the manner directed on the line between sections 25 and 36.

6. In this manner, you will proceed with the survey of each successive section in the first tier, until you arrive at the north boundary of the township, which you will reach in running up a random line between sections 1 and 2. If this random line should not intersect at the corner established for sections 1, 2, 35, and 36, upon the township line, you will note the distance that you fall east or west of the same, from which distance you will

calculate a course that will run a true line south to the corner from which your random started. If the north boundary of a township is a base or standard line, the line between sections 1 and 2 is to be run north as a *true* line, and the closing corner established at the point of intersection with such base or standard line; and in such case, the distance from said closing corner to the nearest section or quarter-section corner on such base or standard line must be carefully measured and noted as a *connection line.*

7. In like manner, proceed with the survey of each successive tier of sections, until you arrive at the fifth tier; and from each section corner which you establish upon this tier you are to run random lines to the corresponding corners established upon the range line forming the western boundary of the township; setting, as you proceed, each *temporary* quarter-section corner at forty chains from the interior section corner, so as to throw the excess or deficiency of measurement on the extreme tier of quarter sections contiguous to the township boundary; and on returning establish the *true* line, and establish thereon the *permanent* quarter-section corner.

8. It is not required that the deputy shall complete the survey of the first tier of sections from south to north, before commencing the survey of the second or any subsequent tier, but the corner on which the random line closes must have been previously established by running the line north on which it is established, except as follows: Where it is impracticable to establish such section corner in the regular manner, it may be established by running the east and west line *east* or *west*, as the case may be, *on a true line*, setting the quarter-section corner at 40 chains and the section corner at 80 chains.

9. Quarter-section corners, both upon north and south and upon east and west lines, are to be established at a point *equidistant* from the corresponding section corners, *except* upon the lines crossing on the north and west

boundaries of the township, and in those situations the quarter-section corners will always be established at precisely *forty chains* to the north or west, as the case may be, of the respective section corners from which those lines respectively *start*, by which procedure the excess or deficiency in the measurements will be thrown, according to law, on the extreme tier of quarter-sections.

Prescribed Limits for Closing and Length of Lines in Certain Cases.—1. Every north-and-south section line, except those terminating in the north boundary of the township, must be *eighty chains* in length.

2. The east-and-west *section lines*, except those terminating in the west boundary of the township, are to be within *eighty links* of the actual distance established on the south bondary line of the township for the width of said tier of sections, and must close within eighty links north or south of the section corner.

3. The north boundary and south boundary of any one section, except in the extreme western tier, are to be within *eighty links* of equal length.

4. The meanders within each fractional section, or between any two meander posts, or of an island in the interior of a section, must close within one chain and fifty links.

5. In running *random* township exteriors, if such random lines fall short or overrun in length, or intersect the eastern or western boundary, as the case may be, of the the township, at more than *three chains* north or south of the true corner, the lines must be *retraced*, even if found necessary to remeasure the meridianal boundaries of the township. One set of chainmen, only, is required in subdividing.

Subdivision of Sections.—Under the provisions of the act of Congress approved February 11, 1805, the course to be pursued in the subdivision of sections is to run straight lines from the established quarter-section corners

—United States surveys—to the opposite corresponding corners, and the point of intersection of the lines so run will be the corner common to the several quarter-sections, or, in other words, the legal center of the section.

In the subdivision of fractional quarter sections, where no opposite corresponding sections have been or can be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east or west lines, as the case may be, to the water-course, Indian boundary line, or other external boundary of such fractional section.

The law presupposes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines, but in actual experience this is not always the case; hence, in order to carry out the spirit of the law, it will be necessary, in running the subdivisional lines through fractional sections, to adopt mean courses where the section *lines* are not due lines, or to run the subdivision line parallel to the section line when there is no opposite section line.

Upon the lines closing on the north and west boundaries of a township, the quarter-section corners are established by the United States deputy surveyors at precisely forty chains to the north or west of the last interior section corners, and the excess or deficiency in the measurement is thrown on the outer tier of lots, as per act of Congress approved May 10, 1800.

In the subdivision of quarter-sections, the quarter-quarter corners are to be placed at points equidistant between the section and quarter-section corners and between the quarter corners and the common centre of the section, *except* on the last half mile of the lines closing on the north or west boundaries of a township, where they should be placed at twenty chains, proportionate measurement, to the north or west of the quarter-section corner.

The subdivisional lines of fractional quarter-sections should be run from points on the section lines intermediate between the section and quarter-section corners due north, south, east, or west, to the lake, water-course, or reservation which renders such tracts fractional.

When there are double sets of section corners on township and range lines, the quarter corners for the sections south of the township lines and east of the range lines are not established in the field by the United States surveyors; but in subdividing such sections, said quarter corners should be so placed as to suit the *calculations of the areas of the quarter-sections adjoining the township boundaries* as expressed upon the official plat, adopting proportionate measurements where the present measurements of the north or west boundaries of the sections differ from the original measurements.

Re-Establishment of Lost Corners.—The original corners, when they can be found, must stand as the true corners they were intended to represent, even though not exactly where strict professional care might have placed them in the first instance.

As has been observed, no existing original corner can be disturbed, and it will be plain that any excess or deficiency in measurements between existing corners cannot in any degree affect the distances beyond said existing corners, but must be added or subtracted proportionately to or from the intervals embraced between the corners which are still standing.

Retracing Township Lines.—If, in subdividing a township, it is found that the exterior boundaries have been improperly run, measured or marked, or the corners established thereon have been obliterated, the deputy will resurvey so much of said exterior boundaries as may be necessary, and establish new corners upon same wherever necessary. Where no subdivions have been made on either side of a township boundary, it will be cor-

rected, if necessary, in point of alignment as well as measurement, by establishing the section corners at lawful distances from the south or east boundaries of the township, as the case may be, and upon a right line extending between the township corners; and in such case, the old corners of said township boundaries will be destroyed.

Where subdivisional lines have been closed upon a township boundary in advance of the preliminary survey of the same, its alignment will not be changed. If it is found necessary to establish new corners on such boundary they will receive only the marks referring to the sections in the township being subdivided, and the marks on the old corners on such boundary, which refer to such sections, will be obliterated.

In all cases, such necessary corrections will be made as will place the section corners at the aforesaid lawful distances from the south or east boundary, in order that a legal subdivision of the township may be made, and where new corners are thus necessarily established, the distance, be it one hundred links or more, and direction between new and old corners must be carefully noted.

New corners on township boundaries must be established by a survey of such lines, and *in no case* will such corners be established from data acquired in running lines closing on such boundaries. One set of chainmen, only, is required in retracing township lines.

If, in the subdivision of part of a township, the lands to be surveyed cannot be reached by lines extending from the south boundary of the township, a line corresponding to the south boundary of the same shall be extended from some section corner on the east boundary of the township to the west boundary thereof, in order that it may constitute the south boundary of the surveyable area; from which subdivisional meridian lines will be projected northward, and the surveys carried forward in the same manner as for the subdivision of a full township, in

order that regular and fractional areas shall occupy their true and legal positions.

Fragmentary portions of surveyable lands lying south of the provisional base last described may be included in the survey by extending lines *south* from the same in harmony with the general system.

When the proper point for the establishment of a section corner is inaccessible, and a witness monument can be erected upon each of the two lines which approach the same at distances not exceeding twenty chains therefrom, the quarter-sections depending thereon will be disposed of in the same manner as if the corner had been regularly established.

The witness monument must be marked as conspicuously as a section corner, and bearing trees used wherever possible.

The deputy will be required to furnish good evidence that the section corner is actually inaccessible.

When township or subdivision lines intersect the boundaries of confirmed private land claims, the latter must be retraced so far as may be necessary to establish the corners to the fractional sections at their proper places, and such corners must be established, in all respects, like meander corners, except that instead of the letters M. C. the letters used to designate such private land claim must be marked on corners. In retracing the boundary of such claim, the deputy must set stakes thereon, at each forty chains, where the ground is level, and on broken ground, at every spur, ridge, or other prominent point, and also at each angle formed by a change in the direction of such boundary.

Field Notes.—The deputy surveyor will provide himself with proper blank books for his field notes, or same will be furnished to him by the surveyor-general, and in such books he must make a faithful, distinct, and minute record of everything officially done and observed by himself and his assistants, pursuant to instructions, in rela-

tion to running, measuring and marking lines, establishing corners, etc., and present, as far as possible, a full and complete topographical description of the country surveyed.

From the *data* thus recorded at the time when the work is done on the ground, the deputy must prepare *true* field notes of the surveys executed by him, in the manner hereinafter prescribed, and return same to the surveyor-general, together with the required sketches, at the earliest practicable date after the completion of his work in the field.

The field notes of the survey of base, meridian, standard, exterior, and subdivision lines are each to be written in separate books.

The first page, or title, of the field-note book is to describe the subject-matter of the same, the locus of the survey, by whom surveyed, date of contract, and the dates of commencement and completion of the work. The second page is to contain the names and duties of the assistants, and the index is to be placed on same or following page. Whenever a new assistant is employed, or the duties of any one of them changed, such facts are to be stated in an appropriate entry immediately preceding the notes taken under such changed arrangements.

The exhibition of every mile of surveying, whether on township or subdivisional lines, and of meanders in each section, must be *complete in itself*, and be separated by a black line drawn across the paper.

The variation of the needle must always occupy a *separate line* preceding the notes of measurements on line.

The description of the surface, soil, minerals, timber, undergrowth, etc., on *each mile* of line, is to follow the notes of survey of such line, and not be mixed up with them.

The date of each day's work must follow immediately after the notes thereof.

No abbreviations of words are allowable, except of such

words as are *constantly* occurring, such as "*sec.*" for "*section*"; "*in. diam.*" for "*inches diameter*"; "*ch.*" for "*chains*"; "*lk.*" for "*links*"; "*dist*" for "*distant*"; "¼ *sec. cor.*" for "*quarter-section corner*"; "*va.*" for "*variation,*" etc.; for 14 inches long, 12 inches wide, and 3 inches thick, in describing a corner stone, use 14 × 12 × 3, being particular always to observe the same order of length, width, and thickness. Proper names must never be abbreviated, however often their recurrence.

When the lines of survey cross hills or ravines, the hight or depth of same, in feet, must be noted as nearly as practicable.

The corners established in previous surveys, from which the lines start, or upon which they close, must be fully described in the field notes. A full description of such corners will in all cases be furnished the deputy from the surveyor-general's office at the date authority is given for commencing work.

In all cases where a corner is re-established, the field notes must describe fully the manner in which it is done.

Field notes of the survey of base, standard, and meridian lines must describe all corners established thereon, how established, the crossings of streams, ravines, hills, and mountains; character of soil, timber, minerals, etc.; and after the description of each township corner established in running such lines, the deputy will note particularly in the "general description" the townships on each side of the lines run.

Field notes of the survey of exterior boundaries of townships must describe the corners and topography, as above required, and the "general description" at the end of such notes must describe the townships as fully as may be, and also state whether or not they should be subdivided. The topography on the *true line* of exterior boundaries must be given, and not that on the random line.

Field notes of the subdivisional survey of townships

must describe the corners and topography as above required, and the "general description" at the end of such notes must state minutely the character of the land, soil, timber, etc., found in such townships.

A blank line must be left at the bottom of each page of the field notes, and the notes must be written in a plain, legible hand, and in clear and precise language, so that the figures, letters, words, and meaning will always be unmistakable, and erasures and interlineations avoided as far as possible.

With the notes of the survey of principal lines forming a tract of 24 miles square, the deputy will submit a plat of the lines run, on a scale of one-half inch to the mile, and with the notes of survey of the exterior lines of townships, a plat of the lines run, on the scale of two inches to the mile, on which are to be noted all the objects of topography on line necessary to illustrate the notes, viz., the distance on line at the crossings of streams, so far as such can be noted on the paper, and the direction of each by an arrow-head pointing down stream; also the intersection of line by prairies, marshes, swamps, ravines, ponds, lakes, hills, mountains, and all other matters indicated by the notes, to the fullest extent practicable.

With the instructions for making subdivisional surveys of townships into sections, the deputy will be furnished by the surveyor-general with a diagram of the *exterior* lines previously established of the townships to be subdivided, on the above-named scale, upon which are carefully to be laid down the measurements of each of the lines on such boundaries whereon he is to close, and the magnetic variation of each mile. And on such diagram, the deputy who subdivides will make appropriate sketches of the various objects of topography as they occur on his lines, so as to exhibit not only the points on line at which the same occur, but also the direction and position of each between the lines, or within each section, as far as prac-

ticable, so that every object of topography may be properly completed or connected in the showing.

Summary of Objects and Data required to be noted.—1. The precise length of every line run, noting all necessary offsets therefrom, with the reason and mode thereof.

2. The kind and diameter of all "*bearing trees*," with the course and distance of the same from their respective corners; and the precise relative position of *witness corners* to the *true corners.*

3. The kind of materials of which corners are constructed.

4. *Trees on line.* The name, diameter, and distance on line to all trees which it intersects.

5. *Intersections* by line of *land objects.* The distance at which the line first intersects and then leaves every *settler's claim and improvement;* prairie, river, creek, or other "bottom"; or swamp, marsh, grove, and windfall, with the course of the same at both points of intersection; also the distances at which you begin to ascend, arrive at the top, begin to descend, and reach the foot of all remarkable hills and ridges, with their courses, and *estimated* hight, in feet, above the level land of the surrounding country, or above the bottom lands, ravines, or waters near which they are situated.

6. *Intersections* by line of *water objects.* All rivers, creeks, and smaller streams of water which the line crosses; the distances on line at the points of intersection, and their *widths on line.* In cases of *navigable* streams, their width will be ascertained between the *meander corners,* as set forth under the proper head.

7. The *surface*—whether level, rolling, broken, or hilly.

8. The *soil*—whether first, second, third, or fourth rate.

9. *Timber*—the several kinds of timber and undergrowth, in the order in which they predominate.

10. *Bottom lands*—to be described as wet or dry, and if subject to inundation, state to what depth.

11. *Springs of water*—whether fresh, saline, or mineral, with the course of the stream flowing from them.

12. *Lakes* and *ponds*—describing their banks and giving their hight, and also depth of water, and whether it be pure or stagnant.

13. *Improvements.* Towns and villages; houses or cabins; fields, or other improvements; sugar-tree groves, sugar camps, mill seats, forges, and factories.

14. *Coal* banks or beds; *peat* or turf grounds; *minerals* and ores; with particular description of the same as to quality and extent, and all *diggings* therefor; also *salt* springs and licks. All reliable information you can obtain respecting these objects, whether they be on your immediate line or not, is to appear in the general description to be given at the end of the notes.

15. *Roads* and *trails*, with their directions, whence and whither.

16. Rapids, cataracts, cascades, or falls of water, with the estimated hight of their fall in feet.

17. Precipices, caves, sink-holes, ravines, stone quarries, ledges of rocks, with the kind of stone they afford.

18. *Natural curiosities*, interesting fossils, petrifactions, organic remains, etc.; also all ancient works of art, as mounds, fortifications, embankments, ditches, or objects of like nature.

19. The *variation* of the needle must be noted at all points or places on the lines where there is found any material *change* of variation, and the position of such points must be perfectly identified in the notes.

20. Besides the ordinary notes taken on line, and which must always be written down on the spot, leaving nothing to be supplied by memory, the deputy will subjoin, at the conclusion of his book, such further description or information touching any matter or thing connected with the township, or other survey, which he may be able to afford, and may deem useful or necessary to be known, with a *general description* of the township in the *aggre-*

gate, as respects the face of the country, its soil and geological features, timber, minerals, waters, etc.

Following the "general description" of the township, is to be "a list of the names of the individuals employed to assist in running, measuring, and marking the lines and corners described in the foregoing field notes of township No. —— of the *base line* of range No. —— of the —— *meridian,* showing the respective capacities in which they acted."

Specimen Field Notes and Plats.—Deputy surveyors are furnished with specimen plats, field notes, and diagrams. The attention of the deputy is particularly directed to these specimens, as indicating not only the method in which his work is to be conducted, but also the order, manner, language, etc., in which his field notes are required to be returned to the surveyor-general's office; and such specimens are to be deemed part of these instructions; and any *departure* from their details, without special authority, in cases where the circumstances are analogous in practice, *will be regarded as a violation of his contract and oath.*

The subdivisions of fractional sections into 40-acre lots, as near as may be, are to be so laid down on the official township plat in dotted black lines as to admit of giving to each a specific designation, if possible, according to its relative position in the fractional section, as per examples afforded by diagram B, as well as by a number, in all cases where the lot cannot properly be designated as a quarter-quarter. Those fractional subdivision lots which are not susceptible of being described according to relative local position, are to be numbered in regular series; those bordering on the north boundary of a township to be numbered progressively from east to west, and those bordering on the west boundary of a township to be numbered progressively from north to south, in each section. As section 6 borders on both the north and west boundaries of the township, the fractional lots in same will be

numbered as follows: Commencing with number 1 in the northeast, thence progressively west to number 4 in the northwest, and south to number 7 in the southwest corner of the section.

In numbering fractional lots, other than those above specified, wherever practicable and as a general rule, the series should commence with number 1 in the northeastern, or the most easterly fractional lot, and continue from east to west, and west to east, alternately, to the end of the series, as shown in diagram B; but such general rule is departed from under circumstances given as examples in said diagram.

Interior lots are to be, as nearly as possible, 20 chains long by 20 chains wide; and the excess or deficiency of measurement is always to be thrown on the lots bordering on the northern and western boundaries of the township, or those made fractional by meander lines.

The official township plat to be returned to the General Land Office is to show on its face, on the right-hand margin, the meanders of navigable streams, islands, and lakes. Such details are wanted in the adjustment of the surveying accounts, but may be omitted in the copy of the township plat to be furnished to the district land office by the surveyor-general. A suitable margin for *binding* is to be preserved on the left-hand side of each plat. Each plat is to be certified, with table annexed, according to the forms subjoined to "Diagram B," and is to show the areas of public land, of private surveys, and of water, with the aggregate area as shown on the diagram.

Each township is to be prepared in *triplicate*: one for the General Land Office, one for the United States district land office, and the third to be retained as the record in the office of the surveyor-general.

The plat for the local land office must not be forwarded until notice is received by the surveyor-general from the

Commissioner of the General Land Office that the survey represented on said plat has been approved.

The plats must be prepared as nearly as possible in accordance with the specimen plat designated as "Diagram B." The use of all fluids, except a preparation of India ink of good quality, must be avoided by the draughtsman in delineations relating to the public surveys. All lines, figures, etc., must be sharply defined. All lettering on the plats must be clear and sharp in outline and design, and ornamentation of any kind is prohibited. These requirements are necessary in order that everything shown upon original plats may be fairly reproduced in making photolithographic copies of the same.

All towns, settlements, permanent buildings, private claims, reservations, water-courses, ditches, lakes, islands, mountains, buttes, cañons, roads, railroads, telegraph lines, canals, etc., will be shown upon the plats and designated by proper names where such are known.

The mean magnetic declinations determined at the date of the survey of the exterior and subdivisional lines will be entered upon each plat in the manner shown in diagram B. This will be ascertained by taking the mean of the greatest and least magnetic declination found at the dates of surveys, excluding such changes as are clearly attributable to local attraction.

All plats are to be drawn to a uniform scale of 40 chains to 1 inch, United States standard.

Surveyors-general will require that the specimen plat shall be closely followed in order that uniformity of appearance and expression of drawings representing the public land surveys may be attained.

The true field books, each bearing the *written approval* of the surveyor-general, are to be substantially bound into volumes of suitable size, and retained in the surveyor-general's office, and certified *transcripts* of such field books, to be of *foolscap* size, are to be prepared and forwarded, from time to time, to the General Land Office.

All transcripts of surveys must be written in a bold, legible hand, with durable black ink; and such transcripts of any series of surveys included in one account forwarded to the General Land Office, must be firmly fastened together at the surveyor-general's office prior to transmittal.

With the copy of each township plat furnished to a district land office, the surveyor-general is required by law to furnish *descriptive notes* as to the character and quality of the soil and timber found on and in the vicinity of each surveyed line, and giving a description of each corner.

Printed blank forms for such notes will be furnished by the General Land Office.

NOTE.—The specimen plats, field notes, and diagrams are furnished by the government to all surveyors in its employ.

Any person desiring them, can procure the full "Manual of Instructions to Surveyors-General" by applying to the Commissioner of the General Land Office, Washington, D. C.

221. Resurvey in Northern Michigan.—Many of the first surveys in the northern part of Michigan were imperfect, and some of them fraudulent. In consequence, a second survey of 155 townships was made, under the following special instructions, issued from the Surveyor General's Office in Detroit, April, 1852:

"SIR: By your contract dated April —, 1852, you are required to resurvey the townships herein enumerated, in conformity with the following instructions, which are deemed necessary in addition to the printed manual of instructions to the deputies of this office, with which you are also furnished. This manual will be your guide in reference to the general subject of surveys,—and you are requested to notice particularly such portions thereof as have been specially marked or amended, as forming a part of your special instructions. To aid you in the preliminary examinations which will be necessary in each township before any resurveys are commenced, you are furnished with a copy of the original plats of the townships in your district, and other information, which enable you the better to understand the nature of your duties.

You are required to use every exertion in searching for the marks of the old surveys, and wherever the original lines and corners can be

found, if regularly established so as to be available in a correctional survey, they are to be respected, retraced, and reestablished by you, and so connected with the new surveys as to form a complete survey of each township.

The destruction of original landmarks is to be avoided as far as possible; but you are not to perpetuate fraudulent surveys,—that is, those which cannot be identified in the field:—where the marks which may be found for the lines are irregular and so inaccurate and inconsistent as to render their perpetuation an evil and their establishment a source of difficulty. Where such surveys exist, you will make regular resurveys without reference to the irregular work, taking care, however, to cut out and efface all the original marks upon the bearing trees, and all marks intended to perpetuate the survey.

The more effectually to cancel such original marks, you are required to indicate in your field notes and upon your diagrams the positions of all the old corners which may be destroyed by you, by giving the course and distance of such old corners from the corners of the new survey, and noting the intersection of the new lines with any marks of the old work.

The marks of your resurveys should be in all respects plain and intelligible, and the marks upon the bearing trees should indicate the resurvey as required in the general instructions, so as to avoid any difficulty hereafter in regard to its identity.

In townships where a portion of the original work only is found sufficiently accurate to be retraced and reestablished, the connection with the new surveys must be made by running closing lines straight from the regularly established corners in the new survey to the corresponding corners in the old survey, and if such lines are fractional in length, the excess or deficiency will be thrown into that portion of the line which closes upon the old corners, by setting the quarter-section posts at the distance of 40 chains from the corners in the new survey in the same manner as if the lines closed upon a boundary or correction line.

The true courses of these and all other lines run or retraced by you must be represented in your field notes, and as you will use the ordinary compass, the courses must be governed by the true variation of the needle, which you will of course ascertain before commencing the surveys in any township, and more frequently if necessary. All errors or defects in the exterior boundaries of the township should be ascertained and adjusted by you in such a manner as not to disturb other regularly established lines and corners.

A jog or crook in any line between two regular corners which may be the result of accident or of intentional fraud on the part of the deputy, may be corrected by making such line straight between the corners. The old crooked lines will be indicated in your field notes

and on your diagrams so that they may be represented upon the plats of your surveys.

It is known that in some cases where a portion of the original work may be found available in a correctional survey, the closing lines on the north and west boundaries of the townships are crooked and irregular, and so indistinctly marked as to render it difficult to detect them. At the same time, it is reported that regular corners for the closing lines are sometimes found upon the township lines at 20, 30, 40, or 60 links from the township corner, without regard to the lines for which they purport to be established. Such corners should be obliterated and such lines, which are doubtless random lines partially marked and not corrected, should be cancelled, and new lines supplied by running random and correcting straight lines back from the township corner to the corresponding section corner which you will have re-established in the old survey. All such irregular lines and all corners which may be found established with no lines closing to them, if not in their proper places should in like manner be cancelled. Your field notes will be taken in the ordinary field book, the largest size of which will probably be large enough to contain all your proceedings in reference to any one township. These proceedings must all be faithfully recorded, and if they should be of such a nature as not properly to belong with the field notes, you will keep a book for the purpose, in which may be entered any remarks respecting your survey, which you will retvrn with your field notes and other papers to this office. If it should be necessary to change the boundaries of any lands which may have been sold, it must be done *only* with the expressed written consent of the owner, duly attested, which will be taken by you, to be returned to this office with your field notes."

A copy of the foregoing instructions was furnished to each of the following deputy surveyors, to wit: William Burt, George H. Cannon, Addison P. Brewer, D. A. Pettibone, W. L. Coffinberry, Artemas Curtis, and Thomas Whelpley.

SECTION IX.

SUBDIVISION OF SECTIONS.

222. Subdivisions of sections are original surveys to be made in the following manner:

1. Section and quarter-section corners set by the government surveyors, and the boundaries actually run by them, as well as the length of all lines as returned in their field notes, are to be taken as correct. (See Sec. 2396 R. S., First and Second. P. 246, Sec. 100.)

2. The corners of half and quarter sections which were not marked on the government surveys, must be placed as nearly as possible equidistant from those two corners which stand on the same line. (Sec. 2396, First. P. 246, Sec. 100.)

This applies to the quarter-posts on the north and west lines of the township which were surveyed previous to 1846; also to those townships which, under the act of 1796, were surveyed into blocks of two miles square, (P. 244, Sec. 99, Third), and to those surveyed under the act of 1800,* where no quarter-section corners were planted on the lines running from south to north.

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* **No. 21.**—An Act to amend the act entitled "An act providing for the sale of the lands of the United States, in the territory northwest of the Ohio, and above the mouth of the Kentucky River."

SEC. 3. *And be it further enacted*, That the surveyor-general shall cause the townships west of the Muskingum, which by the abovementioned act are directed to be sold in quarter townships, to be subdivided into half-sections of three hundred and twenty acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each

3. The boundary lines of sections, (see Page 244, Sec. 99, Third), and of half and quarter sections, which were not actually run and marked, are to be ascertained by running straight lines from the established corners to the opposite corresponding corners. Where no such opposite corners have been or can be fixed, the line should be run from the established corner due north and south or east and west, as the case may be, to the water-course or other external boundary. (P. 246, Sec. 100, Second.) These due lines are to be found by trial of the boundary lines of the section, as actually run by the government surveyor, and the subdivision line, run on a course intermediate between the courses of the section lines which lie parallel with it. (See Instructions of 1881, p. 259.)

The following figure illustrates the manner of subdividing sections. It shows sections 5, 6, 7, and 8, repre-

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other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each *mile on* those running from south to north, and making the marks, notes, and descriptions prescribed to surveyors by the above-mentioned act: And the interior lines of townships intersected by the Muskingum, and of all the townships lying east of that river, which have not been heretofore actually subdivided into sections, shall also be run and marked in the manner prescribed by the said act for running and marking the interior lines of townships directed to be sold in sections of six hundred and forty acres each. And in all cases where the exterior lines of the townships, thus to be subdivided into sections or half-sections, shall exceed or shall not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west, or from south to north; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity And the President of the United States shall fix the compensation of the deputy surveyors, chain-carriers, and axemen: *Provided*, The whole expense of surveying and marking the lines shall not exceed three dollars for every mile that shall be actually run, surveyed, and marked.

senting the four different cases which occur in a township surveyed previous to 1846. In the later surveys, the details would differ a little, owing to the fact that the section and quarter-section corners on the township and range lines are common to the township on each side of and adjoining those lines. The principle of subdivision is, however, the same.

Range Line.

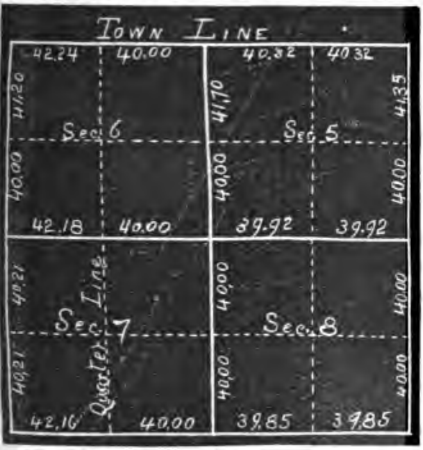

FIG. 125.

CASE 1.—Section 8. All the quarter posts are at equidistant points from the section corners which are on the same line.

CASE 2.—Section 5. Quarter posts on the north and the south are at equidistant points. Those on the east and the west are 40 chains from the south line of the section. The fraction is on the north half of the section.

CASE 3.—Section 7. Quarter posts on the north and the south are placed at 40 chains from the east line of the section. Those on the east and the west are at equidistant points. The west half of the section is fractional.

CASE 4.—Section 6. The quarter posts on the north and the south are placed at 40 chains from the east line of the section. Those on the east and the west are 40 chains from the south line of the section. Fractional both on the north and west.

NOTE.—In 1856, Thomas A. Hendricks, then Commissioner of the General Land Office, gave the following rule for locating the center of a section: "Run a true line from the quarter-section corner on the east boundary, to that in the west boundary, and at the equidistance between them establish the corner for the center of the section."

This was in harmony with an opinion previously given by the Surveyor General of Missouri and Illinois, and was very generally followed by the surveyors in those States. This rule has not been sustained by the courts, nor by any other ruling of the Land Office, so far as we can learn. It was expressly overruled by the Secretary of the Interior in 1868.

Quarter-sections are to be subdivided into **half-quarters** by lines running north and south.

The corners which were not marked are to be placed as nearly as possible equidistant between the two corners of the quarter-section which stand on the same line. Then run straight lines from the established corners to the opposite corresponding corners, (Page 247, Sec. 101.)

Half-quarter sections are to be subdivided into **quarter-quarters** in a similar manner, by east and west lines. (P. 247, Sec. 101.)

It may be well to remark here, that the instructions from the General Land Office have not been uniform in regard to the proper manner of subdividing quarter-sections, and, as might be expected, the practice is not uniform among good surveyors. Commissioners Wilson and Edmunds held that half-quarter and quarter-quarter lines should be "straight lines running through the section" to points on the section line. (See Hawes's Manual, p. 142, and Dunn's Land Laws, p. 19.)

The foregoing rules are those of the statute, and are endorsed by Commissioners Drummond, Williamson, and McFarland.

Commissioner Drummond's instructions are as follows:

"In the subdivision of quarter-sections, the quarter-quarter posts are to be placed at points equidistant, and on straight lines between the section and quarter-section corners, *and between the quarter-corners and the common center of the section*," etc. The difference in the two methods occurs when, as very often happens, the quarter-posts are not in line between the section corners.

223. Fractional sections are to be subdivided according to the Fifth paragraph of Sec. 2395 of the Revised Statutes, under such rules and regulations as may be prescribed by the Secretary of the Interior. (Sec. 99, Ex. Land Laws, and U. S. Instructions, 1881, p. 39.)

Under these regulations,* the fractional quarter-sections lying next to the north line of the township are divided

* NOTE.—"*Circular to Surveyors-General, Nov. 9, 1821.*—SIR: By the first section of the act of April 24, 1820, all the public lands of the United States shall be offered at public sale in half-quarter sections; and

into half-quarters by lines running east and west, parallel with and twenty chains distant from the quarter-section line. (See Keasling v. Truitt, 30 Ind. 506.)

The quarter-sections lying next to the west line of the township are divided into half-quarters by lines running north and south, parallel with and twenty chains distant from the quarter-section line.

224. Section 6 adjoins both the north and the west lines of the township, and is subject to both rules. The north half is divided into half-quarters by an east and west line, and the south half by north and south lines.

The quarter-post on the north side of section six should be placed on the township line at a point 40 chains of original measure west from the northeast corner of the section.

The quarter-post on the west line of section six should be placed at a point on the range line 40 chains of original measure north from the southwest corner of the section. By original measure is meant such measure as was actually laid down on the ground by the deputy surveyors who made the original survey.

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fractional sections containing one hundred and sixty acres and upward shall, as nearly as practicable, be divided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Treasury; but fractional sections containing less than one hundred and sixty acres shall not be divided, etc. By the act of May 10, 1800, section 3, the excess or deficiency of regular sections or quarter-sections in any township is to be thrown on the north and west sides of the township, making fractional sections more or less than one hundred and sixty acres. In subdividing such fractional sections to form a half-quarter section, viz., 80 acres, the Secretary of the Treasury directs that the subdividing line for such fractions as lie on the north side of a township shall be an east and west line, forming the half-quarter section on the south side of the fraction; and for such fractions as lie on the west side, the subdividing line shall be a meridian, forming the half-quarter section on the east side of the fraction. This mode of subdivision will preserve the compactness of the tracts with the general divisions, and will not interfere with the rule adopted relative to fractions formed by a stream, a river, etc."

Sections made fractional by waters, reservations, etc., should be subdivided in such a manner as to produce the same result as would have been produced had the section been full. This may sometimes be done by extending and by measuring the lines on the ice, or over the reservation.

FIG. 126.

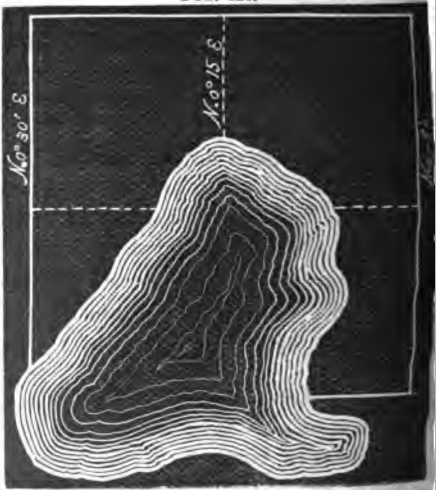

Figure illustrating the Subdivision of a Section fractional on waters.

Commissioner Drummond says, (see Copp's Land Laws, p. 761): "In the subdivision of fractional sections, where no opposite corners have been or can be fixed, the subdivision lines should be ascertained by running lines from the established corners due north, south, east or west, as the case

FIG. 127.

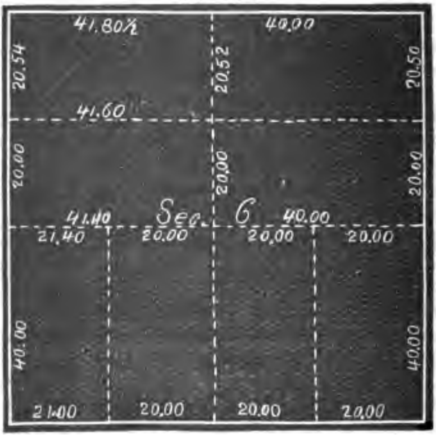

Figure illustrating the Subdivision of Quarter Sections.

may be, to the water-course, Indian boundary line, or other external boundary line of such fractional section. law presupposes the section lines surveyed and ed in the field by the United States deputy survey- be due north and south or east and west lines. actual experience, this is not always the case. in order to carry out the spirit of the law, it will essary in the running of subdivisional lines through onal sections to adopt mean courses where the lines t due lines, or to run the subdivisional line paral- th the section line when there is no opposite section (See Instructions of 1881, *ante* p. 247.)

Irregular Subdivisions of Fractional Sec-—In making irregular subdivisions of fractions ed on streams or lakes, there seems to have been e laid down by the authorities.

has been decided by the Supreme Court of the United es that "the meander lines run in surveying fractional portions of the public lands bordering upon navigable rivers are run not as boundaries of the tract but for the purpose of defining the sinuosities of the stream and as the means of ascertaining the quantity of land in the fraction, and which is to be paid for by the purchaser."

R. R. Co. *v*. Schurmier, 7th Wallace (U.S.) 272.

It is fair to infer that the same lines are to be used in ascertaining the quantity of land in any portion of the fraction. Thus, as often happens, if a deed calls for so many acres off the end of the fraction, the surveyor in making his computations to determine at what point to locate the dividing line, should in the absence of anything showing to the contrary, use the meander line for the purpose of estimating the area of the tract, and lay down the dividing line accordingly. Otherwise there could be no common basis of calculation and as many different results would be arrived at as there were different surveyors to run the line, or different times of survey.

This is especially true of fractions bordering on lakes whose shore lines are subject to great change from natural causes or artificial drainage.

The common law rule for calculating the quantity of land bordering on a non-navigable stream is that no reference is had to what lies between low water mark and the centre of the stream. On navigable waters, high water mark is the line.

Lamb *v.* Rickett, 11 Ohio 311.

226. Exceptional Cases.—In the United States surveys made previous to 1815, there was much irregularity in the practice of the surveyors in carrying on the surveys. The fractional sections were frequently thrown upon the south or east tiers of sections in the township; the surveys being carried on from the north to the south and from the west to the east. Where the township was made fractional by large rivers or lakes, they were frequently so laid off as to throw all the fractions into the sections bordering on the water.

There was even greater irregularity in the manner of subdividing the fractional sections into the lesser tracts. Many of them had no quarter section corners. In some, the government plats show no subdivision; some are subdivided in one way and some in another.

In making resurveys and subdivisions of these and all other exceptional cases, the surveyor must always make his resurvey conform to the plan as shown by the field-notes and plats of the original survey.

227. Other Original Surveys.—In a considerable portion of the United States, the general government never had any ownership of the land.

The surveys were there made by the proprietors upon such system or plan as suited themselves.

The further subdivision of these tracts is original surveying. It is sufficient to say of this work that it should be done with great care, and that *the marks upon the ground which indicate the boundary lines should be of*

the plainest and most permanent character which the circumstances of the case permit,

These marks are intended to fix for all time the boundaries of the tract laid off and *they cannot be too plain or permanent.* Want of due care and precaution in making permanent land marks upon the ground, at the time of the original survey, is the fruitful cause from which arises most of the litigation about boundary lines.

228. Highway surveys, like other surveys, lose much of their value if their corners and lines are not so thoroughly marked as to be readily found at any future time. The centre line of a highway is very commonly used as a boundary line. Good permanent landmarks, well guarded by bearings and distances to the most permanent objects in the vicinity, should be planted at the starting and closing points of the survey, at each angle in its course, and at every crossing of a section line. The distance of the crossing points should be given from the nearest government corners each way on the section line.

229. Surveys for town plats are made upon any system to suit the circumstances of the case, or the views of the owners of the land platted.

In making these surveys, it is important that the work be in every respect carefully done, that full and complete notes be taken, so that the plat when finished shall show every material fact which may be of use to the public or to the future surveyor.

The relation which the lines of the plat bear to the lines of the original boundaries, whether of the government survey or otherwise, should be shown on the plat, and, what is most important of all, *the location of the lines upon the ground should be marked by a sufficient number of permanent monuments so that there may never arise any difficulty in determining the exact position those lines occupy.*

Such monuments should be placed at the corners and angles of the tract platted, and if include the United

States survey, they should be placed at the corners of the legal subdivisions of a section which are included in the plat. Monuments should also be placed so as to define the lines and termini of all streets.

For this purpose, they may be placed either along the centre lines and angles of the streets or along their margins at the corners and angles of blocks. Each method has its advantages and disadvantages. The surveyor should consider the special circumstances of each case, and so locate the monuments that, while effecting the purpose for which they were intended, they shall be the most likely to remain in position and the easiest to refer to.

230. In Michigan, town plats are required by law (Session Laws of 1885) to be made and recorded in the following manner:

The plats must be made on sheets of good muslin backed paper, 18 inches by 24 inches in size, on a scale showing not more than 200 *feet* to an inch.

The plat must have upon it a full, detailed written description of the land embraced in it, showing the township, range, section and subdivision of section of the land platted. If the premises platted are not included in the legal subdivisions of the government survey, then the boundaries are to be defined by metes and bounds and courses.

The plat must contain the full name of the town, city, village or addition platted; the names of the proprietors and of the person making the plat, and the date.

It must be signed by the proprietors and by the person making it, and be witnessed and acknowledged in the same manner as deeds.

The sections and parts of sections must also be designated on the plat by lines with appropriate letters and figures.

There must be a plain designation of the cardinal points of the compass and a correct scale.

When complete and before any copies are made from it, the plat must be submitted to the Auditor General for his approval,

231. The Record.—An exact duplicate of the original plat must be filed in the office of the Register of Deeds for the county in which the land is situated. It must contain all the matter in the original plat and the certificate of the Register of Deeds and the person who made the original plat, that they have separately carefully compared the duplicate with the original plat, and that it is an exact duplicate thereof and of the whole of such plat.

A third copy must be filed in the office of the Auditor General. This copy must contain the certificate of the Register of Deeds and of the person who made the plat, that they have separately compared it with the duplicate plat on record, and that it is a true transcript therefrom and of the whole of such duplicate plat so recorded.

The Register of Deeds receives a fee of $2.00 for recording the plat, and the sum of $1.00 must accompany the plat filed in the Auditor General's office.

SECTION X.

RESURVEYS.

232. In an old settled country, the principal work of the surveyor is to retrace old boundary lines, find old corners, and relocate them when lost. In performing this duty, he exercises, to a certain extent, judicial functions. He usually takes the place of both judge and jury, and acting as arbiter between adjoining proprietors, decides both the law and the facts in regard to their boundary lines. He does this not because of any right or authority he may possess, but because the interested parties voluntarily submit their differences to him as an expert in such matters, preferring to abide by his decision rather than go to law about it.

In making resurveys the surveyor is called upon—

1. To construe descriptions in deeds;

2. To find the location of corners and boundary lines;

3. To renew corner monuments and to mark anew boundary lines.

233. In construing the descriptions the following rules have been laid down by the courts:

RULE 1. The description of boundaries in a deed is to be taken most strongly against the grantor.

Marshall *v.* Niles, 8 Conn. 369.
Ryan *v.* Wilson, 9 Mich. 262.

2. A deed must be construed according to the condition of things at the date thereof.

Crogan *v.* Burling Mills, 124 Mass. 390.

Written descriptions of property are to be interpreted

in the light of the facts known to and in the minds of the parties at the time.

Wiley *v.* Sanders, 36 Mich. 60.
McConnell *v.* Rathbun, 46 Mich. 305.

And should be construed with reference to any plats, facts, and monuments on the ground referred to in the instrument.

Anderson *v.* Baughman, 7 Mich. 77.
Bowen *v.* Earl, 28 Mich. 538.

3. Where the description of the boundaries in a deed are indefinite or uncertain, the construction given by the parties, and manifested by their acts on the ground, is deemed the true one unless the contrary is clearly shown.

Reed *v.* Prop. Locks and Canals, 8 How. (U. S.) 274.

4. Every call in the description of the premises in a deed must be answered if it can be done, and none is to be rejected if all the parts can stand consistently together.

Herrick *v.* Hopkins, 10 Shep. (Me.) 217.

5. Where the boundaries mentioned are inconsistent with each other, those are to be retained which best subserve the prevailing intention manifested on the face of the deed.

Gates *v.* Lewis, 7 Vt. 511.

6. The certain description must prevail over the uncertain, in absence of controlling circumstances.

Richer *v.* Barry, 34 Me., 116.
Tewksbury *v.* French, 44 Mich. 102.
See also 35 N. H. 121, and 11 Conn. 335.

7. When one part of the description in a deed is false and impossible, but by rejecting that a perfect description remains, such false and impossible part should be rejected and the deed held good.

Anderson *v.* Baughman, 7 Mich. 79.
Johnston *v.* Scott, 11 Mich. 232.

8. A deed is to be construed so as to make it effectual rather than void. (*Ibid.*)

9. Where the description in a deed calls for land "owned and occupied," the actual line of occupation is a material

call to be considered in locating the lines of the land bounded therein.

Fahey *v.* Marsh, 40 Mich. 239.
Cronin *v.* Gore, 38 Mich. 386.

10. Where land is described as running a certain distance by measure to a known line, that line will control the measure and determine the extent of the grant.

Flagg *v.* Thurston, 13 Pick. (N. Y.) 145.
See also 13 Wend. (N. Y.) 300, and 7 Irodelle (N. C.) 169 and 310.

11. Not so if the line is obscure, not definitely fixed, marked or known, and therefore likely to be looked upon by the parties as less certain than the measure given.

Howell *v.* Merrill, 30 Mich. 282.

12. In the case of Land Co. *v.* Saunders in 5th Otto (U. S.), the Supreme Court of the United States held the west line of Hart's location to be the boundary of a grant. It was in a mountainous country and had never been surveyed or marked—although capable of being marked—the line being simply marked on the plat of the location. This line is held to be such a monument as would control course and distance.

13. Where land is conveyed "beginning at" and bounding land of "B," the point of beginning and boundary is the true line of B's land, and not the line of occupation as shown by a fence set up and maintained by B before and after the conveyance, with the consent of the owner of the lot conveyed, under the mistaken belief that such was the true line.

Cleveland *v.* Flagg. 4 Cushing (Mass.) 76.

14. A course from corner to corner means *prima facie* a right line; but this may be explained, by other matters in the case, to be a crooked or curved line, as following a ditch, or hedge, or stream.

Baker *v.* Talbott, 6 Mont. (Ky.) 182.

15. "Northward" or "northerly" means due north when nothing is mentioned to show the deflection of the course to the east or west.

Jackson *v.* Reeves, 3 Caines, N. Y. 293.
Brandt *v.* Ogden, 1 Johns. N. Y. 156.

16. The use of the term "about" indicates that exact precision is not intended; but where nothing more certain can be found to control the course and distance, the grantee is limited to the exact course and distance given.

Cutts *v.* King, 3 Greenl. Me. 482

17. Where a given quantity of land is to be laid off on a given base, it must be included in four lines, so that the lines proceeding from the base shall be at right angles with it, and the line opposite the base shall be parallel with it, unless this form is repugnant to the entry.

Massie *v.* Watts, 6 Cranch. (U. S.) 148.
Ker *v.* Watts, 6 Wheat. (U. S.) 550.

18. Seventy acres lying and being in the southwest corner of a section, is a good description, and the land will be in a square.

Walsh *v.* Ringer, 2 Ham. (Ohio) 327.

19. Where lines are laid down on a map or plan, and are referred to in a conveyance of land, the courses, distances, and other particulars appearing on such plan are to be as much considered the true description of the land conveyed as they would if expressly recited in the deed.

Davis *v.* Rainsford, 17 Mass. 211.
See also 14 Mass. 149, and 1st Greenl. Me. 219.

20. A conveyance by metes and bounds will carry all the land included within them, although it be more or less than is stated in the deed.

Butler *v.* Widger, 7 Cow. (N. Y.) 723.
Bratton *v.* Clawson, 3 Strobh. S. C. 127.
Gillman *v.* Riopelle, 18 Mich. 164.

21. A grant of land bounded by a highway takes to the center of the highway. If it be designed to exclude the highway, it must be so stated in explicit terms.

Champlin *v.* Pendleton, 13 Conn. 23.
See also 7 N. H. 275; 6 Shep. Me. 276.
Purkiss *v.* Benson, 28 Mich. 538.

A deed of land lying east of a certain street, and explicitly bounded by the east line of the street, conveys no title to the soil in the street.

G. R. & I. R. R. Co. *v.* Mary Heisel, 38 Mich. 62.

22. The mention of quantity of acres after a definite description by metes and bounds, or by the aliquot part of the section, is matter of description only, and quantity being the least certain, does not control.

Amich *v.* Holman, 13 Strobh. S. C. 132.
McClintock *v.* Rogers, 11 Ills. 279.
Martin *v.* Carlin, 19 Wis. 454.

23. Where boundaries are doubtful, then quantity often becomes a controlling consideration.

Winans *v.* Cheney, 55 Cal. 567.

24. Grants by government are to be construed according to the common law, unless it has done some act to exclude that construction.

Middleton *v.* Pritchard, 3 Scam. Ills. 510.

234. In construing deeds conveying title to **lands bordering on waters**, it will be necessary for the surveyor to inquire into the local laws of the State in which the premises lie, as different States by their laws and courts give different constructions to the word navigable as applied to streams and the smaller lakes. The statute of the United States provides that

"SEC. 440. All navigable rivers, within the territory occupied by the public lands, shall remain and be deemed public highways; and, in all cases where the opposite banks of any streams not navigable belong to different persons, the stream and the bed thereof shall become common to both." (1 Stat. 468; 2 *id.* 235; R. S. 2476.)

It is a universal rule that grants of land bordering on navigable streams take only to high-water mark, while grants on non-navigable streams take to the center of the stream, or the *filum aquæ,* as it is termed.

Now, whether the proprietor in any given case owns the land under water to the center of the stream, or only takes to high-water mark, depends on the local construction given to this word navigable.

Under the *Common Law,* a navigable stream is one in which the tide ebbs and flows. Some exceptions to the rule are made in England.

Under the *Civil Law*, a navigable stream is one capable of being used as a highway of commerce. In the case of the Railroad Co. *v.* Schurmier, (7 Wallace, 272), the Supreme Court of the United States says that "the words navigable and non-navigable were applied by Congress without respect to the ebb and flow of the tide," and in the case of Bowman and Bumley *v.* Wathieu and others, (2d McLean, 276), they say that "the common law doctrine as to the navigableness of streams can have no application in this country, and the fact of navigableness does in no respect depend on the ebb and flow of the tide."

The courts of Pennsylvania, North Carolina, South Carolina and Alabama hold the same view. On the contrary, in Maine, New Hampshire, Massachusetts, Connecticut, New York, Maryland, Virginia, Ohio, Illinois, Indiana, and Michigan, the common law doctrine is held to prevail. (See Angell on Tide Waters, pp. 77 and 78.)

Hence, in applying the principles laid down by the courts in the following decisions, the surveyor will bear in mind the locality in which they are to be applied.

25. Proprietors of lands bordering on navigable rivers, under titles derived from the United States, hold only to the stream, as the express provision is, that all such rivers shall be deemed to be and remain public highways.

R. R. Co. *v.* Schurmeir, 7 Wallace (U. S.) 272.

26. Where a sea or bay is named as a boundary, the line of ordinary high-water mark is always the line, where the common law prevails.

U. S. *v.* Pacheco, 2 Wallace (U. S.) 587.

27. A boundary *on* a stream or *by* or *to* a stream includes flats at least to low-water mark, and in many cases to the middle thread of the river.

Thomas *v.* Hatch, 3 Sumner (U. S.) 170.

28. A boundary on the bank of a river referring to fixed monuments on the bank, limits the grant to the bank and excludes the flats. (*Ibid.*)

See also Hopkins *v.* Kent, 9 Ohio, 13.

29. The words "along the bank" are strong and definite enough to exclude the idea that any part of the river or its bed was granted in the navigable or unnavigable parts of the river.

Howard *v.* Ingersoll, 13 How. (U. S.) 341, 416.

A deed describing the land by a boundary running to a stream, and thence along its bank, and reserving the right to use the river front a specified time, conveys the land to the water's edge and covers the riparian rights to the middle of the stream.

Cole *v.* Wells, 49 Mich. 450.

30. Congress, in making a distinction between streams *navigable* and those *not so*, in the acts relating to the sale of the public lands bordering thereon, intended to provide that the common law rules of riparian ownership should apply to the lands bordering on the *latter*, but that the title to lands bordering on the *former* should stop at the stream.

R. R. Co. *v.* Schurmeir, 7 Wall. (U. S.) 272.

31. In streams which are not navigable, adjacent proprietors own to the center of the stream measured from low-water mark.

Clark *v.* Caupau, 19 Mich. 325.
Moore *v.* Sanborn, 2 Mich. 519.
Lorman *v.* Benson, 8 Mich. 18.
Bay City Gas Light Co. *v.* Ind. Wks., 28 Mich. 182.
Lamb *v.* Ricketts, 11 Ohio, 311.

32. The same principle is applied to Lake Muskegon, in Michigan, (Rice *v.* Ruddeman, 10 Mich. 125), but not applied to a similar lake in Wisconsin, where the court says, (Deidrich *v.* N. W. U. Ry. Co., 42 Wis. 271): "Riparian owners upon a natural lake or pond take only to the shore."

In the case of the State of Indiana *v.* Milk, Circuit Court of the United States, April term, 1882, 11th Bissell, page 197, the court rejects the theory of riparian ownership in the lake, and after presenting its reasons at some

length, concludes with the following: "That while a general grant of land on a river or stream non-navigable extends the line of the grantee to the middle or thread of the current, a grant on a natural pond or lake extends only to the water's edge."

33. Islands in rivers fall under the same rule as to ownership as the soil under water does. If not otherwise lawfully appropriated, they belong to the proprietors on either side of the stream, according to the original dividing line or *filum aquæ* as it would run if the islands were under water. The *filum aquæ* is midway between the lines of ordinary low-water mark, without regard to the channel or depth of water. When the island is appropriated, the boundary is then midway between it and the mainland.

McCullough *v.* Wall, 4 Rich. (S. C.) 68.
Kimball *v.* Schaff, 40 N. H. 190.

34. The grant includes any land between the meander line and the water, in an unnavigable stream.

The same principle applies to unnavigable lakes.

Forsyth *v.* Smale, 7 Biss. (U. S.) 201.

35. High-water mark in the Mississippi River is to be determined from the river bed, and that only is river bed which the river occupies long enough to wrest it from vegetation.

Houghton *v.* Railway Co., 47 Iowa, 370.

36. A bank is the continuous margin where vegetation ceases. The shore is the sandy space between it and low-water mark.

McCullough *v.* Wainwright, 14 Penn. St. 59.

37. Where a levee was shown to have been judiciously located by a competent engineer and agents of the State acting under authority conferred by the State Legislature, it was held that such levee became the boundary line of high water, and that no private ownership could be acquired to land lying between that and the bed of the stream.

Musser *v.* Hershey, 42 Iowa, 356.

38. Grant of a city lot bounded on a river, takes to the center of the stream.

Watson v. Peters, 26 Mich. 508.

Riparian rights, unless expressly limited, extend to the middle of the navigable channel, and cover any shallows or middle ground not shown in the government surveys, but lying between such shallows and the shore, and it makes no difference that the deed conveying the premises to which the rights attach describes them according to a city plat instead of the government entry.

Fletcher v. Thunder Bay Boom Co., 51 Mich. 277.

39. But if the plat plainly indicates the proprietor's intent to reserve the space between the shore and the thread or main channel, the case would be different.

Watson v. Peters, 26 Mich. 508.

40. Riparian rights extend laterally into the stream. Rocks and shoals along the margin of navigable rivers above tide-water belong to the riparian owner.

Moore v. Willamette T. and L. Co., 7 Oregon R. 355.

41. When a navigable stream is meandered in making the public surveys, and the United States has granted to the meander line, the grantee takes to the river. The stream, and not the meander line, is the true boundary of the riparian owner.

Minto v. Delaney, *id.*, 337.

42. Lands patented by the United States on a tide-water stream extend to the meandered line of the stream, which is the line of ordinary high water.

Parker v. Taylor, *id.*, 435.

For further rulings, see *Boundary Lines.*

Second.

235. In locating the corners and boundary lines on the ground, we will consider:

1. General rules which apply to all resurveys;

2. Special applications of these rules to the rectangular system of United States surveys.

GENERAL RULES.

RULE 1.—In locating a deed on the ground, we are to rely—

(1) On the actual lines originally surveyed;

(2) On lines run from acknowledged calls and corners.

(3) On lines run according to the course and distance in the deed.

Avery *v.* Baum, Wright's Ohio, 576.
1 Rich. (S. C.) 491.

2. When the boundaries of lands are fixed, known and unquestionable monuments, though neither courses, distances, nor computed contents correspond, the monuments must govern.

Pernam *v.* Wead, 6 Mass. 131.
Nelson *v.* Hall, 1 McLean (U. S.) 518.

3. Marked lines and corners control courses and distances. Surplus lands do not vitiate a survey, nor does a deficiency of acres called for in a survey operate against it. Wherever the boundaries can be established, they must prevail.

Robinson *v.* Moore, 4 McLean (U. S. C. C.) 279.
Morrow *v.* Whitney, 5 Otto (U. S.) 551.

4. A deed called for posts as corners. The survey was made and the posts set prior to the execution of the deed. It was afterward found that there was a shortage of several acres. Held that proof that posts were set up as corners between adjoining owners controls the call for course and distance.

Alseire *v.* Hulse, 5 Ohio, 534.

5. The rule that courses, distances and quantities must yield to monuments, is not inflexible, especially when the distances are very short, and the monuments artificial ones, as here, a mill-race, etc.

Higinbotham *v.* Stoddard, 77 N. Y. 94.
Ga. R. R. Co. *v.* Hamilton, 59 Ga. 171.

In a case where no mistake could be reasonably supposed in the courses and distances, the reasons of the rule were held to fail, and the rule was not applied.

Davis *v.* Rainsford, 17 Mass. 207.

6. The rule that natural or artificial boundaries will control distances or courses, authorizes no other departure from the course or distance than such as is necessary to effectuate the apparent intent of the grantor.

Distances may be increased and courses departed from in order to preserve the boundary, but the rule authorizes no other departure from the course and distance than such as is necessary to preserve the boundary.

Johnson *v.* McMillan, 1 Strobh. (S, C.) 143.

7. If the courses and distances cannot be otherwise reconciled with the monuments in a description, a line in a survey which has evidently been omitted will be supplied to prevent the obvious intent of the grantor from being frustrated.

Serrano *v.* Rawson, 47 Cal. 52.
See also Schultz *v.* Young, 3 Iredell, N. C. 385,

8. A survey must be closed in some way or other. If this can only be done by following the course the proper distance, then it would seem that distance should prevail; but when the distance falls short of closing, and the course will do it, the reason for observing distance fails, where two lines must be run instead of the one called for, to best conform with the whole description in the deed.

Doe *v*, King, 3 How. Miss. 125.

9. It is a universal rule that course and distance yield to natural and ascertained objects. But where these objects are wanting, and the course and distance can not be reconciled, there is no universal rule that obliges us to prefer the one to the other. Cases may exist in which either one may be preferred, according to the circumstances.

Preston's Heirs *v.* Bowman, 6 Wall. (U. S.) 580.

10. If no principle of location be violated by closing from either of two points, that may be closed from which will be more against the grantor and include the greater quantity of land.

Johnson *v.* McMillan, 1 Strobh, S. C. 143.

11. The boundary line is to be ascertained by running direct lines from one monument to the other.

Melcher *v.* Merryman, 4 Me. 601.

12. A line actually marked must be adhered to, though not a right line from corner to corner. Where a line has been marked only part of the way, the remainder of the line must run direct to the corner called for.

Cowan *v.* Fauntleroy, 2 Bibb (Ky.) 261.

13. A marked line of another tract, when called for in a conveyance, must be run disregarding distance; but where such line can not be established, the distance run must govern.

Gause *v.* Perkins, 2 Jones Law Rep. (N. Y.) 222.

14. Where a line is described as running a certain distance to a particular monument, and that monument has disappeared and its place cannot be ascertained, the course and distance, in the absence of other controlling words, must govern.

Budd *v.* Brooke, 3 Gill. (S. C.) 198.
See also, Bruckner *v.* Lawrence, 1 Douglass (Mich.) 19.

15. Course and distance yield to known, visible and definite objects; but they do not yield unless to calls more material and equally certain.

Shipp *et al.* *v.* Miller's Heirs, 2 Wheat. (U. S.) 316.

Courses and distances in the deed are not to be controlled by monuments or objects variant therefrom and not called for in the description, but they must yield to such objects and monuments as are referred to.

Bruckner's Lessee *v.* Lawrence, 1 Doug., Mich., 29.
Moore *v.* People, 2 Doug., Mich., 424.
Bower *v.* Earle, 18 Mich. 165.

16. Wherever it can be proved that the line was actually run, was marked, and the corners made, the party claiming under the deed will hold accordingly, although there is a mistake in the description in the deed.

Cherry *v.* Slade, 3 Murph. (N. C.) 82.

A sold to B lot 7, informing B, at the time of the sale, that it was four rods wide, and marking it out upon the

ground. He subsequently sold to C lot 8 and a vacated alley one rod in width between lots 7 and 8, informing C, at the time, that lot 8 was four rods wide, and the alley one rod wide, making five rods in all, and pointing out to C the marks previously made by him for the boundary of lot 7, sold to B, as being also the boundary of the alley sold to C. The premises were occupied by B and C in accordance therewith, without dispute. It was subsequently found, by reference to the plat, that lot 7 was five rods wide, and that there was no alley between the lots; whereupon B claimed the additional rod. Held, that to allow B to hold the rod in width of land which she did not purchase or pay for, and to deprive C of land which he did purchase and pay for, would be both bad law and bad morals.

Bolton *v.* Eggleston, Iowa.
N. W. Rep., Vol. 16, p. 62.

17. Boundary may be proved by any evidence which is admissible to establish any other fact.

Smith *v.* Prewitt, 2 A. K. Marsh. (Ky.) 158.

18. Where no bounds were established, the dividing line must be run by aid of the measurements in the deeds, the oldest title receiving its full measure first.

Talbott *v.* Copeland, 38 Me. 333.

19. A long established fence is better evidence of actual boundaries, settled by practical location, than any survey made after the monuments of the original survey have disappeared. A resurvey made after the monuments of the original survey have disappeared, is for the purpose of determining *where they were.* and *not where they ought to have been.*

Diehl *v.* Zauger, 39 Mich. 301.
Hunt's Lessee *v,* ' cHenry and Williams, Wright's (Ohio) 599.

20. Where between the plan and the original survey there is a difference in the location of the lines and monuments, the lines and monuments originally marked as

such are to govern, however much they may differ from those represented on the plan.

Ripley *v.* Barry, 5 Greenl. (Me.) 24.

See also 2 Greenl. (Me.) 214, and 3 Gr. (Me.) 126.

21. But no such rule has obtained where the survey was subsequent to the plan.

Thomas *v.* Patten, 1 Shep. (Me.) 329.

22. Purchasers of town lots have a right to locate them according to the stakes which they find planted and recognized, and no subsequent survey can be allowed to unsettle them. The question afterwards is not where they should have been, in order to make them correspond with the lot lines as they should be if the platting were done with mathematical accuracy, but it is whether they were planted by authority, and the lots were purchased and taken possession of in reliance on them. If such was the case, they must govern, notwithstanding any errors in locating them.

Flynn *v.* Glenny, 51 Mich. 580.

23. Where two surveys call for each other, there can be no vacancy unless the lines marked on the ground contradict the call; and in such case the marked lines must govern.

McGinnis *v.* Porter, 20 Penn. 80.

24. Where two surveys made twenty-three years apart are found to disagree, the probabilities favor the earlier survey when the original corners and witnesses are gone at the time of the last survey, especially if the line of the first survey has remained unquestioned for many years.

Case *v.* Trapp, 49 Mich. 61.

25. When the same grantor conveyed to two persons, to each one a lot of land, limiting each to a certain number of rods from opposite known bounds running in a direction to meet if extended far enough, and by measure the lots do not join—when it appears from the same deeds that it was the intention that they should join, a rule

should be applied which will divide the surplus between the grantees in proportion to the length of the respective lines as stated in their deeds.

Lincoln *v.* Edgecomb, 28 Maine, 275.

26. Where original surveys have been made, and returned as a block into the land office, the location of each tract therein may be proved by proving the location of the block. In ascertaining the location of a tract, the inquiry is not where it should or might have been located, but where it actually was located.

Every mark on the ground tending to show the location of any tract in the block, is some evidence of the location of the whole block, and therefore of each tract therein.

Coal Co. *v.* Clement, 95 Pa. St. 126.

27. Where lots are conveyed by number according to a plat which is made from an actual survey, the corners and lines fixed by that survey are to be respected.

Pyke *v.* Dyke, 2 Greenl., Me., 214.

28. Streets which are well defined, and designated by some natural or artificial monument, must govern course and distance in fixing boundaries of lands; but streets which are not thus defined, and themselves require to be located, would furnish very uncertain guides in arriving at the boundaries of other lands.

Saltenstall *v.* Riley, 28 Ala. 164.

29. When streets have been opened and long acquiesced in, in supposed conformity to the plat, they should be accepted as fixed monuments in locating lots or blocks contiguous thereto or fronting thereon.

Van den Brooks *v.* Correon, 48 Mich. 283.

30. Lands have been laid off into lots and blocks, and platted, before being cleared, when, by reason of inequalities of the surface, logs, and other obstructions, strictly accurate surveys were not and could not be made. Where the blocks and streets were staked out at the time, such

monuments would be fixed and permanent, leaving the excess or shortage to be dealt with by itself.

So where the streets, although not so designated, have by the parties interested or by the public authorities been opened, used, and acquiesced in, they thereby become permanent boundaries and form new starting points in subsequent surveys of the premises.

Twogood *v.* Hoyt, 42 Mich. 609.

31. Ancient reputation and possession in regard to streets in a town are entitled to more respect in deciding on the boundaries of lots than any experimental survey that may be afterwards made.

Ralston *v.* Miller, 3 Rand. (Va.) 44.

32. Where lots are sold by numbers and a plat, any variance in the distance between known and fixed points as found by actual measure on the ground, and the distance between the same points as laid down on the plat, is to be divided between the lots in proportion to the respective lengths as laid down on the plat.

Marsh *v.* Stephenson, 7 Ohio, N. 3, 264.
Quinnin *v.* Reimers, 46 Mich. 605.

Surplus or shortage in a block is to be divided *pro rata* between the lots.

Newcomb *v.* Lewis, 31 Iowa, 488.
O'Brien *v.* McGraw, 27 Wis. 446.

33. Where the accuracy of the starting points taken for test surveys is merely matter of speculation, they cannot be used to fix a disputed boundary between two lots when the dispute arises from a discrepancy which affects all the lots in a block, and must therefore be apportioned among them.

Reimers *v.* Quinnin, 49 Mich. 449.

34. A resurvey is inadmissible in evidence to show that a private boundary is incorrect, if its starting point is outside of and does not belong to the immediate plan or local system by which the original survey was controlled.

Burns *v.* Martin, 45 Mich. 22.

35. If in running the lines of a grant, one line be found which is admitted or proved to be a line of the grant, which will run with a variation from the calls of the grant, if no other marked lines be found, the other calls should be run with the same variation as that found on the marked line.

Sevier *v.* Wilson, Peck. 146.

36. Where a deed conveys lots in a town, and refers to a plat to identify them, and, in describing their lines, calls the points of compass as designated on the plat by its lines and angles, a correct survey cannot be based on any other system; and although the lines there delineated are not conformable to the true meridian, the plat and not the compass should govern.

Bower *v.* Earl, 18 Mich. 367.

37. The boundary lines of water lots fronting on a river extend into the river at right angles with the thread of the stream, without reference to the shape of the shore.

Clark *v.* Campau, 19 Mich. 328.
Bay City Gas Light Co. *v.* Ind. Works, 28 Mich. 182.
Twogood *v.* Hoyt, 42 Mich. 609.
Norris *v.* Hill, 1 Mich. 202.

38. Where a certain distance is called for from a *given point* on a navigable stream to another point on the stream to be ascertained by measurement, such measurement must be made by its meanders, and not in a straight line. The same rule prevails when distance is called for along a travelled highway. A different rule is sometimes adopted when the stream is not navigable.

When a *tract of land* is bounded upon a navigable stream, the distance upon the stream will be ascertained, in the absence of other controlling facts, by measuring in a straight line from the opposite boundaries.

People *v.* Henderson, 40 Cal. 29.

39. In computing the number of acres in a survey, "from," "to," and "with" the bank of a stream mean to low-water mark.

Lamb *v.* Ricketts, 11 Ohio, 311.

40. **Alluvium** means an addition to riparian land gradually and imperceptibly made through causes either natural or artificial by the water to which the land is contiguous. It matters not whether the addition be on streams which overflow their banks, or on those which do not. In each case it is alluvium.

County of St. Clair *v.* Livingston, 23 Wall. (U. S.) 46.

Land formed by alluvium in a river is in general to be divided among the several riparian owners entitled to it, according to the following rule: Measure the whole extent of their ancient line on the river, and ascertain how many feet each proprietor owned on this line. Divide the newly formed river line into an equal number of parts, and appropriate to each owner as many of these parts as he owned feet on the old line; and then draw lines from the points at which the proprietors respectively bounded on the old, to the points thus determined as points of division on the newly formed shore.

This rule is to be modified under particular circumstances; for instance, if the ancient margin has deep indentations or sharp projections, the general available line of the river ought to be taken, and not the actual length of the margin as thus changed by the indentations or projections.

Deerfield *v.* Arms, 17 Pick. Mass. 41.
Jones *et al.* *v.* Johnston, 18 How. (U. S.) 100.

236. Rules applicable to the United States Surveys.—"All the corners marked in the surveys returned by the surveyor-general *shall be established as the proper corners* of the sections or subdivisions of sections which they were intended to designate."

"The boundary lines actually run and marked in the surveys returned by the surveyor-general *shall be established as the proper boundary lines* of the sections or subdivisions for which they were intended; and the length of such lines as returned *shall be held and considered as the true length* thereof."

Sec. 2396, Rev. Stat., *ante* p. 246, Sec. 100.

The preceding quotation from section 2396 of the Revised Statutes of the United States, settles all questions in regard to any change in the corners, lines or measures of the government survey. They are thereby made unchangeable, the statute thus emphasizing the common law, which holds the same doctrine to be true of all original surveys after the land has been conveyed in accordance with them. Hence, in making resurveys, the surveyor must find, if possible, the original corners, and make his courses and distances agree with those of the United States survey.

The following points have been decided by the courts with reference to these surveys:

RULE 1.—The original surveys by which the government sold its land and conveyed it to the purchaser establish the rights of the parties as to the boundaries. No line which will vary the rights thus acquired can afterwards be established without the consent of all parties.

May *v.* Baskins, 12 S. and M. (Miss.) 428.

2. Land sold under the United States surveys pass according to the description of the legal subdivisions, whether those subdivisions contain the legal quantity or not, more or less.

Fulton *v.* Doe, 6 Miss. 751.

3. Each section or a subdivision of a section is independent of any other section in the township and must be governed by its marked and established boundaries. Should they be obliterated, a last recource must be had to the best evidence that can be obtained showing their former situation and place.

Lewen *v.* Smith, 7 Port (Ala.) 428.

4. Field notes must yield to actual monuments erected by the original surveyor. They are only to be relied on as evidence to assist in finding the exact situation of the monuments.

McClintock *v.* Rogers, 11 Ill. 279.

5. Monuments found at the two extremes of a township line are entitled to no more controlling influence in determining the actual location of an intermediate line than the section corners established along the line. All original monuments established in connection with the field notes and plats must be referred to in order to define the locality of the line.

McClintock *v.* Rogers, 11 Ill. 279.

6. The corners established by the original surveyors of public lands by authority of the United States are conclusive as to the boundaries of sections and divisions thereof; and no error in placing them can be corrected by any survey made by individuals or a state surveyor.

Arnier *v.* Wallace, 28 Miss. 556.

In ascertaining the lost corner of a section, recourse must be had to the unobliterated marks of the original survey, the field notes and plats and subsequent surveys made under their guidance. If only a portion of one of the boundary lines leading to the lost corner on a township line has been obliterated, the remaining portion must be considered established as marked, and the corner must be presumed, in the absence of evidence to the contrary, to be at the point where the marked line if continued would intersect the township line But if the lost corner is proved to have been at another point, the lost portion of the boundary must be ascertained by running a straight line from the point where the marks disappear to that corner.

Billingley *v.* Bates, 30 Ala. 378.

7. In determining the line between the quarters of a section, the quarter post established by the government surveyors must govern in all cases where its location can be ascertained.

Vroman *v.* Dewey, 23 Wis. 530.
Britton *v.* Ferry, 14 Mich. 53.

8. In re-establishing a lost quarter post on a section line, any difference in the length of such line by actual

measure as compared with that indicated by the government survey should be divided between the parts in proportion to their respective lengths as shown by that survey.

Jones *v.* Kimble, 19 Wis. 429.

9. If the distance between recognized government corners as originally established overruns or underruns that given in the field notes, it should be divided pro rata between the intervening sections. The original field notes should be the main guide. Section lines being frequently deflected, the true corners must be tested by east and west distances from the recognized government corners yet standing in the same township as well as by north and south distances.

Martz *v.* Williams, 67 Ill. 306.

10. Unknown corners must be found by the corroborative testimony of all known corners with as little departure as may be from the system adopted on the original survey, without giving preponderance to the testimony of any one monument above another.

In re-establishing lost corners between remote corners of the same survey, when the whole length of the line is found to vary from the length called for; we are not permitted to presume that the variance arose from the defective survey of any part, but must conclude in the absence of circumstances showing the contrary that it arose from the imperfect measurement of the whole line, and distribute such variance between the several subdivisions of the whole line in proportion to their respective lengths.

Moreland *v.* Page, 2 Clarkes, Iowa, 139.

11. Quarter posts of the government survey are to be as much respected as the corners of townships or sections however distant from the center line.

Campbell *v.* Clark, 8 Mo. 558.

12. There was a mistake in the government survey of a section by which the quarter section line and the meander

line of a river were shown on the official plat to be one and the same line, being the boundary line of the fractional lots. As a matter of fact they were a considerable distance apart. There was no question as to the location of the quarter section corners. In a suit to determine the ownership of the land between the quarter section line and the river, it was held that the quarter section line should be adhered to as the more certain call, and that where the lines of a survey can be run from well ascertained and established monuments, they are to control and govern a description delineated on a plat, although the quantity in the fraction fell short of the amount laid down in the plat about as much as there was land contained between the quarter line and the river.

Martin *v.* Carlin, 19 Wis. 454.

13. When a deed designates the land conveyed as one of the subdivisions known in the United States survey, as, for instance, a quarter, half-quarter or quarter-quarter section, the presumption is that the parties intend that the tract shall be ascertained in the same manner as is done in the government surveys.

Not so, where the deed conveys a tract of land not known in that system of surveys, as, for instance, the east half of a lot, or of a quarter-quarter section.

Cogan *v.* Cook, 22 Minn. 142.

14. The defendant sold the north half of a lot which is bounded on the west side by the Au Gres river. But the river is not straight at this point, and the north line of the lot is longer than the south line.

The bill demands the north half of the lot, and the north half must mean the north half in quantity divided from the remainder by an east and west line.

Au Gres Boom Co. *v.* Whitney, 26 Mich. 44.

15. It is a question of fact to be determined by all the surrounding circumstances whether the land between the

meander line and the shore of the lake or water course is included in the survey.

Shoemaker *v.* Hatch, 13 Nev. 267.

16. The lines run to divide sections into halves and quarters, if erroneous, may be corrected, for they are subdivided by law; and if the officer in running the subdivision line makes a mistake, it can be corrected by running the line according to law.

Nolin *v.* Palmer, 21 Ala. 66.

17. An original township was divided into sections "by running through the same, each way, parallel lines at the end of every two miles, and making a corner at the end of every mile," and afterward a supplemental survey was made under a subsequent statute, which directed that these two mile blocks should be subdivided by running straight lines from the corners thus marked to the opposite corresponding corners. Held, that where the original mile corners in a certain block can be clearly identified, the courses of lines of subdivision within the block cannot be determined by proof of monuments, blazes, or other witness marks found in other blocks in the township.

Ginn *v.* Brandon, 29 Ohio St. 656.

18. When a navigable stream intervenes in running the lines of a section, the surveyor stops at that point, and does not continue across the river. The fraction thus made is complete, and its contents can be ascertained.

Therefore, when there is a discrepancy between the *corners* of the *section* as established by the United States, and the *lines* as run and marked, the latter do not yield to the former.

Lewen *v.* Smith, 7 Port. (Ala.) 428.

19. In government surveys, the line actually run by the government surveyors is the true line.

Goodman *v.* Myrick, 5 Oregon, 65.

20. In a case where the township lines had been run and marked by the United States survey, but the field

notes of the subdivision lines were fraudulent and rejected by the surveyor-general, because incorrect, no proper survey of them having been made, it was held that the line between sections *one* and *two* must be ascertained by running a straight line from the corner of the sections established on the exterior line of the township to the corresponding corner on the opposite side of the township.

Hamil *v.* Carr, 21 Ohio St. 258.

21. Where the initial point in the description of premises in a deed is the southeast corner of the north half of the southeast quarter, fractional, of a section, and the quarter-section is made fractional by a meandered lake so situated as to cover the eastern and central portions thereof; and the parcel described was carved out of the north half within a year after the same was patented, the southeast corner in question is construed to be the point which constituted the southeast corner of the land as it was surveyed out and platted by the government, which located it on the meandered line of the lake. The fact that the waters of the lake have since receded cannot change the boundaries as previously located.

Verplanck *v.* Hall, 27 Mich. 79.

287. Decisions of the General Land Office with reference to Mineral Surveys.—*Plats and field notes:* Of surveys of mining claims, required to disclose all conflicts with prior surveys, giving areas of all conflicts.

In future, surveyor-general will use no coloring on plats.

Com'r. (N.) Nov. 16, 1882. Circular.

Location (*of mine*)*:* Must be marked on the ground so that its boundaries can be readily traced.

N. Noonday M'g Co *v.* Orient M'g Co., 6 Saw., C. C., 299; Myers *et al.* *v.* Spooner *et al.*, 55 Cal. R., 257; Gleason *v.* N. White M'g Co., 13 Nev. R., 443; Southern Cross G. and S. M'g Co. *v.* Europa M'g Co., 15 *id.*, 383.

Surface line: Agreement by adjoining claimants, fixing surface boundary line between them, must be construed as

extending such line downward, through the dips of the vein or lode, toward the earth's center.

Richmond M'g Co. *v.* Eureka M'g Co., 103 S. C., 389.

Bearings and distances must be given in a survey, from the respective survey corners to the location corners, and the same must be shown on the plat.

Survey: Of a mining claim should show location of all improvements of a municipal nature, as blocks, alleys, etc.

Sec'y. Dec. 18, 1880, and Feb. 3, 1881. Little Nettie Lode.

SECTION XI.

RE-LOCATION OF LOST CORNERS.

238. The general principle to be observed in re-locating lost corners are laid down in the Supreme Court decisions which have already been quoted.

A corner is not lost so long as its position can be determined by evidence of any kind without resorting to surveys from distant corners of the same or other surveys. Often after making a survey from a distant corner, the surveyor will come upon some traces or evidence which will enable him to determine the true position of the corner he is seeking. It is an uncertain way at the best to locate corners by running lines and measuring from distant corners, and should only be resorted to in absence of better proof of the original location of the corner sought.

It will sometimes happen that the exact spot where a lost corner stood cannot be found or shown by evidence, but it can be proved that it stood within certain limits. In these cases, which are not rare, there is no question but that the corner should be placed at that point within the known limits which best agrees with all the evidence in the case.

Failing of better evidence by which to determine the location of a lost corner, we may next resort to the following methods:

General Rule.—Retrace the known lines of the description and find how the lengths and directions of these lines by your survey agree with those of the same lines as laid down in the original description. Then run the

unknown lines and place the lost corners so that they will bear the same relation to the known lines and corners as they are required to do by the description of the original survey.

Example.—The four lines of a description are as follows:

1. North 7° east 12.00 chains.
2. South 83° east 6.00 "
3. South 7° west 12.00 "
4. North 83° west 6.00 "

The first line and its termini are known. We retrace that line and find by our survey that it runs north 7° 30′ east and 12.24 chains.

We would then run the remaining lines, making them as follows:

2. South 82° 30′ east 6.12 chains.
3. South 7° 30′ west 12.24 "
4. North 82° 30′ west 6.12 "

Or the compass may be set on the known line and the vernier so adjusted that the reading of the needle shall be the same as that given in the original description and the remaining lines run accordingly, (Art. 63).

239. Re-location of Lost Corners of the United States Survey.

RULE 1.—On *base lines, correction parallels, township* and *range lines.* Restore the lost corner in line between the nearest known corners on the same line and at distances from them proportional to those laid down in the field notes of the government survey.

This rule supposes the original line to have been a straight line. As a matter of fact this is frequently not the case. If there is reason to suspect the line to have angles in its course, measures from known corners to the right and left of the line will aid in determining its true position.

RULE 2.—**Lost closing section corners** upon a township or range line, where the closing distance from the

adjacent corners is not given in the field notes should be restored by prolonging the known portion of the line to its intersection with the township or range line.

RULE 3. **Lost interior section corners** should be restored at distances from the nearest known corners, north, south, east and west, proportional to those laid down in the field notes of the original survey.

This rule supposes that the measurements of the original survey were uniform on the several adjacent sections. This is frequently not the case, and it will be well for the surveyor to compare his chaining on each section with the original measure between known corners of the same sections, choosing by preference those lines which on the government survey were measured next previous to the portion of the line closing on the lost corner.

RULE 4.—**Lost township corners**, when common to four townships, are to be restored in a similar manner to interior section corners, Rule 3. When common to only two townships, they are to be restored according to Rule 1.

RULE 5.—**Lost quarter section corners** are to be restored in line between the section corners which stand on the same line and at distances between them proportional to those returned in the field notes of the government survey.

RULE 6.—**Lost meander corners** are to be restored by running the line from the nearest known corner the direction and distance called for by the notes of the original survey. When a portion of the line leading to the meander corner is known, it should be prolonged in the same direction. When no portion of the line is known, the surveyor will have to use his own judgment as to what method under the circumstances of the case will most nearly retrace the original line to the corner.

There is no rule which will rigidly and inflexibly apply to all cases for restoring lost corners and boundary lines except this—that the aim of the surveyor should always

be to find the exact spot where the original corner or line was located. *The thing to find out is not where the corner or line ought to have been, but where it actually was.*

There are many cases in which other methods for restoring any of the corners mentioned will prove more satisfactory than the rules heretofore given.

For instance, a half-quarter post properly planted at a time when both the section and quarter-section corners adjacent were known, may be used in restoring either of these corners when lost, by prolonging the line over the known corners and doubling the distance. Any other intermediate corner whose location is definitely known may be used in a similar manner. On a similar principle, the Supreme Court of Illinois decided in the case of Noble *v.* Chrisman (88 Ill. 186) that the northwest corner of section 19 could, in that instance, be better determined by tracing the section lines from known corners east and west of the range line to their intersection with that line, and measuring the jog between the corners, than it could by prorating six miles of the range line.

Most of the difficulties which the surveyor has to contend with in restoring lost corners arise from errors made in the original survey, or in the field notes thereof. He should bear in mind that errors in the original survey cannot be corrected by him. In any case of a lost corner, find as many of the adjacent corners of the original survey as possible, according to the best evidence that can be had to prove their exact location. Having done this, the others may be found according to the rules already laid down. *But do not give up a corner as lost while any means of finding its exact location are left untried.* There is great virtue in a pick and shovel intelligently applied to the finding of corner posts and monuments. This is very important, as it is very difficult, if not impossible, in many cases, to re-locate a lost corner in the exact position it originally occupied, by surveys from distant corners. The following extracts from a paper read by the author

before the Michigan Association of Surveyors and Engineers, treat more fully of the application of the foregoing principles to finding corners of the United States survey in those regions where wooden posts were planted for corner monuments:

"It often happens that one surveyor will fail utterly in finding the marks of an original corner, while another, more apt in discovering the evidences, will strike upon it readily. These evidences are of various kinds, some of which it is the principal aim of this paper to discuss.

I take it that the best possible evidence of the location of an original corner is the monument fixed at that corner when the survey was made. (*Vide* McClintock *v.* Rogers, 11 Ill. 279; also Gratz *v.* Hoover, 16 Penn. State Rep. 232; 16 Ga. 141.) After this come witness trees, fences, distant corners of the same survey, and the testimony of persons.

All these latter kinds of evidence only go to corroborate the first, and may take the place of the first only so far as they may any of them seem to have weight in any particular case.

Many of the corners of the United States survey were marked by planting a post or stake in the ground. These stakes had notches cut in them, were squared at the top, and set in certain regular positions in the ground. These marks tended to distinguish them from other stakes that might chance to be driven in the ground for any purpose. When trees stood conveniently near, two of them were marked, and their directions and distances from the corner were given in the field notes. When no trees were near, a mound was sometimes raised about the post.

Some of the posts have been entirely destroyed, but the bottoms of a great many of them still remain, much decayed, but plainly visible when the surface earth is removed from about them.

To find them, careful manipulation is required. The surveyor first determines as nearly as he can, from extrinsic evidence, the point where the corner post should be looked for. He then, with a shovel, spade or hoe, carefully removes the surface earth, a little at a time, being particular not to strike deep at first into the earth at the level as it was when the stake was set. The best and sometimes the sole evidence of a corner has often been destroyed by an ignorant person striking deep into the ground, expecting to find a sound stake, and casting away the decayed wood and filling up the hole of a rotten one without observing it. If the surveyor is looking in the right place, and the earth has not been previously removed, he will soon come upon the object of his search; but he must be careful lest he mistake it. If the soil is a stiff clay, packed hard, as in a road, or covered with a sward, he will presently find a hole of the size and shape of the stake which

made it. This hole will contain the decayed wood of the stake, and a marking pin may be readily thrust to the bottom. By carefully scraping or cutting away the earth from the top, or cutting down at one side of the hole, its size, shape and direction may be readily discovered. Thus it often happens that the position of a corner is as well and satisfactorily marked by the decayed stake as it was by the sound one. It sometimes happens that new stakes have been driven beside the original stake, so that several different ones will be found by the surveyor. He will seldom have any difficulty in deciding which is the true corner by its appearance, for the first stake will be more completely decayed and of a darker color.

As a rule, it will be driven deeper and straighter down than the newer stakes. Then, too, the original stakes were generally round, being cut from whole timber, while the later ones were often cut from rails or other split timber, the sharp corners of which can be readily seen in the holes made by them.

There is thus in the appearance of the stakes of the United States survey such peculiarities and such likeness to each other, even when far gone in decay, that the experienced surveyor will be impressed with the appearance of truthfulness pervading them, and will seldom be deceived. This appearance of truthfulness about a stake, which to a surveyor is one of the most valuable parts of the testimony of these silent witnesses, is something that courts and juries can seldom take cognizance of, because, *first*, they speak in a language that courts and juries do not understand, and *secondly*, the evidence is itself destroyed by the surveyor in the taking, and does not come before court or jury in all its freshness, truth and purity. These decayed stakes may be best observed in the light-colored subsoil after the black surface mould has been removed. In sandy soil, the cavity made by the stake is gradually filled by the falling sand as the wood decays, but rotten wood discolors the sand so that where it has not been disturbed the position, size and shape of the stake may be readily traced. In the black muck of our marshes and river bottoms it is more difficult to distinguish the stake near the surface, but as the ground is soft and wet the stakes were driven deep, and we may sometimes find in the wet, peaty subsoil the bottom of the stake so perfectly preserved that even the scratches made in the wood by nicks in the axe are plainly to be seen. When the stakes are constantly wet, they do not decay.

Next we consider the bearing or witness trees. These are marked and their directions and distances noted, in order to assist in finding the corner posts set on the survey. These bearing trees are marked with a blaze and a notch near the ground on the side facing the corner. The measures were taken from this notch. At this time most of the living witness trees have grown to such an extent that only a scar remains in sight, to indicate the point where the notch was cut. In order

to get at the notch, the superincumbent wood, which is in some cases a foot in thickness, will have to be cut away. It will not often be necessary to do this, as we can come sufficiently near the correct point to find the stake without it. But if the stake has been destroyed, or there are several stakes near, we shall need to be exact, and measure from the notch. If the tree has been cut down, and a sound stump remains, the marks will be easily exposed. Sometimes the mark is gone, but a part of the stump is left. At others the stump is gone, but a dish-like cavity remains in the earth to show where the tree once stood. We can almost always find under and around these cavities places where the large roots have penetrated the subsoil, and thus be able to locate within a foot or so the position of the bole of the tree when standing. In looking for a corner post, we may frequently assume for the time being that a certain stump or a cavity where a tree had stood was the stump of or the place occupied by a bearing tree. If we then measure the required direction and distance, and find a stake, we may reasonably conclude that our assumption was correct. Such assumptions are frequently of great assistance in finding corners. There may be, and I know there are cases, where the original corner stakes have been destroyed, and can be more nearly restored to their original position by measurements from old stump bottoms or holes in the ground than in any other way. But bearing trees, however good their condition, are by no means infallible witnesses as to the location of a corner. Mistakes in laying down their direction or distance, or both, are not rare. (See McClintock *v.* Rogers, 11 Ills, 279.) A direction may be given as north instead of south, east instead of west, or *vice versa*. The limb may have been wrongly read 64° for 56°. The figures denoting the bearing may have been transposed in setting down, as 53 for 35. So, too, the chain may have been wrongly read, as 48 for 52, the links having been counted from the wrong end. Or they may have counted from the wrong tag, as 48 for 38. Mistakes of the nature of these mentioned are common, so that in working from a bearing tree to find a corner, and not finding the stake at the place indicated in the notes, it will be well to test all these sources of error before giving up the search, for as I have said before, the *post planted at the time of the original survey is the best evidence* of the corner it was intended to indicate.

I next consider fences in their relations to corners. (Potts *v.* Everhart, 26 Penn. St. Rep., 493.) Whether any particular fence may be depended on to indicate the true line will depend on the particular circumstances attending that case. In a general and rough way, a fence will indicate to the surveyor where to begin looking for his corner. But the practice has been, and still is common, for the first settlers on a section to clear and fence beyond the line in order to have a clear place on which to set their permanent fence when they get ready to

build it. Afterward they forget where the line is and set the new fence where the old one stood. Many fences, too, were set without any survey or any accurate knowledge where the line was and left there to await a convenient time to have the line established. So, too, where the land has been long settled and occupied, it is a common custom for adjoining land owners by consent to set the fence on one side of the true line, there to remain until they are ready to rebuild, the one party to have the use of the land for that time in consideration of clearing out and subduing the old fence row. The original parties frequently sell out or die, and the new owners have no knowledge of the agreement and suppose the fence to be on the true line. For these reasons, fences should be looked on with suspicion, unless corroborated by other evidence, and the surveyor should enquire pretty closely into the history of a fence before placing any great reliance on it to determine the position of a corner. It may be the best of evidence, or it may be utterly worthless.

It not unfrequently happens that there are no trustworthy marks near a corner to direct the surveyor in his search for the post or from which to replace it if it be destroyed. In these cases, he must visit the nearest corners he can find in each direction (varying with the circumstances whether it be section corner or quarter post he wishes to find or restore), go through the process of identification with each of them, and then make his point so that it will bear the same relation to these corners as did the original corner post. Many very intelligent gentlemen suppose that if the surveyor can but find one of the corners of the original United States survey he can readily determine the position of all the rest from it. They were never more mistaken in their lives. The continual change in the direction of the magnetic needle, the uncertainty as to what its direction was when any particular line was run, the difference in the lengths of chains, and the difference in the men who use them, introduce so many elements of uncertainty into the operation as to render it one of little value, and not to be resorted to except in the absence of trustworthy evidence nearer at hand.

If it be a section corner you desire to find or replace, and have adjacent quarter posts in each direction to work from, you will not be likely on the one hand to fall more than a rod or two out of the way, and on the other hand will not be likely to come within a foot or two of the right place. This method will assist you in seaching for the original stake, and if that be destroyed, and no better evidence presents itself, may be used to determine the point where the corner stake shall be placed. The chief difficulty in applying this method to determine corners arises from the fact that the measurements made on the original surveys were not uniform in length on different sections, and frequently not on different parts of the same section. I have measured sections 22 and 23 on a level prairie, along the line of high-

ways, where no obstacles of any kind interfered to prevent accurate work. I took the greatest possible care in the chaining to have it as accurate as chain work can be done. On the north line of section 22 my chaining tallied exactly with that of the United States survey, viz., 79.60. On the north line of section 23, my measure was 80.96, that of the United States survey, 80.40—a difference of 56 links. Fortunately, all the corners of the original survey on this two miles of line were well preserved, and the distance between quarter post and section corners was uniform on the same section in both sections. But suppose that a part of them had been lost, and it was required to restore the middle section corner (n. e. of 22) from the remaining ones. Omit all consideration of corners, north or south, and there remain four different solutions of the problem, depending on which corners were lost and which preserved. Of these different solutions, one would place the corner 9⅛ links, one 14 links, one 18⅜ links, and one 28 links, all east of the true corner. This is not by any means an extreme instance, as I have observed discrepancies twice as great. It is given simply to show how unreliable is the evidence drawn from distant corners of the United States survey.

Lastly, I shall consider the evidence of living persons. [Weaver *v.* Robinett, 17 Mo., 459; Chapman *v.* Twitchell, 37 Maine, 59; Dagget *v.* Wiley, 6 Florida, 482: Lewen *v.* Smith, 7 Port. (Ala), 428; McCoy *v.* Galloway, 3 Han. (Ohio), 283; and Stover *v.* Freeman, 6 Mass., 441.] Conceding all men to be equally honest in their evidence, there is a vast deal of difference among them with regard to their habits of observation and their ability to determine localities. Some have an exceedingly acute sense of locality, if we may so call it, and can determine very accurately the position of any object which they have been accustomed to see; while others seem to have little or no capacity of that sort. I have found many men who would describe accurately the sort of monument used to perpetuate a corner, and who would tell you that they could put their foot on the very spot to look for it; but when the trial came I have found but few of them who could locate the point within several feet, unless they had some object near at hand to assist the memory, and even then they would frequently fail.

It may happen where a corner post has been destroyed, that its location can be more nearly determined by the testimony of persons who were familiar with it when standing and can testify to its relations to other objects in its vicinity, than in any other way. But the surveyor in receiving this testimony should ascertain as far as possible what are the habits of accurate observation and the memory of localities possessed by the person testifying, in order to know how much weight to give his testimony."

SECTION XII.

MISCELLANEOUS.

240. Questions of Practice.—Answers to most if not all questions which arise in the surveyor's practice will be found in the Supreme Court decisions which have been quoted. The following questions which have been raised in several surveyors' associations, are given with the answers adopted in each case, or a reference to the law decision or principle which governs it.

1. An interior section has its quarter posts out of line and not at equidistant points between the section corners. How shall the centre be determined?

Ans. At the intersection of straight lines from each quarter section corner to its opposite corresponding corner.

See page 246, Sec. 100, Second.

2. How shall the quarter posts on the north and west lines of the township which were not established by the U. S. survey be located?

Ans. The corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from two corners which stand on the same line.

See page 246, Sec. 100, First.

Section 6 is an exception to this rule.

See page 299.

3. Posts for lines closing on the north and west boundaries of townships are often off the boundary line to one side or the other. Shall the boundary line be deflected to pass through these posts?

Ans. No. The posts serve to show the position of the section line, but the line itself stops at the township boundary.*

Mich. Surv. Rep., 1881.

4. Are the station or line trees marked on the government surveys and returned in the field notes, monuments of the lines?

Ans. Yes.

See page 246, Sec. 100, Second.

Billingsley *v.* Bates, 30 Ala. 378.

5. How shall the east and west quarter line of section 30 be located, there having been no quarter post set on the east side of the section by the U. S. survey, because of a lake?

Ans. Locate the west quarter post as directed in the answer to question 2. Then run the quarter line east on a course which is intermediate between the courses of the north and the south lines of the section.

See page 296.

6. A closing corner on the north or west boundary of the township is lost. The field notes do not give the distance between the closing corner and the adjacent corner on the boundary. How shall it be restored?

Ans. Prolong the known portion of the line to its intersection with the boundary and there set the corner.

See Billingsley *v.* Bates, 30 Ala. 378; *ante* p. 106.

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* Note.—The author has not met with any judicial determination of this question. Some very able surveyors hold a different view from that expressed above. But suppose that the deputy surveyor, not finding the standard corner, as frequently happens, ran his line directly over it, and planted his closing corner in the section line beyond. It would then be impossible to deflect the township line so as to pass through both corners.

It would seem to be a safe way for the surveyor, in making a survey on a section, to locate his lines with reference to the corners established for that section only; and leave any question of title, raised by overlapping or non-closing lines, to be settled by the courts.

7. Should section lines running north and south be run in a straight line between known corners to locate lost corners on interior sections?

Ans. Not unless the original lines were actually straight lines between the known points, which they seldom are.

See Moreland *v.* Page, 2 Clarkes, Iowa, 139; *ante* p. 108.
Martz *v.* Williams, 67 Ill., 306; *ante* p. 107.

8. How shall the half-quarter corner on the quarter line be located on those quarter sections which adjoin the north and west lines of the township?

Ans. Measure the distance from the centre of the section to the quarter post on the township line.

Then place the corner on the quarter line at a distance of twenty chains proportionate measurement from the centre of the section. In order to prorate the distance, your own measure should be compared with a distance which is a mean between the distances given in the field notes as the length of the corresponding lines of the section on either side. For example, on section 3 the distance by U. S. survey from the east ¼ post to township line is 42.18; from the west ¼ post to township line is 43.20; which gives a mean distance of 42.69.

Commissioner McFarland gives the following reply to a similar question:

DEPARTMENT OF THE INTERIOR,
GENERAL LAND OFFICE,
Washington, D. C., February 11, 1882.

Isaac Teller, Esq., Webberville, Ingham County, Michigan:

SIR—I am in receipt of your letter of the 5th instant requesting information in regard to the proper method of locating the quarter-quarter corners north of the legal centres of the northern tier of sections in a township when the present measurement of the east and west boundaries of the section differs from the original measurement.

In reply, I have to state that the length of the quarter line from the south quarter corner to the township line is to be considered as the mean of the east and west boundaries of the section as given in the field notes, and where the present measurement of the section lines differs from the original measurement, the rule of proportionate measurement applies to the quarter line as well as to the section lines in the establishment of quarter-quarter corners on the half mile closing

on the township boundary. See enclosed circular dated November 1, 1879.

The mean width of the north half of the section in the case stated by you is 40.18 chains, while by your chaining it is 42.42 chains (calling the distance to the east and west quarter line 40.00 chains), therefore the proportion will be as 40.18 : 42.42 :: 20.00 : 21.11 chains, the distance north of the centre of the section at which by your chaining the quarter-quarter corner should be located.

Very respectfully,

N. C. McFARLAND, Commissioner.

9. In surveying sections fractional on the township line, to restore lost quarter section corners, should the lines be divided pro rata according to the U. S. field notes, or should the south or east quarters be made full and the entire excess or deficiency be thrown into the fraction?

Ans. Any difference between your measure and the government measure must be distributed proportionally between the different parts of the section.

See p. 246, Sec. 100, Second.
Moreland *v.* Page, 2 Clarkes, Iowa 139.
Jones *v.* Kimble, 19 Wis. 429.
Martz *v.* Williams, 67 Ill. 306,

In Missouri, the Supreme Court holds (Knight *v.* Elliott, 57 Mo. 317) a different view, viz., that the difference in measure is all to be thrown into the fraction.

It is difficult to see upon what grounds this decision can be upheld in view of the fact that all rights to the land were acquired and held under the law of Congress, which expressly states that the length of such lines as returned by the surveyor-general shall be held and considered as the true length thereof.

Northwest Quarter, Sec. 18.

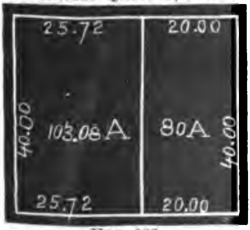

FIG. 128.

10. The accompanying figure is a copy of the plat of the U.S. survey of this quarter section.

A owns the whole quarter. He sells to B the W. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of section 18, containing $91\frac{54}{100}$ acres. At about the same time he sells to C the E. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of section 18, containing $91\frac{54}{100}$ acres.

Where shall the surveyor run the dividing line between B and C?

Ans. The language of the deed clearly shows the intention of A to sell and of B and C to purchase each the half of the area of the quarter section. The surveyor should so locate the line as to carry out the evident intent of the parties. See rule 2, p. 306 and rule 14, p. 327. The fact that the quarter is differently subdivided on the government plat has no bearing on this case.

11. Certain early surveyed townships had three sets of corners on the range lines. (1) Those set when the range lines were run; (2) Those set as closing corners running east; (3) Those set as closing corners running west. What use is made of each set of corners?

Ans. The first corners set determine the location of the range line. The second and third sets of corners determine the location of their respective section lines which close on and terminate at the range line.

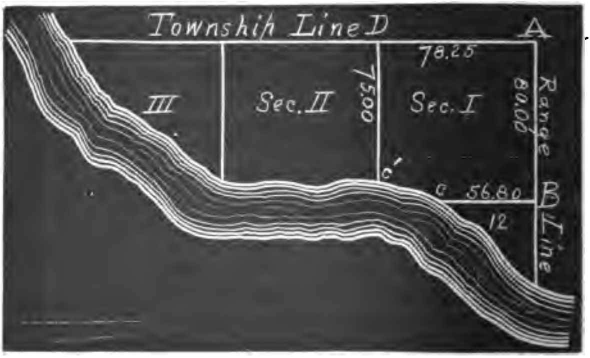

FIG. 129.

12. This figure shows a fractional township on the Ohio River. The figures show the dimensions of section 1, as shown by the field notes of the United States survey. By a subsequent measure,

$$AB = 82.25 \text{ chains, and}$$
$$AD = 79.50 \quad ``$$

How shall the northeast quarter of section 1 be laid off, no quarter-posts having been planted?

Ans. Place the quarter-section corners on the north and east sides of the section in line with and midway between their respective section corners. Make the east and west quarter-line parallel with the south line of the section, placing the west quarter-post at the point where the quarter-line thus run intersects the section line. From the north quarter-post run the quarter-line south on a course which is a mean between the courses of the east and the west lines of the section, placing the south quarter post at the intersection of the section and quarter-section lines.

The exceptional features of this case are that no quarter-posts were set on the United States survey, and that the east line of the section is just 80 chains in length, having been run from the north to the south.

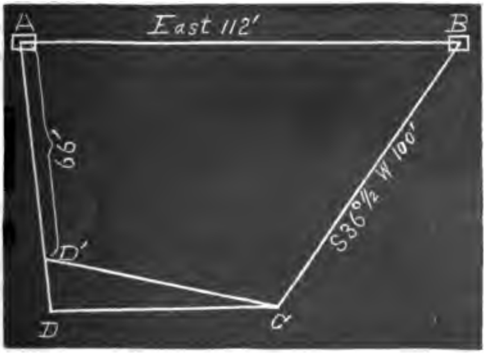

FIG. 130.

13. The description in the deed runs: "Beginning at a stone (A), at the N.W. corner of lot 401; thence east 112 ft. to a stone (B); thence S. 36½° W. 100 ft.; thence west parallel with AB to the west line of said lot 401; thence north on west line of said lot, 66 ft., to the place of beginning." The points A and B, and angle ABC, are fixed. C, by

construction and in fact, is $80\frac{18}{100}$ ft. distant, at right angles from the line *AB*.

1. Shall I locate *CD* parallel with *AB*, or locate *D* 66 ft. from *A*?

2. Have I any right to consider any apparent intention to locate 66 ft. or $80\frac{18}{100}$ ft. from *A*?

3. Have I, if I know it, any authority to consider the actual intention of the grantor to locate *CD*?

4. If the distance *AB* should actually measure 114 ft., am I to use it, or shall I make *B* 112 ft. from *A*?

Ans. 1. The answer to this question will depend upon the state of facts brought out in answer to questions 2 and 3. If there be evidence showing what the intention and understanding of the parties to the conveyance was as to which of the two lines should be taken, that evidence would settle the question. If not, that construction may be given to the deed which will operate most strongly against the grantor and give the grantee the greater amount of land. So far as anything is shown in the question, the deeds to the adjacent land might furnish the necessary evidence.

2 and 3. Yes. Judge Cooley says, (see "Judicial Functions of Surveyors"): "The surveyor must inquire into all the facts, giving due prominence to the acts of parties concerned, and always keeping in mind * * * * that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and the same rules that will govern theirs."

4. The monument controls the distance.

14. A piece of land is sold, and described as commencing at the north quarter-post of section 15, and running thence east 100 rods; thence south 160 rods; thence west 100 rods; thence north 160 rods, to the place of beginning; containing 100 acres, according to the United States survey.

Ques. How shall it be set off?

Ans. The deed clearly indicates the understanding of the parties to the conveyance to be that the land should pass according to the rules that govern the United States survey. One of these rules is, that "the length of the boundary lines as returned by the surveyor-general shall be held and considered as the true length thereof." Hence in this case, measure east from the quarter-post along the section line 25 chains of just such measure as the United States surveyors gave; or in other words, of *pro rata* measurement. Suppose the distance by the field notes to be 40.32 chains from quarter-post to section corner. Then lay off $\frac{25.00}{40.32}$ of that distance. Proceed in a similar manner, running east on the quarter-line from the center of the section, and the two points thus located will be the corners of the 100 acres. To get the length of the south line of the N. E. $\frac{1}{4}$ of the section by the United States survey, take the half sum of the measure given on the north and the south lines of the section. Supposing it to be 40 32 on the north, and 40.18 on the south, then the distance on the quarter-line would be equal to

$$\frac{40.32 + 40.18}{2} = 40.25,$$

and you should measure off $\frac{25.00}{40.25}$ of this distance for the corner.

241. The Rights, Duties and Responsibilities of Surveyors.—Surveyors, by the consent and acquiescence of the parties concerned, are usually the arbiters of disputed boundaries, and their decisions, when thus acquiesced in by the parties, become in time as binding, and as much respected by the authorities, as the decisions of juries and courts of law. It is probable that at least ninety-nine per cent. of all questions of disputed bound-

aries are thus settled by the interested parties themselves, in accordance with the decision of the surveyor.

Surveyors, from constantly exercising this seeming authority, come at last in many cases to believe it to be absolute and final, something which must be respected, overlooking the fact that the only force their decisions have comes from the consent of the parties. When that consent is withheld, the case goes to the courts for settlement; and thus the courts have in some cases felt called upon to define the surveyor's standing before the law. They say:

1. "Surveyors have no more authority than other men to determine boundaries, of their own motion. All bounds and starting points are questions of fact to be determined by testimony. Surveyors may or may not have in certain cases means of judgment not possessed by others, but the law can not and does not make them arbiters of private rights.

Cronin *v.* Gore, 38 Mich. 381.

2. The law recognizes surveyors as useful assistants in doing the mechanical work of measurement, and calculation, and also allows such credit to their judgment as belongs to any experience which may give it value in cases where better means of information do not exist. But the determination of facts belongs exclusively to courts and juries. Where a section line or other starting point actually exists, is always a question of fact, and cannot be left to the opinion of an expert for final decision. And where, as is generally the case in an old community, boundaries have been fixed by long use and acquiescence, it would be contrary to all reason to have them interfered with on any abstract notion of science.

Stewart *v.* Carleton, 31 Mich. 273.

Gregory *v.* Knight, 50 Mich. 61.

3. New surveys disturbing old boundaries are not to be encouraged.

Toby *v.* Secor, Wisconsin. N. W. Reporter, Vol. 19, p. 79.

4. Lines long unquestioned ought not to be disturbed upon a mere disagreement among surveyors, especially when the last survey is made under the unfavorable circumstances of corner posts and witness trees being gone, which it is probable to suppose were in existence at the time of the first survey.

Case *v.* Trapp, 49 Mich. 59.

5. County surveyors' certificates are not admissible in evidence unless they contain all the particulars required by the statute to be entered in the surveyor's record.

Smith *v.* Rich, 37 Mich. 549.

The statute of Michigan requires the length of all lines run, the area of lands surveyed, and other particulars, to be entered in the county surveyor's record. In the above case the survey was solely to find the location of a corner post. As the surveyor's certificate did not show any area of land surveyed, it was not admitted in evidence.

6. A surveyor was called on to survey the line of a highway. He performed the work so unskillfully as to render a new survey necessary. A large amount of road constructed at great expense, on the line designated by the surveyor before the mistake was discovered, had to be abandoned. Action was brought to recover damages. Held, that whether the defendant was a professional or official surveyor, or represented himself as such, his undertaking was that he should bring to the work the necessary knowledge and skill to perform the same properly and correctly; and if he failed to do so, and the plaintiff suffered damage in consequence of such failure, the plaintiff will be entitled to recover.

Commissioner of Highways *v.* Beebe, Mich. Sup. Court. N. W. Rep., Vol. 20, No. 16.

The following paper, by Chief Justice Cooley, of the Supreme Court of Michigan, discusses more fully the surveyor's functions:

242. The Judicial Functions of Surveyors.—When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan, all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes; and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is, that the lines were very carelessly run, the monuments inaccurately placed; and, as the recorded witnesses to these were many times wanting in permanency, it is often the case that when the monument was not correctly placed, it is impossible to determine by the record, by the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them, however, were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate

the importance of having correct lines to determine boundaries when land should become dear. The fact probably is, that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the State of Michigan were made under authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold, and the work of improvement began. A generation has passed away since they were converted into cultivated farms, and few if any of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing was substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining seventy; for parties buy, or are supposed to buy, in reference to these monuments, and are entitled to what is within their lines, and no more, be it more or less

While the witness trees remain, there can generally be no difficulty in determining the locality of the stakes.

When the witness-trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men, whose theoretical education is thought to make them experts, who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and would have been, if placed correctly. This is a serious mistake. The problem is now the same that it was before: To ascertain by the best lights of which the case admits, where the original lines were. The mistake above alluded to, is supposed to have found expression in our legislation; though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869, (Compiled Laws, § 593), amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

"*Second.* Extinct interior section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

"*Third.* Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line."

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monument was set for an interior section corner, but did

not happen to be "at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between the government subdivisions.

There are two senses in which the word extinct may be used in this connection: One, the sense of physical disappearance; the other, the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man loses his deed, he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and federal law; it is a question of property right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether State or federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of a surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark,

which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them, not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have disappeared. (Stewart *v.* Carleton, 31 Mich. Reports, 270; Diehl *v.* Zanger, 39 Mich. Reports, 601.) And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession most often appear in cities and villages. In towns the block and lot stakes soon disappear; there are no witness trees, and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely to be soon marked off by fences, and the lots in a block will be measured off from these, without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was the starting point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and all lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done; the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and used for many years,

and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor, that he may make sure his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was so or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will allow the new survey to unsettle their possessions, but there is always a probability of finding some one disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years. In fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that shall be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. Public policy requires that such lines be not lightly disturbed, or disturbed at all after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were at an-

other. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong if not conclusive evidence of such settlement. The peace of the community absolutely requires this rule. It is not long since, that in one of the leading cities of the State an attempt was made to move houses two or three rods into a street, on the ground that a survey under which the street had been located for many years, had been found on a more recent survey to be erroneous.

From the foregoing, it will appear that the duty of the surveyor where boundaries are in dispute must be varied by the circumstances. 1. He is to search for original monuments, or for the places where they were originally located, and allow these to control if he finds them, unless he has reason to believe that agreements of the parties, express or implied, have rendered them unimportant. By monuments in the case of government surveys we mean of course the corner and quarter-stakes; blazed lines or marked trees on the lines are not monuments: they are merely guides or finger posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. 2. If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any State statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts: giving due promi-

nence to the acts of parties concerned, and always keeping in mind, *first*, that neither his opinion nor his survey can be conclusive upon parties concerned; and, *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his action by the same lights and the same rules that will govern theirs.

It is always possible, when corners are extinct, that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course, it is desirable that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander Lines.—The subject to which allusion will now be made, is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless, it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are somewhat connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers, as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and that all in front of them remains unsold. Of course this is erroneous. There was was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the

middle line of the river; and while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the State, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this State, the rule that the center line is the boundary line, is applied to all our great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation, in some cases, with the stream in the main shallow, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander line to the center line of the river. Of course, the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream, or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and, in some cases, if it were continued in the same direction to the center line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of a line crossing it at right angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the center line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and is not limited to government surveys, but applies as well to water lots which appear as such on town plats. (Bay City Gas Light Co. *v.* The Industrial Works, 28 Mich. Reports, 182.) It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so that the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what shall be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive; but if he adopts one that follows as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to have a line on

the legal boundary, namely, the center line of the stream proportioned to the length of his line on the shore; and the problem in each case is, how this is to be given him. Alluvion, when a river imperceptibly changes its course, will be apportioned by the same rules.

The existence of islands in a stream when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purposes of government sale afterward, for the reason that the rights of the bank owners are fixed by their purchase; when making that they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which they purchase would include under it cannot afterward be taken from them. It is believed, however, that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the great lakes.

On all the little lakes of the state which are mere expansions near their mouths of the rivers passing through them—such as the Muskegon, Pere Marquette and Manistee—the same rule of bed ownership has been judicially applied that is applied to the rivers themselves; and the division lines are extended under the water in the same way. (Rice *v.* Ruddiman, 10 Mich., 125.) If such a lake were circular, the lines would converge to the center; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be

important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as such. It is sometimes important, however, to run the lines out for considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. (Lorman *v.* Benson, 8 Mich., 18; People's Ice Co. *v.* Steamer Excelsior, recently decided.)

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the state, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

1. To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?

2. What public rights exist in them?

3. If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the great lakes. Perhaps it should be held that the boundary is at low water mark, but improvements beyond this would

only become unlawful when they became nuisances. Islands in the great lakes would belong to the United States until sold, and might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public like the right of navigation, but is to be exercised in such manner as not to interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the state, not to the United States; and so, it is believed, does the bed of a lake also. (Pollord *v.* Hagan, 3 Howard's U. S. Reports.) But such rights are not generally considered proper subjects of sale, but like the right to make use of the public highways, they are held by the state in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the state; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size—say, even a mile across—there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have have any rights in it whatever. Where it is not so included, it is believed they have rights

of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the states may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the state should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but the state could not drain, even for this purpose, against the will of the shore owners, unless their rights were appropriated and paid for.

Upon many questions that might arise between the state as owner of the bed of a little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and cannot be judicial officers, but in a great many cases they act in a *quasi* judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value.

TABLES.

LOGARITHMS OF NUMBERS

FROM 1 TO 10,000.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
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3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	419
4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	396
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	382
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	369
8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	351
4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9467	9868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272
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1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	254
1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	214
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7430	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198
No	0	1	2	3	4	5	6	7	8	9	Diff

No	0	1	2	3	4	5	6	7	8	9	Diff.
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1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
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2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
8	411620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
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1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
No.	0	1	2	3	4	5	6	7	8	9	Diff

No.	0	1	2	3	4	5	6	7	8	9	Diff
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
9	460898	461048	461198	461348	461499	1649	1799	1948	2098	2248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	147
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
1	9828	9959	520090	520221	520353	520484	520615	520745	520876	521007	131
2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
5	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	1351	128
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
5	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	120
4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
5	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
1	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
5	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	1191	1299	1408	1517	1625	1734	1843	1951	109
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036	108
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	623249	623353	623456	693559	623663	623766	623869	623973	624076	624179	103
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
5	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	610021	630123	630224	630326	102
7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	101
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
5	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	99
6	9486	9586	9686	9785	9885	9984	640084	640183	640283	640382	99
7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
7	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Dif.
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
9	680336	680426	680517	680607	680698	680789	680879	0970	1060	1151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
No.	0	1	2	3	4	5	6	7	8	9	Dif.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	80
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
2	9736	9814	9891	9968	750045	750123	750200	759277	750354	750431	77
3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Dif
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
No.	0	1	2	3	4	5	6	7	8	9	Dif.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
9	2245	2312	2379	2145	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
1	820201	820267	820333	820399	829464	820530	820595	0661	0727	0792	65
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	66
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
6	9947	830011	830075	830139	830204	830268	839332	830396	830460	830525	64
7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	D.
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	[illegible]
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	[illegible]
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	[illegible]
5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	[illegible]
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	6
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	6
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	6
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	6
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
6	4913	4934	5034	5095	5156	5216	5277	5337	5398	5459	61
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3262	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
9	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
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No.	0	1	2	3	4	5	6	7	8	9	Diff.
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
8	2003	2057	2112	1266	2221	2275	2329	2384	2438	2492	54
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
5	1158	1211	1264	1317	1331	1424	1477	1530	1584	1637	53
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
2	9995	960042	960090	960138	960185	960233	960280	960328	960376	960423	48
3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
6	1276	1322	1369	1415	1421	1508	1554	1601	1647	1693	46
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
No.	0	1	2	3	4	5	6	7	8	9	Diff.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
9	1819	1864	1901	1954	2000	2045	2090	2135	2181	2226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	45
7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

TABLE II.

NATURAL SINES AND COSINES.

	0°		1°		2		3°		4°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.39642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	89°		88°		87°		86°		85°		

	5°		6°		7°		8°		9°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15292	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos	Sine.	M.
	84°		83°		82°		81°		80°		

M.	10°		11°		12°		13°		14°		M.
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24953	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	79°		78°		77°		76°		75°		

M.	15° Sine.	15° Cos.	16° Sine.	16° Cos.	17° Sine.	17° Cos.	18° Sine.	18° Cos.	19° Sine.	19° Cos.	M.
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
15	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94325	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	47
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.39008	.91648	.41602	.90936	25
36	.35183	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	49
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.99692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	69°		68°		67°		66°		65°		

	25°		26°		27°		28°		29°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.43367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.41972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.43330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.83743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82748	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84345	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.21179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79582	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.10212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79015	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos	Sine.	M.
	54°		53°		52°		51°		50°		

	40°		41°		42°		43°		44°		
M.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	M.
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
M.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	Cos.	Sine.	M.
	49°		48°		47°		46°		45°		

TABLE III.

NATURAL TANGENTS

AND

COTANGENTS.

	0°		1°		2°		3°		
M.	Tang.	Cotang.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang.	M.
0	.00000	Infinite	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180 932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
M.	Cotang.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	89°		88°		87°		86°		

	4°		5°		6°		7°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	85°		84°		83°		82°		

	8°		9°		10°		11°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang	Tang.	Cotang	M.
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.29164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
42	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
M.	Cotang.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang	Tang.	M.
	81°		80°		79°		78°		

	12°		13°		14°		15°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
M.	Cotang	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	77°		76°		75°		74°		

	16°		17°		18°		19°		
M.	Tang.	Cotang.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang.	M.
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.24693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2 76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
M.	Cotang.	Tang.	Cotang	Tang.	Cotang.	Tang.	Cotang	Tang.	M
	73°		72°		71°		70°		

	20°		21°		22°		23°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56486	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	3.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	69°		68°		67°		66°		

	24°		25°		26°		27°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	65°		64°		63°		62°		

	28°		29°		30°		31°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	61°		60°		59°		58°		

M.	32° Tang.	32° Cotang.	33° Tang.	33° Cotang.	34° Tang.	34° Cotang.	35° Tang.	35° Cotang.	M.
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
M.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	57°		56°		55°		54°		

	36°		37°		38°		39°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
M.	Cotang	Tang.	Cotang.	Tang,	Cotang.	Tang.	Cotang.	Tang.	M.
	53°		52°		51°		50°		

	40°		41°		42°		43°		
M.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	M.
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
M	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	M.
	49°		48°		47°		46°		

	44°				44°				44°		
M.	Tang.	Cotang.	M.	M.	Tang.	Cotang.	M.	M.	Sang.	Cotang.	M.
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
M.	Cotang	Tang.	M.	M.	Cotang	Tang.	M.	M.	Cotang.	Tang.	M.
	45°				45°				45°		

TABLE IV.

LOGARITHMIC SINES, COSINES,

TANGENTS,

AND

COTANGENTS.

0° 179°

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	Inf. neg.		0.000000	.00	Inf. neg.		Infinite.	60
1	6.463726	5017.17	.000000	.00	6.463726	5017.17	13.536274	59
2	.764756	2934.85	.000000	.00	.764756	2934.83	.235244	58
3	.940847	2082.31	.000000	.00	.940847	2082.31	.059153	57
4	7.065786	1615.17	.000000	.00	7.065786	1615.17	12.934214	56
5	.162696	1319.68	.000000	.00	.162696	1319.69	.837304	55
6	.241877	1115.75	9.999999	.01	.241878	1115.78	.758122	54
7	.308824	966.53	.999999	.01	.308825	996.53	.691175	53
8	.366816	852.54	.999999	.01	.366817	852.54	.633183	52
9	.417968	762.63	.999999	.01	.417970	762.63	.582030	51
10	7.463726	689.88	9.999998	.01	7.463727	689.88	12.536273	50
11	.505118	629.81	.999998	.01	.505120	629.81	.494880	49
12	.542906	579.36	.999997	.01	.542909	579.33	.457091	48
13	.577668	536.41	.999997	.01	.577672	536.42	.422328	47
14	.609853	499.38	.999996	.01	.609857	499.39	.390143	46
15	.639816	467.14	.999996	.01	.639820	467.15	.360180	45
16	.667845	438.81	.999995	.01	.667849	438.82	.332151	44
17	.694173	413.72	.999995	.01	.694179	413.73	.305821	43
18	.718997	391.35	.969994	.01	.719003	391.36	.280997	42
19	.742477	371.27	.999993	.01	.742484	371.28	.257516	41
20	7.764754	353.15	9.999993	.01	7.764761	351.36	12.235239	40
21	.785943	336.72	.999992	.01	.785951	336.73	.214049	39
22	.806146	321.75	.999991	.01	.806155	321.76	.193845	38
23	.825451	308.05	.999990	.01	.825460	308.06	.174540	37
24	.843934	295.47	.999989	.02	.843944	295.49	.156056	36
25	.861662	283.88	.999988	.02	.861674	283.90	.138326	35
26	.878695	273.17	.999988	.02	.878708	273.18	.121292	34
27	.895085	263.23	.999987	.02	.895099	263.25	.104901	33
28	.910879	253.99	.999986	.02	.910894	254.01	.089106	32
29	.926119	245.38	.999985	.02	.926134	245.40	.073866	31
30	7.940842	237.33	9.999983	.02	7.940858	237.35	12.059142	30
31	.955082	229.80	.999982	.02	.955100	229.81	.044900	29
32	.968870	222.73	.999981	.02	.968889	222.75	.031111	28
33	.982233	216.08	.999980	.02	.982253	216.10	.017747	27
34	.995198	209.81	.999979	.02	.995219	209.83	.004781	26
35	8.007787	203.90	.999977	.02	8.007809	203.92	11.992191	25
36	.020021	198.31	.999976	.02	.020045	198.33	.979955	24
37	.031919	193.02	.999975	.02	.031945	193.05	.968055	23
38	.043501	188.01	.999973	.02	.043527	188.03	.956473	22
39	.054781	183.25	.999972	.02	.054809	183.27	.945191	21
40	8.065776	178.72	9.999971	.02	8.065806	178.74	11.934194	20
41	.076500	174.41	.999969	.02	.076531	174.44	.923469	19
42	.086965	170.31	.999968	.02	.086997	170.34	.913003	18
43	.097183	166.39	.999966	.02	.097217	166.42	.902783	17
44	.107167	162.65	.999964	.03	.107202	162.68	.892797	16
45	.116926	159.08	.999963	.03	.116963	159.10	.883037	15
46	.126471	155.66	.999961	.03	.126510	155.68	.873490	14
47	.135810	152.38	.999959	.03	.135851	152.41	.864149	13
48	.144953	149.24	.999958	.03	.144996	149.27	.855004	12
49	.153907	146.22	.999956	.03	.153952	146.27	.846048	11
50	8.162681	143.33	9.999954	.03	8.162727	143.36	11.837273	10
51	.171280	140.54	.999952	.03	.171328	140.57	.828672	9
52	.179713	137.86	.999950	.03	.179763	137.90	.820237	8
53	.187985	135.29	.999948	.03	.188036	135.32	.811964	7
54	.196102	132.80	.999946	.03	.196156	132.84	.803844	6
55	.204070	130.41	.999944	.03	.204126	130.44	.795874	5
56	.211895	128.10	.999942	.04	.211953	128.14	.788047	4
57	.219581	125.87	.999940	.04	.219641	125.90	.780359	3
58	.227134	123.72	.999938	.04	.227195	123.76	.772805	2
59	.234557	121.64	.999936	.04	.234621	121.68	.765379	1
60	.241855		.999934		.241921		.758079	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

90° 89°

M.	Sine.	D.1″.	Cosine.	D.1″	Tang.	D.1″.	Cotang.	M.
0	8.241855	119.63	9.999934	.04	8.241921	119.67	11.758079	60
1	.249033	117.68	.999932	.04	.249102	117.72	.750898	59
2	.256094	115.80	.999929	.04	.256165	115.84	.743835	58
3	.263042	113.98	.999927	.04	.263115	114.02	.736885	57
4	.269881	112.21	.999925	.04	.269956	112.25	.730044	56
5	.276614	110.50	.999922	.04	.276691	110.54	.723309	55
6	.283243	108.83	.999920	.04	.283323	108.87	.716677	54
7	.289773	107.21	.999918	.04	.289856	107.26	.710144	53
8	.296207	105.65	.999915	.04	.296292	105.70	.703708	52
9	.302546	104.13	.999913	.04	.302634	104.18	.697366	51
10	8.308794	102.66	9.999910	.04	8.308884	102.70	11.691116	50
11	.314954	101.22	.999907	.04	.315046	101.26	.684954	49
12	.321027	99.82	.999905	.04	.321122	99.87	.678878	48
13	.327016	98.47	.999902	.04	.327114	98.51	.672886	47
14	.332924	97.14	.999899	.05	.333025	97.19	.666975	46
15	.338753	95.86	.999897	.05	.338856	95.90	.661144	45
16	.344504	94.60	.999894	.05	.344610	94.65	.655390	44
17	.350181	93.38	.999891	.05	.350289	93.43	.649711	43
18	.355783	92.19	.999888	.05	.355895	92.24	.644105	42
19	.361315	91.03	.999885	.05	.361430	91.08	.638570	41
20	8.366777	89.90	9.999882	.05	8.366895	89.95	11.633105	40
21	.372171	88.80	.999879	.05	.372292	88.85	.627708	39
22	.377499	87.72	.999876	.05	.377622	87.77	.622378	38
23	.382762	86.67	.999873	.05	.382889	86.72	.617111	37
24	.387962	85.64	.999870	.05	.388092	85.70	.611908	36
25	.393101	84.64	.999867	.05	.393234	84.70	.606766	35
26	.398179	83.66	.999864	.05	.398315	83.71	.601685	34
27	.403199	82.71	.999861	.05	.403338	82.76	.596662	33
28	.408161	81.77	.999858	.05	.408304	81.82	.591696	32
29	.413068	80.86	.999854	.05	.413213	80.91	.586787	31
30	8.417919	79.96	9.999851	.06	8.418068	80.02	11.581932	30
31	.422717	79.09	.999848	.06	.422869	79.14	.577131	29
32	.427462	78.23	.999844	.06	.427618	78.30	.572382	28
33	.432156	77.40	.999841	.06	.432315	77.45	.567685	27
34	.436800	76.57	.999838	.06	.436962	76.63	.563038	26
35	.441394	75.77	.999834	.06	.441560	75.83	.558440	25
36	.445941	74.99	.999831	.06	.446110	75.05	.553890	24
37	.450440	74.22	.999827	.06	.450613	74.28	.549387	23
38	.454893	73.46	.999823	.06	.455070	73.52	.544930	22
39	.459301	72.73	.999820	.06	.459481	72.79	.540519	21
40	8.463665	72.00	9.999816	.06	8.463849	72.06	11.536151	20
41	.467985	71.29	.999812	.06	.468172	71.35	.531828	19
42	.472263	70.60	.999809	.06	.472454	70.66	.527546	18
43	.476498	69.91	.999805	.06	.476693	69.98	.523307	17
44	.480693	69.24	.999801	.06	.480892	69.31	.519108	16
45	.484848	68.59	.999797	.06	.485050	68.65	.514950	15
46	.488963	67.94	.999793	.07	.489170	68.01	.510830	14
47	.493040	67.31	.999790	.07	.493250	67.38	.506750	13
48	.497078	66.69	.999786	.07	.497293	66.76	.502707	12
49	.501080	66.08	.999782	.07	.501298	66.15	.498702	11
50	8.505045	65.48	9.999778	.07	8.505267	65.55	11.494733	10
51	.508974	64.89	.999774	.07	.509200	64.96	.490800	9
52	.512867	64.31	.999769	.07	.513098	64.39	.486902	8
53	.516726	63.75	.999765	.07	.516961	63.82	.483039	7
54	.520551	63.19	.999761	.07	.520790	63.26	.479210	6
55	.524343	62.64	.999757	.07	.524586	62.72	.475414	5
56	.528102	62.11	.999753	.07	.528349	62.18	.471651	4
57	.531828	61.58	.999748	.07	.532080	61.65	.467920	3
58	.535523	61.06	.999744	.07	.535779	61.13	.464221	2
59	.539186	60.55	.999740	.07	.539447	60.62	.460553	1
60	.542819		.999735		.543084		.456916	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

2° 177°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	8.542819	60.04	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.07	.546691	59.62	.453309	59
2	.549995	59.06	.999726	.07	.550268	59.14	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.66	.446183	57
4	.557054	58.11	.999717	.08	.557336	58.19	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.73	.439172	55
6	.563999	57.19	.999708	.08	.564291	57.27	.435709	54
7	.567431	56.74	.999704	.08	.567727	56.82	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	8.577566	55.44	9.999689	.08	8.577877	55.52	11.422123	50
11	.580892	55.02	.999685	.08	.581208	55.10	.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.19	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.79	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.39	.999665	.08	.594283	53.47	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.61	.999655	.08	.600677	52.70	.399323	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.86	.999645	.09	.606978	51.94	.393022	41
20	8.609734	51.49	9.999640	.09	8.610094	51.58	11.389906	40
21	.612823	51.12	.999635	.09	.613189	51.21	.386811	39
22	.615891	50.76	.999629	.09	.616262	50.85	.383738	38
23	.618937	50.41	.999624	.09	.619313	50.50	.380687	37
24	.621962	50.06	.999619	.09	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.09	.625352	49.81	.374648	35
26	.627948	49.38	.999608	.09	.628340	49.47	.371660	34
27	.630911	49.04	.999603	.09	.631308	49.13	.368692	33
28	.633854	48.71	.999597	.09	.634256	48.80	.365744	32
29	.636776	48.39	.999592	.09	.637184	48.48	.362816	31
30	8.639680	48.06	9.999586	.09	8.640093	48.16	11.359907	30
31	.642563	47.75	.999581	.09	.642982	47.84	.357018	29
32	.645428	47.43	.999575	.09	.645853	47.53	.354147	28
33	.648274	47.12	.999570	.09	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.09	.651537	46.91	.348463	26
35	.653911	46.52	.999558	.10	.654352	46.61	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.31	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.44	.334567	21
40	8.667689	45.06	9.999529	.10	8.668160	45.26	11.331840	20
41	.670393	44.79	.999524	.10	.670870	44.88	.329130	19
42	.673080	44.51	.999518	.10	.673563	44.61	.326437	18
43	.675751	44.24	.999512	.10	.676239	44.34	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.17	.321100	16
45	.681043	43.70	.999500	.10	.681544	43.80	.318456	15
46	.683665	43.44	.999493	.10	.684172	43.54	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	8.693998	42.42	9.999469	.10	8.694529	42.52	11.305471	10
51	.696543	42.17	.999463	.11	.697081	42.28	.302919	9
52	.699073	41.92	.999456	.11	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.11	.702139	41.79	.297861	7
54	.704090	41.44	.999443	.11	.704646	41.55	.295354	6
55	.706577	41.21	.999437	.11	.707140	41.32	.292860	5
56	.709049	40.97	.999431	.11	.709618	41.08	.290382	4
57	.711507	40.74	.999424	.11	.712083	40.85	.287917	3
58	.713952	40.51	.999418	.11	.714534	40.62	.285465	2
59	.716383	40.29	.999411	.11	.716972	40.40	.283028	1
60	.718800		.999404		.719396		.280604	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

92° 87°

3° **176°**

M	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.718800	40.06	9.999404	.11	8.719396	40.17	11.280604	60
1	.721204	39.84	.999398	.11	.721806	39.95	.278194	59
2	.723595	39.62	.999391	.11	.724204	39.74	.275796	58
3	.725972	39.41	.999384	.11	.726588	39.52	.273412	57
4	.728337	39.19	.999378	.11	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.11	.731317	39.09	.268683	55
6	.733027	38.77	.999364	.12	.733663	38.89	.266337	54
7	.735354	38.57	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.36	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.16	.999343	.12	.740626	38.27	.259374	51
10	8.742259	37.96	9.999336	.12	8.742922	38.07	11.257078	50
11	.744536	37.76	.999329	.12	.745207	37.87	.254793	49
12	.746802	37.56	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.49	.250260	47
14	.751297	37.17	.999308	.12	.751989	37.29	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.79	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.61	.999286	.12	.758668	36.73	.241332	43
18	.760151	36.42	.999279	.12	.760872	36.55	.239128	42
19	.762337	36.24	.999272	.12	.763065	36.36	.236935	41
20	8.764511	36.06	9.999265	.12	8.765246	36.18	11.234754	40
21	.766675	35.88	.999257	.12	.767417	36.00	.232583	39
22	.768828	35.70	.999250	.13	.769578	35.83	.230422	38
23	.770970	35.53	.999242	.13	.771727	35.65	.228273	37
24	.773101	35.35	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.18	.999227	.13	.775995	35.31	.224005	35
26	.777333	35.01	.999220	.13	.778114	35.14	.221886	34
27	.779434	34.84	.999212	.13	.780222	34.97	.119778	33
28	.781524	34.67	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.51	.999197	.13	.784408	34.64	.215592	31
30	8.785675	34.31	9.999189	.13	8.786486	34.47	11.213514	30
31	.787736	34.18	.999181	.13	.788554	34.31	.211446	29
32	.789787	34.02	.999174	.13	.790613	34.14	.209387	28
33	.791828	33.86	.999166	.13	.792662	33.99	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.54	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.39	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.23	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.08	.999126	.13	.802765	33.22	.197235	32
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	8.805852	32.78	9.999110	.13	8.806742	32.92	11.193258	20
41	.807819	32.63	.999102	.13	.808717	32.78	.191283	19
42	.809777	32.49	.999094	.14	.810683	32.62	.189317	18
43	.811726	32.34	.999086	.14	.812641	32.48	.187359	17
44	.813667	32.19	.999077	.14	.814589	32.33	.185411	16
45	.815599	32.05	.990069	.14	.816529	32.19	.183471	15
46	.817522	31.91	.999061	.14	.818461	32.05	.181539	14
47	.819436	31.77	.999053	.14	.820384	31.91	.179616	12
48	.821343	31.63	.999044	.14	.822298	31.77	.177702	12
49	.823240	31.49	.999036	.14	.824205	31.63	.175795	11
50	8.825130	31.35	9.999027	.14	8.826103	31.50	11.173897	10
51	.827011	31.22	.999019	.14	.827992	31.36	.172008	9
52	.828884	31.08	.999010	.14	.829874	31.23	.170126	8
53	.830749	30.95	.999002	.14	.831748	31.10	.168252	7
54	.832607	30.82	.998993	.14	.833613	30.96	.166387	6
55	.834456	30.69	.998984	.14	.835471	30.83	.164529	5
56	.836297	30.56	.998976	.14	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.57	.160837	3
58	.839956	30.30	.998958	.15	.840998	30.45	.159002	2
59	.841774	30.17	.998950	.15	.842825	30.32	.157175	1
60	.843585		.998941		.844644		.155356	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

93° **86°**

4° 175°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	8.843585	30.05	9.998941	.15	8.844644	30.19	11.155356	60
1	.845387	29.92	.998932	.15	.846455	30.07	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	67
4	.850751	29.55	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.31	.998887	.15	.855403	29.46	.144597	54
7	.856049	29.19	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.07	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.96	.998860	.15	.860686	29.11	.139314	51
10	8.861283	28.84	9.998851	.15	8.862433	29.00	11.137567	50
11	.863014	28.73	.998841	.15	.864173	28.88	.135827	49
12	.864738	28.61	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.16	.867632	28.66	.132368	47
14	.868165	28.39	.998813	.16	.869351	28.54	.130649	46
15	.869868	28.28	.998804	.16	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.16	.872770	28.32	.127230	44
17	.873255	28.06	.998785	.16	.874469	28.21	.125531	43
18	.874938	27.95	.998776	.16	.876162	28.11	.123838	42
19	.876615	27.86	.998766	.16	.877849	28.00	.122151	41
20	8.878285	27.73	9.998757	.16	8.879529	27.89	11.120471	40
21	.879949	27.63	.998747	.16	.881202	27.79	.118798	39
22	.881607	27.52	.998738	.16	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.16	.884530	27.58	.115470	37
24	.884903	27.31	.998718	.16	.886185	27.47	.113815	36
25	.886542	27.21	.998708	.16	.887833	27.37	.112167	25
26	.888174	27.11	.998699	.16	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.16	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.16	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	8.894643	26.70	9.998659	.17	8.895984	26.87	11.104016	30
31	.896246	26.60	.998649	.17	.897596	26.77	.102404	29
32	.897842	26.51	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.41	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.31	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.29	.094430	24
37	.905736	26.03	.998589	.17	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.84	.998568	.17	.910285	26.01	.089715	21
40	8.910404	25.75	9.998558	.17	8.911846	25.92	11.088154	20
41	.911949	25.66	.998548	.17	.913401	25.83	.086599	19
42	.913488	25.56	.998537	.17	.914951	25.74	.085049	18
43	.915022	25.47	.998527	.17	.916495	25.65	.083505	17
44	.916550	25.38	.998516	.18	.918034	26.56	.081966	16
45	.918073	25.29	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.18	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.30	.077381	13
48	.922610	25.03	.998474	.18	.924136	25.21	.075864	12
49	.924112	24.94	.998464	.18	.925649	25.12	.074351	11
50	8.925609	24.86	9.998453	.18	8.927156	25.03	11.072844	10
51	.927100	24.77	.998442	.18	.928658	24.95	.071342	9
52	.928587	24.69	.998431	.18	.930155	24.86	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.61	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	2
58	.937398	24.19	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.11	.998355	.18	.940494	24.30	.059506	1
60	.940296		.998344		.941952		.058048	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

94° 85°

5° 174°

M.	Sine.	D.1''.	Cosine.	D.1''.	Tang.	D.1''.	Cotang.	M.
0	8.940296	24.03	9.998344	.19	8.941952	24.21	11.058048	60
1	.941738	23.94	.998333	.19	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.19	.944852	24.05	.055148	58
3	.944606	23.79	.998311	.19	.946295	23.97	.053705	57
4	.946034	23.71	.998300	.19	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.19	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.19	.950597	23.74	.049403	54
7	.950287	23.48	.998266	.19	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.19	.953441	23.60	.046559	52
9	.953100	23.32	.998243	.19	.954856	23.51	.045144	51
10	8.954499	23.25	9.998232	.19	8.956267	23.44	11.043733	50
11	.955894	23.17	.998220	.19	.957674	23.37	.042326	49
12	.957284	23.10	.998209	.19	.959075	23.29	.040925	48
13	.958670	23.02	.998197	.19	.960473	23.23	.039537	47
14	.960052	22.95	.998186	.19	.961866	23.14	.038134	46
15	.961429	22.88	.998174	.19	.963255	23.07	.036745	45
16	.962801	22.80	.998163	.19	.964639	23.00	.035361	41
17	.964170	22.73	.998151	.19	.966019	22.93	.033981	43
18	.965534	22.66	.998139	.20	.967394	22.86	.032606	42
19	.966893	22.59	.998128	.20	.968766	22.79	.031234	41
20	8.968249	22.52	9.998116	.20	8.970133	22.71	11.029867	40
21	.969600	22.44	.998104	.20	.971496	22.65	.028504	39
22	.970947	22.38	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.31	.998080	.20	.974209	22.51	.025791	37
24	.973628	22.24	.998068	.20	.975560	22.44	.024440	36
25	.974962	22.17	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.921752	34
27	.977619	22.03	.998032	.20	.979586	22.23	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	8.981573	21.83	9.997996	.20	8.983577	22.04	11.016423	30
31	.982883	21.77	.997984	.20	.984899	21.97	.015101	29
32	.984189	21.70	.997972	.20	.986217	21.91	.913783	28
33	.985491	21.63	.997959	.20	.987532	21.84	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.50	.997935	.21	.990149	21.71	.009851	25
36	.989374	21.44	.997922	.21	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.21	.992750	21.58	.007250	23
38	.991943	21.31	.997897	.21	.994045	21.52	.005955	22
39	.993222	21.25	.997885	.21	.995337	21.46	.004663	21
40	8.994497	21.19	9.997872	.21	8.996624	21.40	11.003376	20
41	.995768	21.12	.997860	.21	.997908	21.34	.002092	19
42	.997036	21.06	.997847	.21	.999188	21.27	.000812	18
43	.998299	21.00	.997835	.21	9.000465	21.21	10.999535	17
44	.999560	20.94	.997822	.21	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.21	.003007	21.09	.996993	15
46	.002069	20.82	.997797	.21	.004272	21.03	.995728	14
47	.003318	20.76	.997784	.21	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.21	.006792	20.91	.993208	12
49	.005805	20.64	.997758	.21	.008047	20.85	.991953	11
50	9.007044	20.58	9.997745	.21	9.009298	20.80	10.990702	10
51	.008278	20.52	.997732	.21	.010546	20.74	.989454	9
52	.009510	20.46	.997719	.21	.011790	20.68	.988210	8
53	.010737	20.40	.997706	.21	.013031	20.62	.986969	7
54	.011962	20.34	.996693	.22	.014268	20.56	.985732	6
55	.013182	20.29	.997680	.22	.015502	20.51	.984498	5
56	.014400	20.23	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.17	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.06	.997628	.22	.020403	20.28	.979597	1
60	.019235		.997614		.021620		.978380	0
M.	Cosine.	D.1''.	Sine.	D.1''.	Cotang.	D.1''.	Tang.	M.

95° 84°

6° 173°

M.	Sine.	D.1".	Cosine.	D.1".	Tang.	D.1".	Cotang.	M.
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.89	.997588	.22	.024044	20.12	.975956	58
3	.022825	19.84	.997574	.22	.025251	20.06	.974749	57
4	.024016	19.78	.997561	.22	.026455	20.01	.973545	56
5	.025203	19.73	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.67	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.23	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.79	.968763	52
9	.029918	19.51	.997493	.23	.032425	19.74	.967575	51
10	9.031089	19.47	9.997480	.23	9.033609	19.69	10.966391	50
11	.032257	19.41	.997466	.23	.034791	19.64	.965209	49
12	.033421	19.36	.997452	.23	.035969	19.58	.964031	48
13	.034582	19.30	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.38	.959349	44
17	.039197	19.10	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	9.042625	18.95	9.997341	.23	9.945284	19.18	10.954716	40
21	.043762	18.90	.997327	.24	.046434	19.13	.953566	39
22	.044895	18.85	.997313	.24	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.24	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.24	.949869	18.98	.950131	36
25	.048279	18.70	.997271	.24	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.24	.052144	18.89	.947856	34
27	.050519	18.60	.997242	.24	.053277	18.84	.946723	33
28	.051635	18.55	.997228	.24	.054407	18.79	.945593	32
29	.052749	18.50	.997214	.24	.055535	18.74	.944465	31
30	9.053859	18.46	9.997199	.24	9.056659	18.70	10.943341	30
31	.054966	18.41	.997185	.24	.057781	18.65	.942219	29
32	.056071	18.36	.997170	.24	.058900	18.60	.941100	28
33	.057172	18.31	.997156	.24	.060016	18.56	.939984	27
34	.058271	18.27	.997141	.24	.061130	18.51	.938870	26
35	.059367	18.22	.997127	.24	.062240	18.46	.937760	25
36	.060460	18.17	.997112	.24	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.24	.064453	18.37	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.33	.934444	22
39	.963724	18.04	.997068	.25	.066655	18.28	.933345	21
40	9.064806	17.99	9.997053	.25	9.067752	18.24	10.932248	20
41	.065885	17.95	.997039	.25	.068846	18.19	.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.86	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.81	.996994	.25	.072113	18.06	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.72	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.68	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.64	.996934	.25	.076432	17.89	.923568	12
49	.074424	17.59	.996919	.25	.077505	17.84	.922495	11
50	9.075480	17.55	9.996904	.25	9.078576	17.80	10.921424	10
51	.076533	17.51	.996889	.25	.079644	17.76	.920356	9
52	.077583	17.46	.996874	.25	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.25	.082833	17.63	.917167	6
55	.080719	17.34	.996828	.25	.083891	17.59	.916109	5
56	.081759	17.29	.996812	.26	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.26	.086000	17.51	.914000	3
58	.083832	17.21	.996782	.26	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.26	.088098	17.43	.911902	1
60	.085894		.996751		.089144		.910856	0
M.	Cosine.	D.1".	Sine.	D.1".	Cotang.	D.1".	Tang.	M.

7° 172°

M.	Sine.	D. 1″.	Cosine.	D. 1″	Tang.	D. 1″.	Cotang.	M.
0	9.085894	17.13	9.996751	.26	9.089144	17.39	10.910856	60
1	.086922	17.09	.996735	.26	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.26	.091228	17.31	.908772	58
3	.088970	17.00	.996704	.26	.092266	17.27	.907734	57
4	.089990	16.96	.996688	.26	.093302	17.23	.906698	56
5	.091008	16.92	.996673	.26	.094336	17.19	.905664	55
6	.092024	16.88	.996657	.26	.095367	17.15	.904633	54
7	.093037	16.84	.996641	.26	.096395	17.11	.903605	53
8	.094047	16.80	.996625	.26	.097422	17.07	.902578	52
9	.095056	16.76	.996610	.26	.098446	17.03	.901554	51
10	9.096062	16.73	9.996594	.26	9.099468	16.99	10.900532	50
11	.097065	16.68	.996578	.27	.100487	16.95	.899513	49
12	.098066	16.65	.996562	.27	.101504	16.91	.898496	48
13	.099065	16.61	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.84	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.49	.996498	.27	.105550	16.76	.894450	44
17	.103037	16.46	.996482	.27	.106556	16.72	.893444	43
18	.104025	16.43	.996465	.27	.107559	16.69	.892441	42
19	.105010	16.38	.996449	.27	.108560	16.65	.891440	41
20	9.105992	16.34	9.996433	.27	9.109559	16.61	10.890441	40
21	.106973	16.30	.996417	.27	.110556	16.58	.889444	39
22	.107951	16.27	.996400	.27	.111551	16.54	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.19	.996368	.27	.113533	16.47	.886467	36
25	.110873	16.16	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.27	.115507	16.39	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.36	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.32	.882528	32
29	.114737	16.01	.996285	.28	.118452	16.29	.881548	31
30	9.115698	15.98	9.996269	.28	9.119429	16.25	10.880571	30
31	.116656	15.94	.996252	.28	.120404	16.22	.879596	29
32	.117613	15.90	.996235	.28	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.966202	.28	.123317	16.11	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.76	.996168	.28	.125249	16.04	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.01	.873789	23
38	.123306	15.69	.996134	.28	.127172	15.98	.872828	22
39	.124248	15.66	.996117	.28	.128130	15.94	.871870	21
40	9.125187	15.62	9.996100	.29	9.129087	15.91	10.870913	20
41	.126125	15.59	.996083	.29	.130041	15.87	.869959	19
42	.127060	15.56	.996066	.29	.130994	15.84	.869006	18
43	.127993	15.52	.996049	.29	.131944	15.81	.868056	17
44	.128925	15.49	.996032	.29	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.29	.133839	15.74	.866161	15
46	.130781	15.42	.995998	.29	.134784	15.71	.865216	14
47	.131706	15.39	.995980	.29	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.29	.136667	15.64	.863333	12
49	.133551	15.32	.995946	.29	.137605	15.61	.862395	11
50	9.134470	15.29	9.995928	.29	9.138542	15.58	10.861458	10
51	.135387	15.26	.995911	.29	.139476	15.55	.860524	9
52	.136303	15.22	.995894	.29	.140409	15.51	.859591	8
53	.137216	15.19	.995876	.29	.141340	15.48	.858660	7
54	.138128	15.16	.995859	.29	.142269	15.45	.857731	6
55	.139037	15.13	.995841	.29	.143196	15.42	.856804	5
56	.139944	15.09	.995823	.29	.144121	15.39	.855879	4
57	.140850	15.06	.995806	.29	.145044	15.36	.854956	3
58	.141754	15.03	.995788	.29	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.29	.146885	15.29	.853115	1
60	.143555		.995753		.147803		.852197	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

97° 82°

8° 171°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.143555	14.97	9.995753	.30	9.147803	15.26	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.87	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.84	.995681	.30	.151454	15.14	.848546	56
5	.148026	14.81	.995664	.30	.152363	15.11	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.75	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.30	.155077	15.02	.844923	52
9	.151569	14.69	.995591	.30	.155978	14.99	.844022	51
10	9.152451	14.66	9.995573	.30	9.156877	14.96	10.843123	50
11	.153330	14.63	.995555	.30	.157775	14.93	.842225	49
12	.154208	14.60	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.54	.995501	.31	.160457	14.84	.839543	46
15	.156830	14.51	.995482	.31	.161347	14.81	.838653	45
16	.157700	14.48	.995464	.31	.162236	14.78	.837764	44
17	.158569	14.45	.995446	.31	.163123	14.75	.836877	43
18	.159435	14.42	.995427	.31	.164008	14.73	.835992	42
19	.160301	14.39	.995409	.31	.164892	14.70	.835108	41
20	9.161164	14.36	9.995390	.31	9.165774	14.67	10.834226	40
21	.162025	14.33	.995372	.31	.166654	14.64	.833346	39
22	.162885	14.30	.995353	.31	.167532	14.61	.832468	38
23	.163743	14.27	.995334	.31	.168409	14.58	.831591	37
24	.164600	14.24	.995316	.31	.169284	14.56	.830716	36
25	.165454	14.22	.995297	.31	.170157	14.53	.829843	35
26	.166307	14.19	.995278	.31	.171029	14.50	.828971	34
27	.167159	14.16	.995260	.31	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.44	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	9.169702	14.07	9.995203	.32	9.174499	14.39	10.825501	30
31	.170547	14.05	.995184	.32	.175362	14.36	.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	13.99	.995146	.32	.177084	14.31	.822916	27
34	.173070	13.96	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.94	.995108	.32	.178799	14.25	.821201	25
36	.174744	13.91	.995089	.32	.179655	14.23	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.17	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.15	.817789	21
40	9.178072	13.80	9.995013	.32	9.183059	14.12	10.816941	20
41	.178900	13.77	.994993	.32	.183907	14.09	.816093	19
42	.179726	13.75	.994974	.32	.184752	14.07	.815248	18
43	.180551	13.72	.994955	.32	.185597	14.04	.814403	17
44	.181374	13.69	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	.994916	.33	.187280	13.99	.812720	15
46	.183016	13.64	.994896	.33	.188120	13.97	.811880	14
47	.183834	13.61	.994877	.33	.188958	13.94	.811042	13
48	.184651	13.59	.994857	.33	.189794	13.91	.810206	12
49	.185466	13.56	.994838	.33	.190629	13.89	.809371	11
50	9.186280	13.54	9.994818	.33	9.191462	13.86	10.808538	10
51	.187092	13.51	.994798	.33	.192294	13.84	.807706	9
52	.187903	13.48	.994779	.33	.193124	13.81	.806876	8
53	.188712	13.46	.994759	.33	.193953	13.79	.806047	7
54	.189519	13.43	.994739	.33	.194780	13.76	.805220	6
55	.190325	13.41	.994719	.33	.195606	13.74	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.71	.803570	4
57	.191933	13.36	.994680	.33	.197253	13.69	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.66	.801926	2
59	.193534	13.31	.994640	.33	.198894	13.64	.801106	1
60	.194332		.994620		.199713		.800287	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

98° 81°

9° 170°

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.194332	13.28	9.994620	.33	9.199713	13.62	10.800287	60
1	.195129	13.26	.994600	.33	.200529	13.59	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.21	.994560	.34	.202159	13.54	.797841	57
4	.197511	13.18	.994540	.34	.202971	13.52	.797029	56
5	.198302	13.16	.994519	.34	.203782	13.49	.796218	55
6	.199091	13.13	.994499	.34	.204592	13.47	.795408	54
7	.199879	13.11	.994479	.34	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.34	.206207	13.42	.793793	52
9	.201451	13.06	.994438	.34	.207013	13.40	.792987	51
10	9.202234	13.04	9.994418	.34	9.207817	13.38	10.792183	50
11	.203017	13.01	.994398	.34	.208619	13.35	.791381	49
12	.203797	12.99	.994377	.34	.209420	13.33	.7[illegible]580	48
13	.204577	12.96	.994357	.34	.210220	13.31	.789780	47
14	.205354	12.94	.994336	.34	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.34	.211815	13.26	.788185	45
16	.206906	12.89	.994295	.34	.212611	13.24	.787389	44
17	.207679	12.87	.994274	.35	.213405	13.21	.786595	43
18	.208452	12.85	.994254	.35	.214198	13.19	.785802	42
19	.209222	12.82	.994233	.35	.214989	13.17	.785011	41
20	9.209992	12.80	9.994212	.35	9.215780	13.15	10.784220	40
21	.210760	12.78	.994191	.35	.216568	13.12	.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.08	.781858	37
24	.213055	12.71	.994129	.35	.218926	13.06	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.66	.994087	.35	.220492	13.01	.779508	34
27	.215338	12.64	.994066	.35	.221272	12.99	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.59	.994024	.35	.222830	12.95	.777170	31
30	9.217609	12.57	9.994003	.35	9.223607	12.92	10.776393	30
31	.218363	12.55	.993982	.35	.224382	12.90	.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.86	.774071	27
34	.220618	12.48	.993918	.36	.226700	12.84	.773300	26
35	.221367	12.46	.993897	.36	.227471	12.82	.772529	25
36	.222115	12.44	.993875	.36	.228239	12.79	.771761	24
37	.222861	12.42	.993854	.36	.229007	12.77	.770993	23
38	.223606	12.39	.993832	.36	.229773	12.75	.770227	22
39	.224349	12.37	.993811	.36	.230539	12.73	.769461	21
40	9.225092	12.35	9.993789	.36	9.231302	12.71	10.768698	20
41	.225833	12.33	.993768	.36	.232065	12.69	.767935	19
42	.226573	12.31	.993746	.36	.232826	12.67	.767174	18
43	.227311	12.29	.993725	.36	.233586	12.65	.766414	17
44	.228048	12.26	.993703	.36	.234345	12.63	.765655	16
45	.228784	12.24	.993681	.36	.235103	12.60	.764897	15
46	.229518	12.22	.993660	.36	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.36	.236614	12.56	.763386	13
48	.230984	12.18	.993616	.36	.237368	12.54	.762632	12
49	.231715	12.16	.993594	.36	.238120	12.52	.761880	11
50	9.232444	12.14	9.993572	.36	9.238872	12.50	10.761128	10
51	.233172	12.12	.993550	.37	.239622	12.48	.760378	9
52	.233899	12.10	.993528	.37	.240371	12.46	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.44	.758882	7
54	.235349	12.05	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.01	.993440	.37	.243354	12.38	.756646	4
57	.237515	11.99	.993418	.37	.244097	12.36	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.34	.755161	2
59	.238953	11.95	.993374	.37	.245579	12.32	.754421	1
60	.239670		.993351		.216319		.753681	0
M	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

99° 80°

10° 169°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240386	11.91	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.89	.993307	.37	.247794	12.26	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.24	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.22	.750736	56
5	.243237	11.83	.993240	.37	.249998	12.20	.750002	55
6	.243947	11.81	.993217	.38	.250730	12.18	.749270	54
7	.244656	11.79	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.75	.993149	.38	.252920	12.13	.747080	51
10	9.246775	11.73	9.993127	.38	9.253648	12.11	10.746352	50
11	.247478	11.71	.993104	.38	.254374	12.09	.745626	49
12	.248181	11.69	.993081	.38	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.01	.742731	45
16	.250980	11.61	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.59	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.58	.992944	.38	.259429	11.96	.740571	42
19	.253067	11.56	.992921	.38	.260146	11.94	.739854	41
20	9.253761	11.54	9.992898	.38	9.260863	11.92	10.739137	40
21	.254453	11.52	.992875	.38	.261578	11.90	.738422	39
22	.255144	11.50	.992852	.38	.262292	11.89	.737708	38
23	.255834	11.48	.992829	.39	.263005	11.87	.736995	37
24	.256523	11.46	.992806	.39	.263717	11.85	.736283	36
25	.257211	11.44	.992783	.39	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.39	.265138	11.81	.734862	34
27	.258583	11.41	.992736	.39	.265847	11.79	.734153	33
28	.259268	11.39	.992713	.39	.266555	11.78	.733445	32
29	.259951	11.37	.992690	.39	.267261	11.76	.732739	31
30	9.260633	11.35	9.992666	.39	9.267967	11.74	10.732033	30
31	.261314	11.33	.992643	.39	.268671	11.72	.731329	29
32	.261994	11.31	.992619	.39	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.39	.270077	11.69	.729923	27
34	.263351	11.28	.992572	.39	.270779	11.67	.729221	26
35	.264027	11.26	.992549	.39	.271479	11.65	.728521	25
36	.264703	11.24	.992525	.39	.272178	11.64	.727822	24
37	.265377	11.22	.992501	.39	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.19	.992454	.40	.274269	11.58	.725731	21
40	9.267395	11.17	9.992430	.40	9.274964	11.57	10.725036	20
41	.268065	11.15	.992406	.40	.275658	11.55	.724342	19
42	.268734	11.13	.992382	.40	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.51	.722957	17
44	.270069	11.11	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.06	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.05	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.40	.280488	11.43	.719512	12
49	.273388	11.01	.992214	.40	.281174	11.41	.718826	11
50	9.274049	10.99	9.992190	.40	9.281858	11.40	10.718142	10
51	.274708	10.98	.992166	.40	.282542	11.38	.717458	9
52	.275367	10.96	.992142	.40	.283225	11.36	.716775	8
53	.276025	10 94	.992118	.41	.283907	11.35	.716093	7
54	.276681	10.92	.992093	.41	.284588	11.33	.715412	6
55	.277337	10.91	.992069	.41	.285268	11.31	.714732	5
56	.277991	10.89	.992044	.41	.285947	11.30	.714053	4
57	.278645	10.87	.992020	.41	.286624	11.28	.713376	3
58	.279297	10.86	.991996	.41	.287301	11.26	.712699	2
59	.279948	10.84	.991971	.41	.287977	11.25	.712023	1
60	.280599		.991947		.288652		.711348	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

100° 79°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.280599	10.82	9.991947	.41	9.288652	11.23	10.711348	60
1	.281248	10.81	.991922	.41	.289326	11.22	.710674	59
2	.281897	10.79	.991897	.41	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.41	.290671	11.18	.709329	57
4	.283190	10.76	.991848	.41	.291342	11.17	.708658	56
5	.283836	10.74	.991823	.41	.292013	11.15	.707987	55
6	.284480	10.72	.991799	.41	.292682	11.14	.707318	54
7	.285124	10.71	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.69	.991749	.42	.294017	11.11	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.09	.705316	51
10	9.287048	10.66	9.991699	.42	9.295349	11.07	10.704651	50
11	.287688	10.64	.991674	.42	.296013	11.06	.703987	49
12	.288326	10.63	.991649	.42	.296677	11.04	.703323	48
13	.288964	10.61	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.59	.991599	.42	.298001	11.01	.701999	46
15	.290236	10.58	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.56	.991549	.42	.299322	10.98	.700678	44
17	.291504	10.55	.991524	.42	.299980	10.97	.700020	43
18	.292137	10.53	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.51	.991473	.42	.301295	10.93	.698705	41
20	9.293399	10.50	9.991448	.42	9.301951	10.92	10.698049	40
21	.294029	10.48	.991422	.42	.302607	10.90	.697393	39
22	.294658	10.47	.991397	.42	.303261	10.89	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.87	.696086	37
24	.295913	10.43	.991346	.43	.304567	10.86	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.84	.694782	35
26	.297164	10.40	.991295	.43	.305869	10.83	.694131	34
27	.297788	10.39	.991270	.43	.306519	10.81	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.36	.991218	.43	.307816	10.78	.692184	31
30	9.299655	10.34	9.991193	.43	9.308463	10.77	10.691537	30
31	.300276	10.33	.991167	.43	.309109	10.76	.690891	29
32	.300895	10.31	.991141	.43	.309754	10.74	.690246	28
33	.301514	10.30	.991115	.43	.310399	10.73	.689601	27
34	.302132	10.28	.991090	.43	.311042	10.71	.688958	26
35	.302748	10.26	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.22	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.64	.685753	21
40	9.305819	10.19	9.990934	.44	9.314885	10.62	10.685115	20
41	.306430	10.17	.990908	.44	.315523	10.61	.684477	19
42	.307041	10.16	.990882	.44	.316159	10.60	.683841	18
43	.307650	10.14	.990855	.44	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.44	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.44	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.44	.318697	10.54	.681303	14
47	.310080	10.09	.990750	.44	.319330	10.53	.680670	13
48	.310685	10.07	.990724	.44	.319961	10.51	.680039	12
49	.311289	10.06	.990697	.44	.320592	10.50	.679408	11
50	9.311893	10.04	9.990671	.44	9.321222	10.48	10.678778	10
51	.312495	10.03	.990645	.44	.321851	10.47	.678149	9
52	.313097	10.01	.990618	.44	.322479	10.46	.677521	8
53	.313698	10.00	.990591	.44	.323106	10.44	.676894	7
54	.314297	9.98	.990565	.44	.323733	10.43	.676267	6
55	.314897	9.97	.990538	.44	.324358	10.41	.675642	5
56	.315495	9.96	.990511	.45	.324983	10.40	.675017	4
57	.316092	9.94	.990485	.45	.325607	10.39	.674393	3
58	.316689	9.93	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.91	.990431	.45	.326853	10.36	.673147	1
60	.317879		.990404		.327475		.672525	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

12° 167°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.317879	9.90	9.990404	.45	9.327475	10.35	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.86	.990324	.45	.329334	10.31	.670666	57
4	.320249	9.84	.990297	.45	.329953	10.29	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.45	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.79	.990188	.45	.332418	10.24	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.23	.666967	51
10	9.323780	9.76	9.990134	.45	9.333646	10.21	10.666354	50
11	.324366	9.75	.990107	.46	.334259	10.20	.665741	49
12	.324950	9.73	.990079	.46	.334871	10.19	.665129	48
13	.325534	9.72	.990052	.46	.335482	10.17	.664518	47
14	.326117	9.70	.990025	.46	.336093	10.16	.663907	46
15	.326700	9.69	.989997	.46	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.46	.337311	10.14	.662689	44
17	.327862	9.66	.989942	.46	.337919	10.12	.662081	43
18	.328442	9.65	.989915	.46	.338527	10.11	.661473	42
19	.329021	9.64	.989887	.46	.339133	10.10	.660867	41
20	9.329599	9.62	9.989860	.46	9.339739	10.08	10.660261	40
21	.330176	9.61	.989832	.46	.340344	10.07	.659656	39
22	.330753	9.60	.989804	.46	.340948	10.06	.659052	38
23	.331329	9.58	.989777	.46	.341552	10.05	.658448	37
24	.331903	9.57	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.56	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.54	.989693	.47	.343358	10.01	.656642	34
27	.333624	9.53	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.52	.989637	.47	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	9.335337	9.49	9.989582	.47	9.345755	9.96	10.654245	30
31	.335906	9.48	.989553	.47	.346353	9.95	.653647	29
32	.336475	9.46	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.92	.652455	27
34	.337610	9.44	.989469	.47	.348141	9.91	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.41	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.47	.349922	9.87	.650078	23
38	.339871	9.39	.989356	.47	.350514	9.86	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	9.340996	9.36	9.989300	.47	9.351697	9.84	10.648303	20
41	.341558	9.35	.989271	.47	.352287	9.82	.647713	19
42	.342119	9.34	.989243	.47	.352876	9.81	.647124	18
43	.342679	9.32	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.31	.989186	.47	.354053	9.79	.645947	16
45	.343797	9.30	.989157	.47	.354640	9.78	.645360	15
46	.344355	9.29	.989128	.48	.355227	9.76	.644773	14
47	.344912	9.27	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.26	.989071	.48	.356398	9.74	.643602	12
49	.346024	9.25	.989042	.48	.356982	9.73	.643018	11
50	9.346579	9.24	9.989014	.48	9.357566	9.72	10.642434	10
51	.347134	9.22	.988985	.48	.358149	9.70	.641851	9
52	.347687	9.21	.988956	.48	.358731	9.69	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.68	.640687	7
54	.348792	9.19	.988898	.48	.359893	9.67	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.66	.639526	5
56	.349893	9.16	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.49	.361632	9.63	.638368	3
58	.350992	9.14	.988782	.49	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.49	.362787	9.61	.637213	1
60	.352088		.988724		.363364		.636636	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

102° 77°

13° 166°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.352088	9.11	9.988724	.49	9.363364	9.60	10.636636	60
1	.352635	9.10	.988695	.49	.363940	9.59	.636060	59
2	.353181	9.09	.988666	.49	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.49	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.49	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.49	.366237	9.54	.633763	55
6	.355358	9.04	.988548	.49	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.49	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.49	.367953	9.51	.632047	52
9	.356984	9.01	.988460	.49	.368524	9.50	.631476	51
10	9.357524	8.99	9.988430	.49	9.369094	9.49	10.630906	50
11	.358064	8.98	.988401	.49	.369663	9.48	.630337	49
12	.358603	8.97	.988371	.49	.370232	9.47	.629768	48
13	.359141	8.96	.988342	.49	.370799	9.45	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.44	.628633	46
15	.360215	8.94	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.50	.372499	9.42	.627501	44
17	.361287	8.91	.988223	.50	.373064	9.41	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.89	.988163	.50	.374193	9.39	.625807	41
20	9.362889	8.88	9.988133	.50	9.374756	9.38	10.625244	40
21	.363422	8.87	.988103	.50	.375319	9.37	.624681	39
22	.363954	8.86	.988073	.50	.375881	9.36	.624119	38
23	.364485	8.84	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.81	.987953	.50	.378122	9.31	.621878	34
27	.366604	8.80	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.79	.987892	.50	.379239	9.29	.620761	32
29	.367659	8.78	.987862	.50	.379797	9.28	.620203	31
30	9.368185	8.76	9.987832	.51	9.380354	9.27	10.619646	30
31	.368711	8.75	.987801	.51	.380910	9.26	.619090	29
32	.369236	8.74	.987771	.51	.381466	9.25	.618534	28
33	.369761	8.73	.987740	.51	.382020	9.24	.617980	27
34	.370285	8.72	.987710	.51	.382575	9.23	.617425	26
35	.370808	8.71	.987679	.51	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.51	.383682	9.21	.616318	24
37	.371852	8.69	.987618	.51	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.51	.384786	9.19	.615214	22
39	.372894	8.66	.987557	.51	.385337	9.18	.614663	21
40	9.373414	8.65	9.987526	.51	9.385888	9.17	10.614112	20
41	.373933	8.64	.987496	.51	.386438	9.16	.613562	19
42	.374452	8.63	.987465	.51	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.51	.387536	9.14	.612464	17
44	.375487	8.61	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.11	.611369	15
46	.376519	8.59	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.58	.987310	.52	.389724	9.09	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.56	.987248	.52	.390815	9.07	.609185	11
50	9.378577	8.55	9.987217	.52	9.391360	9.06	10.608640	10
51	.379089	8.53	.987186	.52	.391903	9.05	.608097	9
52	.379601	8.52	.987155	.52	.392447	9.04	.607553	8
53	.380113	8.51	.987124	.52	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.02	.606469	6
55	.381134	8.49	.987061	.52	.394073	9.01	.605927	5
56	.381643	8.48	.987030	.52	.394614	9.00	.605386	4
57	.382152	8.47	.986998	.52	.395154	8.99	.604846	3
58	.382661	8.46	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.52	.396233	8.97	.603767	1
60	.383675		.986904		.396771		.603229	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

103° 76°

14° **165°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.383675	8.44	9.986904	.53	9.396771	8.96	10.603229	60
1	.384182	8.43	.986873	.53	.397309	8.96	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.41	.986809	.53	.398383	8.94	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.39	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.53	.399990	8.91	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.36	.986651	.53	.401058	8.89	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	9.388711	8.34	9.986587	.53	9.402124	8.87	10.597876	50
11	.389211	8.33	.986555	.53	.402656	8.86	.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.31	.986491	.53	.403718	8.84	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.83	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.82	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.81	.594692	44
17	.392199	8.26	.986363	.54	.405836	8.80	.594164	43
18	.392695	8.25	.986331	.54	.406364	8.79	.593636	42
19	.393191	8.24	.986299	.54	.406892	8.78	.593108	41
20	9.393685	8.23	9.986266	.54	9.407419	8.77	10.592581	40
21	.394179	8.22	.986234	.54	.407945	8.76	.592055	39
22	.394673	8.21	.986202	.54	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.54	.408997	8.74	.591003	37
24	.395658	8.19	.986137	.54	.409521	8.74	.590479	36
25	.396150	8.18	.986104	.54	.410045	6.73	.589955	35
26	.396641	8.17	.986072	.54	.410569	8.72	.589431	34
27	.397132	8.17	.986039	.54	.411092	8.71	.588908	33
28	.397621	8.16	.986007	.54	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.54	.412137	8.69	.587863	31
30	9.398600	8.14	9.985942	.54	9.412658	8.68	10.587342	30
31	.399088	8.13	.985909	.55	.413179	8.67	.586821	29
32	.399575	8.12	.985876	.55	.413699	8.66	.586301	28
33	.400062	8.11	.985843	.55	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.09	.985778	.55	.415257	8.64	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402489	8.06	.985679	.55	.416810	8.61	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	9.403455	8.04	9.985613	.55	9.417842	8.59	10.582158	20
41	.403938	8.03	.985580	.55	.418358	8.58	.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.01	.985514	.55	.419387	8.56	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.56	.580099	16
45	.405862	7.99	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.56	.420927	8.54	.579073	14
47	.406820	7.97	.985380	.56	.421440	8.53	.578560	13
48	.407299	7.96	.985347	.56	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.56	.422463	8.51	.577537	11
50	9.408254	7.94	9.985280	.56	9.422974	8.50	10.577026	10
51	.408731	7.94	.985247	.56	.423484	8.49	.576516	9
52	.409207	7.93	.985213	.56	.423993	8.49	.576007	8
53	.409682	7.92	.985180	.56	.424503	8.48	.575497	7
54	.410157	7.91	.985146	.56	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.56	.425519	8.46	.574481	5
56	.411106	7.89	.985079	.56	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.56	.426534	8.44	.573466	3
58	.412052	7.87	.985011	.56	.427041	8.43	.572959	2
59	.412524	7.86	.984978	.56	.427547	8.43	.572453	1
60	.412996		.984944		.428052		.571948	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

104° **75°**

15° | 164°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.412996	7.85	9.984944	.57	9.428052	8.42	10.571948	60
1	.413467	7.84	.984910	.57	.428558	8.41	.571442	59
2	.413938	7.84	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.39	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.81	.984774	.57	.430573	8.38	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.79	.984706	.57	.431577	8.36	.568423	53
8	.416751	7.78	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.77	.984637	.57	.432580	8.34	.567420	51
10	9.417684	7.76	9.984603	.57	9.433080	8.33	10.566920	50
11	.418150	7.75	.984569	.57	.433580	8.33	.566420	49
12	.418615	7.75	.984535	.57	.434080	8.32	.565920	48
13	.419079	7.74	.984500	.57	.434579	8.31	.565421	47
14	.419544	7.73	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.29	.564424	45
16	.420470	7.71	.984397	.58	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.69	.984328	.58	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.26	.562437	41
20	9.422318	7.67	9.984259	.58	9.438059	8.25	10.561941	40
21	.422778	7.67	.984224	.58	.438554	8.24	.561446	39
22	.423238	7.66	.984190	.58	.439048	8.24	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.23	.560457	37
24	.424156	7.64	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.21	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.61	.984015	.58	.441514	8.20	.558486	33
28	.425987	7.61	.983981	.58	.442006	8.19	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	9.426899	7.59	9.983911	.58	9.442988	8.17	10.557012	30
31	.427354	7.58	.983875	.58	.443479	8.16	.556521	29
32	.427809	7.57	.983840	.59	.443968	8.16	.556032	28
33	.428263	7.56	.983805	.59	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.59	.444947	8.14	.555053	26
35	.429170	7.54	.983735	.59	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.59	.445923	8.13	.554077	24
37	.430075	7.52	.983664	.59	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.59	.446898	8.11	.553102	22
39	.430978	7.51	.983594	.59	.447384	8.10	.552616	21
40	9.431429	7.50	9.983558	.59	9.447870	8.09	10.552130	20
41	.431879	7.49	.983523	.59	.448356	8.09	.551644	19
42	.432329	7.49	.983487	.59	.448841	8.08	.551159	18
43	.432778	7.48	.983452	.59	.449326	8.07	.550674	17
44	.433226	7.47	.983416	.59	.449810	8.06	.550190	16
45	.433675	7.46	.983381	.59	.450294	8.06	.549706	15
46	.434122	7.45	.983345	.59	.450777	8.05	.549223	14
47	.434569	7.44	.983309	.59	.451260	8.04	.548740	13
48	.435016	7.44	.983273	.60	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.03	.547775	11
50	9.435908	7.42	9.983202	.60	9.452706	8.02	10.547294	10
51	.436353	7.41	.983166	.60	.453187	8.01	.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.39	.983058	.60	.454628	7.99	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.36	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.36	.982914	.60	.456542	7.96	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	.440338		.982842		.457496		.542504	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

105° | 74°

16° 163°

M.	Sine	D.1".	Cosine.	D.1"	Tang.	D.1".	Cotang.	M.
0	9.440338	7.34	9.982842	.60	9.457496	7.94	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.94	.542027	59
2	.441218	7.32	.982769	.61	.458449	7.93	.541551	58
3	.441658	7.31	.982733	.61	.458925	7.92	.541075	57
4	.442096	7.31	.982696	.61	.459400	7.91	.540600	56
5	.442535	7.30	.982660	.61	.459875	7.91	.540125	55
6	.442973	7.29	.982624	.61	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.61	.460823	7.89	.539177	53
8	.443847	7.27	.982551	.61	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.61	.461770	7.88	.538230	51
10	9.444720	7.26	9.982477	.61	9.462242	7.87	10.537758	50
11	.445155	7.25	.982441	.61	.462714	7.86	.537286	49
12	.445590	7.24	.982404	.61	.463186	7.86	.536814	48
13	.446025	7.24	.982367	.61	.463658	7.85	.536342	47
14	.446459	7.23	.982331	.61	.464129	7.84	.535871	46
15	.446893	7.22	.982294	.61	.464599	7.83	.535401	45
16	.447326	7.21	.982257	.61	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.81	.533992	42
19	.448623	7.19	.982146	.62	.466476	7.81	.533524	41
20	9.449054	7.18	9.982109	.62	9.466945	7.80	10.533055	40
21	.449485	7.17	.982072	.62	.467413	7.79	.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.16	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.14	.981924	.62	.469280	7.76	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.76	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.62	.470676	7.74	.529324	32
29	.452915	7.11	.981774	.62	.471141	7.74	.528859	31
30	9.453342	7.10	9.981737	.62	9.471605	7.73	10.528395	30
31	.453768	7.10	.981700	.62	.472068	7.72	.527932	29
32	.454194	7.09	.981662	.63	.472532	7.71	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.71	.527005	27
34	.455044	7.07	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.63	.473919	7.69	.526081	25
36	.455893	7.06	.981512	.63	.474381	7.69	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.04	.981436	.63	.475303	7.67	.524697	32
39	.457162	7.04	.981399	.63	.475763	7.67	.524237	21
40	9.457584	7.03	9.981361	.63	9.476223	7.66	10.523777	20
41	.458006	7.02	.981323	.63	.476683	7.65	.523317	19
42	.458427	7.01	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.01	.981247	.63	.477601	7.64	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	6.99	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.64	.479432	7.61	.520568	13
48	.460946	6.97	.981057	.64	.479889	7.61	.520111	12
49	.461364	6.96	.981019	.64	.480345	7.60	.519655	11
50	9.461782	6.96	9.980981	.64	9.480801	7.59	10.519199	10
51	.462199	6.95	.980942	.64	.481257	7.59	.518743	9
52	.462616	6.94	.980904	.64	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.64	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.64	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.64	.483075	7.56	.516925	5
56	.464279	6.91	.980750	.64	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.64	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.64	.484435	7.54	.515565	2
59	.465522	6.89	.980635	.64	.484887	7.53	.515113	1
60	.465935		.980596		.485339		.514661	0
M.	Cosine.	D.1".	Sine.	D.1".	Cotang.	D.1".	Tang.	M.

106° 73°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.
0	9.465935	6.88	9.980596	.64	9.485339	7.53	10.514661
1	.466348	8.88	.980558	.64	.485791	7.52	.514209
2	.466761	6.87	.980519	.65	.486242	7.51	.513758
3	.467173	6.86	.980480	.65	.486693	7.51	.513307
4	.467585	6.85	.980442	.65	.487143	7.50	.512857
5	.467996	6.85	.980403	.65	.487593	7.50	.512407
6	.468407	6.84	.980364	.65	.488043	7.49	.511957
7	.468817	6.83	.980325	.65	.488492	7.48	.511508
8	.469227	6.83	.980286	.65	.488941	7.48	.511059
9	.469637	6.82	.980247	.65	.489390	7.47	.510610
10	9.470046	6.81	9.980208	.65	9.489838	7.46	10.510162
11	.470455	6.81	.980169	.65	.490286	7.46	.509714
12	.470863	6.80	.980130	.65	.490733	7.45	.509267
13	.471271	6.79	.980091	.65	.491180	7.44	.508820
14	.471679	6.78	.980052	.65	.491627	7.44	.508373
15	.472086	6.78	.980012	.65	.492073	7.43	.507927
16	.472492	6.77	.979973	.65	.492519	7.43	.507481
17	.472898	6.76	.979934	.66	.492965	7.42	.507035
18	.473304	6.76	.979895	.66	.493410	7.41	.506590
19	.473710	6.75	.979855	.66	.493854	7.41	.506146
20	9.474115	6.74	9.979816	.66	9.494299	7.40	10.505701
21	.474519	6.74	.979776	.66	.494743	7.39	.505257
22	.474923	6.73	.979737	.66	.495186	7.39	.504814
23	.475327	6.72	.979697	.66	.495630	7.38	.504370
24	.475730	6.72	.979658	.66	.496073	7.38	.503927
25	.476133	6.71	.979618	.66	.496515	7.37	.503485
26	.476536	6.70	.979579	.66	.496957	7.36	.503043
27	.476938	6.69	.979539	.66	.497399	7.36	.502601
28	.477340	6.69	.979499	.66	.497841	7.35	.502159
29	.477741	6.68	.979459	.66	.498282	7.34	.501718
30	9.478142	6.67	9.979420	.66	9.498722	7.34	10.501278
31	.478542	6.67	.979380	.66	.499163	7.33	.500837
32	.478942	6.66	.979340	.67	.499603	7.33	.500397
33	.479342	6.65	.979300	.67	.500042	7.32	.499958
34	.479741	6.65	.979260	.67	.500481	7.31	.499519
35	.480140	6.64	.979220	.67	.500920	7.31	.499080
36	.480539	6.63	.979180	.67	.501359	7.30	.498641
37	.480937	6.63	979140	.67	.501797	7.30	.498203
38	.481334	6.62	.979100	.67	.502235	7.29	.497765
39	.481731	6.61	.979059	.67	.502672	7.28	.497328
40	9.482128	6.61	9.979019	.67	9.503109	7.28	10.496891
41	.482525	6.60	.978979	.67	.503546	7.27	.496454
42	.482921	6.59	.978939	.67	.503982	7.27	.496018
43	.483316	6.59	.978898	.67	.504418	7.26	.495582
44	.483712	6.58	.978858	.67	.504854	7.25	.495146
45	.484107	6.57	.978817	.67	.505289	7.25	.494711
46	.484501	6.57	.978777	.67	.505724	7.24	.494276
47	.484895	6.56	.978736	.68	.506159	7.24	.493841
48	.485289	6.55	.978696	.68	.506593	7.23	.493407
49	.485682	6.55	.978655	.68	.507027	7.23	.492973
50	9.486075	6.54	9.978615	.68	9.507460	7.22	10.492540
51	.486467	6.54	.978574	.68	.507893	7.21	.492107
52	.486860	6.53	.978533	.68	.508326	7.21	.491674
53	.487251	6.52	.978493	.68	.508759	7.20	.491241
54	.487643	6.52	.978452	.68	.509191	7.20	.490809
55	.488034	6.51	.978411	.68	.509622	7.19	.490378
56	.488424	6.50	.978370	.68	.510054	7.18	.489946
57	.488814	6.50	.978329	.68	.510485	7.18	.489515
58	.489204	6.49	.978288	.68	.510916	7.17	.489084
59	.489593	6.48	.978247	.68	.511346	7.17	.488654
60	.489982		.978206		.511776		.488224
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.

18° **161°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.489982	6.48	9.978206	.68	9.511776	7.16	10.488224	60
1	.490371	6.47	.978165	.69	.512206	7.16	.487794	59
2	.490759	6.46	.978124	.69	.512635	7.15	.487365	58
3	.491147	6.46	.978083	.69	.513064	7.14	.486936	57
4	.491535	6.45	.978042	.69	.513493	7.14	.486507	56
5	.491922	6.45	.978001	.69	.513921	7.13	.486079	55
6	.492308	6.44	.977959	.69	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.69	.514777	7.12	.485223	53
8	.493081	6.43	.977877	.69	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.69	.515631	7.11	.484369	51
10	9.493851	6.41	9.977794	.69	9.516057	7.10	10.483943	50
11	.494236	6.41	.977752	.69	.516484	7.10	.483516	49
12	.494621	6.40	.977711	.69	.516910	7.09	.483090	48
13	.495005	6.39	.977669	.69	.517335	7.09	.482665	47
14	.495388	6.39	.977628	.69	.517761	7.08	.482239	46
15	.495772	6.38	.977586	.69	.518185	7.08	.481815	45
16	.496154	6.38	.977544	.70	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.36	.977461	.70	.519458	7.06	.480542	42
19	.497301	6.36	.977419	.70	.519882	7.05	.480118	41
20	9.497682	6.35	9.977377	.70	9.520305	7.05	10.479695	40
21	.498064	6.34	.977335	.70	.520728	7.04	.479272	39
22	.498444	6.34	.977293	.70	.521151	7.04	.478849	38
23	.498825	6.33	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.31	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.31	.977083	.70	.523259	7.01	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.01	.476320	32
29	.501099	6.30	.976999	.70	.524100	7.00	.475900	31
30	9.501476	6.29	9.976957	.70	9.524520	6.99	10.475480	30
31	.501854	6.28	.976914	.71	.524939	6.99	.475061	29
32	.502231	6.28	.976872	.71	.525359	6.98	.474641	28
33	.502607	6.27	.976830	.71	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.71	.526197	6.97	.473803	26
35	.503360	6.26	.976745	.71	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.71	.527033	6.96	.472967	24
37	.504110	6.25	.976660	.71	.527451	6.96	.472549	23
38	.504485	6.24	.976617	.71	.527868	6.95	.472132	22
39	.504860	6.24	.976574	.71	.528285	6.95	.471715	21
40	9.505234	6.23	9.976532	.71	9.528702	6.94	10.471298	20
41	.505608	6.22	.976489	.71	.529119	6.94	.470881	19
42	.505981	6.22	.976446	.71	.529535	6.93	.470465	18
43	.506354	6.21	.976404	.71	.529950	6.93	.470050	17
44	.506727	6.21	.976361	.71	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.91	.469219	15
46	.507471	6.19	.976275	.72	.531196	6.91	.468804	14
47	.507843	6.19	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.89	.467561	11
50	9.508956	6.17	9.976103	.72	9.532853	6.89	10.467147	10
51	.509326	6.16	.976060	.72	.533266	6.88	.466734	9
52	.509696	6.16	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.72	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.14	.975887	.72	.534916	6.86	.465084	5
56	.511172	6.14	.975844	.72	.535328	6.86	.464672	4
57	.511540	6.13	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.12	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.72	.536561	6.84	.463439	1
60	.512642		.975670		.536972		.463028	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

108° **71°**

19°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.
0	9.512642	6.11	9.975670	.73	9.536972	6.84	10.463028
1	.513009	6.11	.975627	.73	.537382	6.83	.462618
2	.513375	6.10	.975583	.73	.537792	6.83	.462208
3	.513741	6.09	.975539	.73	.538202	6.82	.461798
4	.514107	6.09	.975496	.73	.538611	6.82	.461389
5	.514472	6.08	.975452	.73	.539020	6.81	.460980
6	.514837	6.08	.975408	.73	.539429	6.81	.460571
7	.515202	6.07	.975365	.73	.539837	6.80	.460163
8	.515566	6.07	.975321	.73	.540245	6.80	.459755
9	.515930	6.06	.975277	.73	.540653	6.79	.459347
10	9.516294	6.05	9.975233	.73	9.541061	6.79	10.458939
11	.516657	6.05	.975189	.73	.541468	6.78	.458532
12	.517020	6.04	.975145	.73	.541875	6.78	.458125
13	.517382	6.04	.975101	.73	.542281	6.77	.457719
14	.517745	6.03	.975057	.73	.542688	6.77	.457312
15	.518107	6.03	.975013	.74	.543094	6.76	.456906
16	.518468	6.02	.974969	.74	.543499	6.76	.456501
17	.518829	6.02	.974925	.74	.543905	6.75	.456095
18	.519190	6.01	.974880	.74	.544310	6.75	.455690
19	.519551	6.00	.974836	.74	.544715	6.74	.455285
20	9.519911	6.00	9.974792	.74	9.545119	6.74	10.454881
21	.520271	5.99	.974748	.74	.545524	6.73	.454476
22	.520631	5.99	.974703	.74	.545928	6.73	.454072
23	.520990	5.98	.974659	.74	.546331	6.72	.453669
24	.521349	5.98	.974614	.74	.546735	6.72	.453265
25	.521707	5.97	.974570	.74	.547138	6.71	.452862
26	.522066	5.97	.974525	.74	.547540	6.71	.452460
27	.522424	5.96	.974481	.74	.547943	6.70	.452057
28	.522781	5.95	.974436	.74	.548345	6.70	.451655
29	.523138	5.95	.974391	.75	.548747	6.69	.451253
30	9.523495	5.94	9.974347	.75	9.549149	6.69	10.450851
31	.523852	5.94	.974302	.75	.549550	6.68	.450450
32	.524208	5.93	.974257	.75	.549951	6.68	.450049
33	.524564	5.93	.974212	.75	.550352	6.67	.449648
34	.524920	5.92	.974167	.75	.550752	6.67	.449248
35	.525275	5.92	.974122	.75	.551152	6.67	.448848
36	.525630	5.91	.974077	.75	.551552	6.66	.448448
37	.525984	5.90	.974032	.75	.551952	6.66	.448048
38	.526339	5.90	.973987	.75	.552351	6.65	.447649
39	.526693	5.89	.973942	.75	.552750	6.65	.447250
40	9.527046	5.89	9.973897	.75	9.553149	6.64	10.446851
41	.527400	5.88	.973852	.75	.553548	6.64	.446452
42	.527753	5.88	.973807	.75	.553946	6.63	.446054
43	.528105	5.87	.973761	.75	.554344	6.63	.445656
44	.528458	5.87	.973716	.76	.554741	6.62	.445259
45	.528810	5.86	.973671	.76	.555139	6.62	.444861
46	.529161	5.86	.973625	.76	.555536	6.61	.444464
47	.529513	5.85	.973580	.76	.555933	6.61	.444067
48	.529864	5.85	.973535	.76	.556329	6.60	.443671
49	.530215	5.84	.973489	.76	.556725	6.60	.443275
50	9.530565	5.83	9.973444	.76	9.557121	6.59	10.442879
51	.530915	5.83	.973398	.76	.557517	6.59	.442483
52	.531265	5.82	.973352	.76	.557913	6.59	.442087
53	.531614	5.82	.973307	.76	.558308	6.58	.441692
54	.531963	5.81	.973261	.76	.558702	6.58	.441298
55	.532312	5.81	.973215	.76	.559097	6.57	.440903
56	.532661	5.80	.973169	.76	.559491	6.57	.440509
57	.533009	5.80	.973124	.76	.559885	6.56	.440115
58	.533357	5.79	.973078	.77	.560279	6.56	.439721
59	.533704	5.79	.973032	.77	.560673	6.55	.439327
60	.534052		.972986		.561066		.438934
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.

109°

20° 159°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.78	.972940	.77	.561459	6.54	.438541	59
2	.534745	5.77	.972894	.77	.561851	6.54	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.54	.437756	57
4	.535438	5.76	.972802	.77	.562636	6.53	.437364	56
5	.535783	5.76	.972755	.77	.563028	6.53	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.52	.436581	54
7	.536474	5.75	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.74	.972617	.77	.564202	6.51	.435798	52
9	.537163	5.74	.972570	.77	.564592	6.51	.435408	51
10	9.537507	5.73	9.972524	.77	9.564983	6.50	10.435017	50
11	.537851	5.73	.972478	.77	.565373	6.50	.434627	49
12	.538194	5.72	.972431	.78	.565763	6.50	.434237	48
13	.538538	5.71	.972385	.78	.566153	6.49	.433847	47
14	.538880	5.71	.972338	.78	.566542	6.49	.433458	46
15	.539223	5.70	.972291	.78	.566932	6.48	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.69	.972198	.78	.567709	6.47	.432291	43
18	.540249	5.69	.972151	.78	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.46	.431514	41
20	9.540931	5.68	9.972058	.78	9.568873	6.46	10.431127	40
21	.541272	5.67	.972011	.78	.569261	6.46	.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.66	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.66	.971870	.78	.570422	6.44	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.44	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.64	.971729	.79	.571581	6.43	.428419	33
28	.543649	5.64	.971682	.79	.571967	6.43	.428033	32
29	.543987	5.63	.971635	.79	.572352	6.42	.427648	31
30	9.544325	5.63	9.971588	.79	9.572738	6.42	10.427262	30
31	.544663	5.62	.971540	.79	.573123	6.41	.426877	29
32	.545000	5.62	.971493	.79	.573507	6.41	.426493	28
33	.545338	5.61	.971446	.79	.573892	6.40	.426108	27
34	.545674	5.61	.971398	.79	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.79	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.79	.575044	6.39	.424956	24
37	.546683	5.59	.971256	.79	.575427	6.39	.424573	23
38	.547019	5.59	.971208	.79	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.79	.576193	6.38	.423807	21
40	9.547689	5.58	9.971113	.79	9.576576	6.37	10.423424	20
41	.548024	5.57	.971066	.80	.576958	6.37	.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.56	.970970	.80	.577723	6.36	.422277	17
44	.549027	5.56	.970922	.80	.578104	6.36	.421896	16
45	.549360	5.55	.970874	.80	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	.550026	5.55	.970779	.80	.579248	6.34	.420752	13
48	.550359	5.54	.970731	.80	.579629	6.34	.420371	12
49	.550692	5.54	.970683	.80	.580009	6.34	.419991	11
50	9.551024	5.53	9.970635	.80	9.580389	6.33	10.419611	10
51	.551356	5.53	.970586	.80	.580769	6.33	.419231	9
52	.551687	5.52	.970538	.80	.581149	3.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.51	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.51	.970394	.81	.582286	6.31	.417714	5
56	.553010	5.50	.970345	.81	.582665	6.31	.417335	4
57	.553341	5.50	.970297	.81	.583043	6.30	.416957	3
58	.553670	5.49	.970249	.81	.583422	6.30	.416578	2
59	.554000	5.49	.970200	.81	.583800	6.30	.416200	1
60	.554329		.970152		.584177		.415823	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

110° 69°

21° 158°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.554329	5.48	9.970152	.81	9.584177	6.29	10.415823	60
1	.554658	5.48	.970103	.81	.584555	6.29	.415445	59
2	.554987	5.47	.970055	.81	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.81	.585309	6.28	.414691	67
4	.555643	5.46	.969957	.81	.585686	6.28	.414314	56
5	.555971	5.46	.969909	.81	.586062	6.27	.413938	55
6	.556299	5.45	.969860	.81	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.81	.586815	6.26	.413185	53
8	.556953	5.44	.969762	.81	.587190	6.26	.412810	52
9	.557280	5.44	.969714	.81	.587566	6.26	.412434	51
10	9.557606	5.44	9.969665	.82	9.587941	6.25	10.412059	50
11	.557932	5.43	.969616	.82	.588316	6.25	.411684	49
12	.558258	5.43	.969567	.82	.588691	6.24	.411309	48
13	.558583	5.42	.969518	.82	.589066	6.24	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.24	.410560	46
15	.559234	5.41	.969420	.82	.589814	6.23	.410186	45
16	.559558	5.41	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.39	.969223	.82	.591308	6.22	.408692	41
20	9.560855	5.39	9.969173	.82	9.591681	6.21	10.408319	40
21	.561178	5.38	.969124	.82	.592054	6.21	.407946	39
22	.561501	5.38	.969075	.82	.592426	6.20	.407574	38
23	.561824	5.37	.969025	.82	.592798	6.20	.407202	37
24	.562146	5.37	.968976	.83	.593170	6.20	.406829	36
25	.562468	5.37	.968926	.83	.593542	6.19	.406458	35
26	.562790	5.36	.968877	.83	.593914	6.19	.406086	34
27	.563112	5.36	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.35	.968777	.83	.594656	6.18	.405344	32
29	.563755	5.35	.968728	.83	.595027	6.18	.404973	31
30	9.564075	5.34	9.968678	.83	9.595398	6.17	10.404602	30
31	.564396	5.34	.968628	.83	.595768	6.17	.404232	29
32	.564716	5.33	.968578	.83	.596138	6.16	.403862	28
33	.565036	5.33	.968528	.83	.596508	6.16	.403492	27
34	.565356	5.32	.968479	.83	.596878	6.16	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.31	.968329	.83	.597985	6.15	.402015	23
38	.566632	5.31	.968278	.84	.598354	6.14	.401646	22
39	.566951	5.30	.968228	.84	.598722	6.14	.401278	21
40	9.567269	5.30	9.968178	.84	9.599091	6.13	10.400909	20
41	.567587	5.29	.968128	.84	.599459	6.13	.400541	19
42	.567904	5.29	.968078	.84	.599827	6.13	.400173	18
43	.568222	5.28	.968027	.84	.600194	6.12	.399806	17
44	.568539	5.28	.967977	.84	.600562	6.12	.399438	16
45	.568856	5.28	.967927	.84	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.84	.601296	6.11	.398704	14
47	.569488	5.27	.967826	.84	.601662	6.11	.398338	13
48	.569804	5.26	.967775	.84	.602029	6.10	.397971	12
49	.570120	5.26	.967725	.84	.602395	6.10	.397605	11
50	9.570435	5.25	9.967674	.84	9.602761	6.10	10.397239	10
51	.570751	5.25	.967624	.84	.603127	6.09	.396873	9
52	.571066	5.24	.967573	.85	.603493	6.09	.396507	8
53	.571380	5.24	.967522	.85	.603858	6.09	.396142	7
54	.571695	5.24	.967471	.85	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.23	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.22	.967319	.85	.605317	6.07	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.21	.967217	.85	.606046	6.06	.393954	1
60	.573575		.967166		.606410		.393590	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

111° 68°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.573575	5.21	9.967166	.85	9.606410	6.06	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.06	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.85	.607500	6.05	.392500	57
4	.574824	5.19	.966961	.85	.607863	6.05	.392137	56
5	.575136	5.19	.966910	.85	.608225	6.04	.391775	55
6	.575447	5.18	.966859	.86	.608588	6.04	.391412	54
7	.575758	5.18	.966808	.86	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.86	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.86	.609674	6.03	.390326	51
10	9.576689	5.17	9.966653	.86	9.610036	6.02	10.389964	50
11	.576999	5.16	.966602	.86	.610397	6.02	.389603	49
12	.577309	5.16	.966550	.86	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.86	.611120	6.01	.388880	47
14	.577927	5.15	.966447	.86	.611480	6.01	.388520	46
15	.578236	5.14	.966395	.86	.611841	6.01	.388159	45
16	.578545	5.14	.966344	86	.612201	6.00	.387799	44
17	.578853	5.14	.966292	.86	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.86	.612921	6.00	.387079	42
19	.579470	5.13	.966188	.86	.613281	5.99	.386719	41
20	9.579777	5.12	9.966136	.87	9.613641	5.99	10.386359	40
21	.580085	5.12	.966085	.87	.614000	5.98	.386000	39
22	.580392	5.11	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.11	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.11	.965928	.87	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.09	.965772	.87	.616151	5.96	.383849	33
28	.582229	5.09	.965720	.87	.616509	5.96	.383491	32
29	.582535	5.09	.965668	.87	.616867	5.96	.383133	31
30	9.582840	5.08	9.965615	.87	9.617224	5.95	10.382776	30
31	.583145	5.08	.965563	.87	.617582	5.95	.382418	29
32	.583449	5.07	.965511	.87	.617939	5.95	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.94	.381705	27
34	.584058	5.06	.965406	.88	.618652	5.94	.381348	26
35	.584361	5.06	.965353	.88	.619008	5.94	.380992	25
36	.584665	5.06	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.05	.965248	.88	.619721	5.93	.380280	23
38	.585272	5.05	.965195	.88	.620076	5.93	.379924	22
39	.585574	5.04	.965143	.88	.620432	5.92	.379568	21
40	9.585877	5.04	9.965090	.88	9.620787	5.92	10.379213	20
41	.586179	5.04	.965037	.88	.621142	5.92	.378858	19
42	.586482	5.03	.964984	.88	.621497	5.91	.378503	18
43	.586783	5.03	.964931	.88	.621852	5.91	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.91	.377793	16
45	.587386	5.02	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.01	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.01	.964720	.88	.623269	5.90	.376731	13
48	.588289	5.01	.964666	.89	.623623	5.89	.376377	12
49	.588590	5.00	.964613	.89	.623976	5.89	.376024	11
50	9.588890	5.00	9.964560	.89	9.624330	5.89	10.375670	10
51	.589190	4.99	.964507	.89	.624683	5.88	.375317	9
52	.589489	4.99	.964454	.89	.625036	5.88	.374964	8
53	.589789	4.99	.964400	.89	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.89	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.89	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.89	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.89	.626797	5.86	.373203	3
58	.591282	4.97	.964133	.89	.627149	5.86	.372851	2
59	.591580	4.96	.964080	.89	.627501	5.86	.372499	1
60	.591878		.964026		.627852		.372148	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

23°

M.	Sine.	D.1″.	Cosine.	D.1″	Tang.	D.1″.	Cotang.
0	9.591878	4.96	9.964026	.89	9.627852	5.85	10.372148
1	.592176	4.95	.963972	.89	.628203	5.85	.371797
2	.592473	4.95	.963919	.90	.628554	5.85	.371446
3	.592770	4.95	.963865	.90	.628905	5.84	.371095
4	.593067	4.94	.963811	.90	.629255	5.84	.370745
5	.593363	4.94	.963757	.90	.629606	5.84	.370394
6	.593659	4.93	.963704	.90	.629956	5.83	.370044
7	.593955	4.93	.963650	.90	.630306	5.83	.369694
8	.594251	4.93	.963596	.90	.630656	5.83	.369344
9	.594547	4.92	.963542	.90	.631005	5.82	.368995
10	9.594842	4.92	9.963488	.90	9.631355	5.82	10.368645
11	.595137	4.91	.963434	.90	.631704	5.82	.368296
12	.595432	4.91	.963379	.90	.632053	5.81	.367947
13	.595727	4.91	.963325	.90	.632401	5.81	.367599
14	.596021	4.90	.963271	.90	.632750	5.81	.367250
15	.596315	4.90	.963217	.90	.633098	5.80	.366902
16	.596609	4.89	.963163	.91	.633447	5.80	.366553
17	.596903	4.89	.963108	.91	.633795	5.80	.366205
18	.597196	4.89	.963054	.91	.634143	5.79	.365857
19	.597490	4.88	.962999	.91	.634490	5.79	.365510
20	9.597783	4.88	9.962945	.91	9.634838	5.79	10.365162
21	.598075	4.88	.962890	.91	.635185	5.78	.364815
22	.598368	4.87	.962836	.91	.635532	5.78	.364468
23	.598660	4.87	.962781	.91	.635879	5.78	.364121
24	.598952	4.86	.962727	.91	.636226	5.78	.363774
25	.599244	4.86	.962672	.91	.636572	5.77	.363428
26	.599536	4.86	.962617	.91	.636919	5.77	.363081
27	.599827	4.85	.962562	.91	.637265	5.77	.362735
28	.600118	4.85	.962508	.91	.637611	5.76	.362389
29	.600409	4.84	.962453	.92	.637956	5.76	.362044
30	9.600700	4.84	9.962398	.92	9.638302	5.76	10.361698
31	.600990	4.84	.962343	.92	.638647	5.75	.361353
32	.601280	4.83	.962288	.92	.638992	5.75	.361008
33	.601570	4.83	.962233	.92	.639337	5.75	.360663
34	.601860	4.83	.962178	.92	.639682	5.74	.360318
35	.602150	4.82	.962123	.92	.640027	5.74	.359973
36	.602439	4.82	.962067	.92	.640371	5.74	.359629
37	.602728	4.81	.962012	.92	.640716	5.73	.359284
38	.603017	4.81	.961957	.92	.641060	5.73	.358940
39	.603305	4.81	.961902	.92	.641404	5.73	.358596
40	9.603594	4.80	9.961846	.92	9.641747	5.73	10.358253
41	.603882	4.80	.961791	.92	.642091	5.72	.357909
42	.604170	4.79	.961735	.92	.642434	5.72	.357566
43	.604457	4.79	.961680	.93	.642777	5.72	.357223
44	.604745	4.79	.961624	.93	.643120	5.71	.356880
45	.605032	4.78	.961569	.93	.643463	5.71	.356537
46	.605319	4.78	.961513	.93	.643806	5.71	.356194
47	.605606	4.78	.961458	.93	.644148	5.70	.355852
48	.605892	4.77	.961402	.93	.644490	5.70	.355510
49	.606179	4.77	.961346	.93	.644832	5.70	.355168
50	9.606465	4.76	9.961290	.93	9.645174	5.69	10.354826
51	.606751	4.76	.961235	.93	.645516	5.69	.354484
52	.607036	4.76	.961179	.93	.645857	5.69	.354143
53	.607322	4.75	.961123	.93	.646199	5.69	.353801
54	.607607	4.75	.961067	.93	.646540	5.68	.353460
55	.607892	4.74	.961011	.93	.646881	5.68	.353119
56	.608177	4.74	.960955	.93	.647222	5.68	.352778
57	.608461	4.74	.960899	.94	.647562	5.67	.352438
58	.608745	4.73	.960843	.94	.647903	5.67	.352097
59	.609029	4.73	.960786	.94	.648243	5.67	.351757
60	.609313		.960730		.648583		.351417
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.

113°

24° 155°

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.609313	4.73	9.960730	.94	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.94	.648923	5.66	.351077	59
2	.609880	4.72	.960618	.94	.649263	5.66	.350737	58
3	.610164	4.72	.960561	.94	.649602	5.66	.350398	57
4	.610447	4.71	.960505	.94	.649942	5.65	.350058	56
5	.610729	4.71	.960448	.94	.650281	5.65	.349719	55
6	.611012	4.71	.960392	.94	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.94	.650959	5.64	.349041	53
8	.611576	4.70	.960279	.94	.651297	5.64	.348703	52
9	.611858	4.69	.960222	.94	.651636	5.64	.348364	51
10	9.612140	4.69	9.960165	.95	9.651974	5.64	10.348026	50
11	.612421	4.69	.960109	.95	.652312	5.63	.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.95	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.66	.959711	.95	.654674	5.61	.345326	42
19	.614665	4.66	.959654	.95	.655011	5.61	.344989	41
20	9.614944	4.65	9.959596	.95	9.655348	5.61	10.344652	40
21	.615223	4.65	.959539	.95	.655684	5.61	.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.64	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.64	.959368	.96	.656692	5.60	.343308	36
25	.616338	4.64	.959310	.96	.657028	5.59	.342972	35
26	.616616	4.63	.959253	.96	.657364	5.59	.342636	34
27	.616894	4.63	.959195	.96	.657699	5.59	.342301	33
28	.617172	4.63	.959138	.96	.658034	5.58	.341966	32
29	.617450	4.62	.959081	.96	.658369	5.58	.341631	31
30	9.617727	4.62	9.959023	.96	9.658704	5.58	10.341296	30
31	.618004	4.61	.958965	.96	.659039	5.58	.340961	29
32	.618281	4.61	.958908	.96	.659373	5.57	.340627	28
33	.618558	4.61	.958850	.96	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.96	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.96	.660376	5.56	.339624	25
36	.619386	4.60	.958677	.96	.660710	5.56	.339290	24
37	.619662	4.59	.958619	.97	.661043	5.56	.338957	23
38	.619938	4.59	.958561	.97	.661377	5.56	.338623	22
39	.620213	4.59	.958503	.97	.661710	5.55	.338290	21
40	9.620488	4.58	9.958445	.97	9.662043	5.55	10.337957	20
41	.620763	4.58	.958387	.97	.662376	5.55	.337624	19
42	.621038	4.58	.958329	.97	.662709	5.54	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.54	.336958	17
44	.621587	4.57	.958213	.97	.663375	5.54	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.54	.336293	15
46	.622135	4.56	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.56	.958038	.97	.664371	5.53	.335629	13
48	.622682	4.56	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.53	.334965	11
50	9.623229	4.55	9.957863	.97	9.665366	5.52	10.334634	10
51	.623502	4.54	.957804	.98	.665697	5.52	.334303	9
52	.623774	4.54	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.54	.957687	.98	.666360	5.51	.333640	7
54	.624319	4.53	.957628	.98	.666691	5.51	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.51	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.51	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.50	.332318	3
58	.625406	4.52	.957393	.98	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	.625948		.957276		.668672		.331328	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

114° 65°

25° **154°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.625948	4.51	9.957276	.98	9.668673	5.50	10.331327	60
1	.626219	4.51	.957217	.98	.669002	5.49	.330998	59
2	.626490	4.51	.957158	.98	.669332	5.49	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.49	.330339	57
4	.627030	4.50	.957040	.99	.669991	5.49	.330009	56
5	.627300	4.50	.956981	.99	.670320	5.48	.329680	55
6	.627570	4.49	.956921	.99	.670649	5.48	.329351	54
7	.627840	4.49	.956862	.99	.670977	5.48	.329023	53
8	.628109	4.49	.956803	.99	.671306	5.47	.328694	52
9	.628378	4.48	.956744	.99	.671635	5.47	.328366	51
10	9.628647	4.48	9.956684	.99	9.671963	5.47	10.328037	50
11	.628916	4.48	.956625	.99	.672291	5.47	.327709	49
12	.629185	4.47	.956566	.99	.672619	5.46	.327381	48
13	.629453	4.47	.956506	.99	.672947	5.46	.327053	47
14	.629721	4.47	.956447	.99	.673274	5.46	.326726	46
15	.629989	4.46	.956387	.99	.673602	5.46	.326398	45
16	.630257	4.46	.956327	.99	.673929	5.45	.326071	44
17	.630524	4.46	.956268	.99	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	1.00	.674910	5.45	.325089	41
20	9.631326	4.45	9.956089	1.00	9.675237	5.44	10.324763	40
21	.631593	4.44	.956029	1.00	.675564	5.44	.324436	39
22	.631859	4.44	.955969	1.00	.675890	5.44	.324110	38
23	.632125	4.44	.955909	1.00	.676216	5.44	.323784	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.43	.955789	1.00	.677869	5.43	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.42	.322480	33
28	.633454	4.42	.955609	1.00	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	9.633984	4.41	9.955488	1.00	9.678496	5.42	10.321504	30
31	.634249	4.41	.955428	1.01	.678821	5.41	.321179	29
32	.634514	4.41	.955368	1.01	.679146	5.41	.320854	28
33	.634778	4.40	.955307	1.01	.679471	5.41	.320529	27
34	.635042	4.40	.955247	1.01	.679795	5.41	.320205	26
35	.635306	4.40	.955186	1.01	.680120	5.40	.319880	25
36	.635570	4.39	.955126	1.01	.680444	5.40	.319556	24
37	.635834	4.39	.955065	1.01	.680768	5.40	.319232	23
38	.636097	4.39	.955005	1.01	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.01	.681416	5.39	.318584	21
40	9.636623	4.38	9.954883	1.01	9.681740	5.39	10.318260	20
41	.636886	4.38	.954823	1.01	.682063	5.39	.317937	19
42	.637148	4.37	.954762	1.01	.682387	5.39	.317613	18
43	.637411	4.37	.954701	1.01	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.36	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.36	.954518	1.02	.683679	5.38	.316321	14
47	.638458	4.36	.954457	1.02	.684001	5.37	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	9.639242	4.35	9.954274	1.02	9.684968	5.37	10.315032	10
51	.639503	4.34	.954213	1.02	.685290	5.36	.314710	9
52	.639764	4.34	.954152	1.02	.685612	5.36	.314388	8
53	.640024	4.34	.954090	1.02	.685934	5.36	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.36	.313745	6
55	.640544	4.33	.953968	1.02	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.32	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.03	.687540	5.35	.312460	2
59	.641584	4.32	.953722	1.03	.687861	5.35	.312139	1
60	.641842		.953660		.688182		.311818	0
M	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

26° 153°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.641842	4.32	9.953660	1.03	9.688182	5.34	10.311818	60
1	.642101	4.31	.953599	1.03	.688502	5.34	.311498	59
2	.642360	4.31	.953537	1.03	.688823	5.34	.311177	58
3	.642618	4.31	.953475	1.03	.689143	5.34	.310857	57
4	.642877	4.30	.953413	1.03	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.30	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.29	.953228	1.03	.690423	5.33	.309577	53
8	.643908	4.29	.953166	1.03	.690742	5.32	.309258	52
9	.644165	4.29	.953104	1.03	.691062	5.32	.308938	51
10	9.644423	4.28	9.953042	1.03	9.691381	5.32	10.308619	50
11	.644680	4.28	.952980	1.04	.691700	5.32	.308300	49
12	.644936	4.28	.952918	1.04	.692019	5.31	.307981	48
13	.645193	4.27	.952855	1.04	.692338	5.31	.307662	47
14	.645450	4.27	.952793	1.04	.692656	5.31	.307344	46
15	.645706	4.27	.952731	1.04	.692975	5.31	.307025	45
16	.645962	4.26	.952669	1.04	.693293	5.30	.306707	44
17	.646218	4.26	.952606	1.04	.693612	5.30	.306388	43
18	.646474	4.26	.952544	1.04	.693930	5.30	.306070	42
19	.646729	4.26	.952481	1.04	.694248	5.30	.305752	41
20	9.646984	4.25	9.952419	1.04	9.694566	5.29	10.305434	40
21	.647240	4.25	.952356	1.04	.694883	5.29	.305117	39
22	.647494	4.25	.952294	1.04	.695201	5.29	.304799	38
23	.647749	4.24	.952231	1.04	.695518	5.29	.304482	37
24	.648004	4.24	.952168	1.05	.695836	5.29	.304164	36
25	.648258	4.24	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.28	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	9.649527	4.22	9.951791	1.05	9.697736	5.27	10.302264	30
31	.649781	4.22	.951728	1.05	.698053	5.27	.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.21	.951602	1.05	.698685	5.26	.301315	27
34	.650539	4.21	.951539	1.05	.699001	5.26	.300999	26
35	.650792	4.21	.951476	1.05	.699316	5.26	.300684	25
36	.651044	4.20	.951412	1.05	.699632	5.26	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.26	.300053	23
38	.651549	4.20	.951286	1.06	.700263	5.25	.299737	22
39	.651800	4.19	.951222	1.06	.700578	5.25	.299422	21
40	9.652052	4 19	9.951159	1.06	9.700893	5.25	10.299107	20
41	.652304	4.19	.951096	1.06	.701208	5.25	.298792	19
42	.652555	4.18	.951032	1.06	.701523	5.24	.298477	18
43	.652806	4.18	.950968	1.06	.701837	5.24	.298163	17
44	.653057	4.18	.950905	1.06	.702152	5.24	.297848	16
45	.653308	4.18	.950841	1.06	.702466	5.24	.297534	15
46	.653558	4.17	.950778	1.06	.702781	5.24	.297219	14
47	.653808	4.17	.950714	1.06	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.06	.703409	5.23	.296591	12
49	.654309	4.16	.950586	1.06	.703723	5.23	.296273	11
50	9.654558	4.16	9.950522	1.07	9.704036	5.23	10.295964	10
51	.654808	4.16	.950458	1.07	.704350	5.22	.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704977	5.22	.295023	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.14	.950138	1.07	.705916	5.21	.294084	4
57	.656302	4.14	.950074	1.07	.706228	5.21	.293772	3
58	.656551	4.14	.950010	1.07	.706541	5.21	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.21	.293146	1
60	.657047		.949881		.707166		.292834	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

116° 63°

27° **152°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.657047	4.13	9.949881	1.07	9.707166	5.20	10.292834	60
1	.657295	4.13	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.12	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5 20	.291586	56
5	.658284	4.12	.949558	1.08	.708726	5.19	.291274	55
6	.658531	4.11	.949494	1.08	.709037	5.19	.290963	54
7	.658778	4.11	.949429	1.08	.709349	5.19	.290651	53
8	.659025	4.11	.949364	1.08	.709660	5.19	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	9.659517	4.10	9.949235	1.08	9.710282	5.18	10.289718	50
11	.659763	4.10	.949170	1.08	.710593	5.18	.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.09	.949040	1.08	.711215	5.18	.288785	47
14	.660501	4.09	.948975	1.08	.711525	5.17	.288475	46
15	.660746	4.09	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.09	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.09	.712766	5.17	.287234	42
19	.661726	4.08	.948650	1.09	.713076	5.16	.286924	41
20	9.661970	4.07	9.948584	1.09	9.713386	5.16	10.286614	40
21	.662214	4.07	.948519	1.09	.713696	5.16	.286304	39
22	.662459	4.07	.948454	1.09	.714005	5.16	.285995	38
23	.662703	4.06	.948388	1.09	.714314	5.15	.285686	37
24	.662946	4.06	.948323	1.09	.714624	5.15	.285376	36
25	.663190	4.06	.948257	1.09	.714933	5.15	.285067	35
26	.663433	4.05	.948192	1.09	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.09	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.09	.715860	5.14	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.14	.283832	31
30	9.664406	4.04	9.947929	1.10	9.716477	5.14	10.283523	30
31	.664648	4.04	.947863	1.10	.716785	5.14	.283215	29
32	.664891	4.04	.947797	1.10	.717093	5.14	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.10	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.10	.718017	5 13	.281983	25
36	.665859	4.03	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.02	.947467	1.10	.718633	5.13	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.12	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	9.666824	4.01	9.947269	1.10	9.719555	5.12	10.280445	20
41	.667065	4.01	.947203	1.11	.719862	5.12	.280138	19
42	.667305	4.01	.947136	1.11	.720169	5.11	.279831	18
43	.667546	4.01	.947070	1.11	.720476	5.11	.279524	17
44	.667786	4.00	.947004	1.11	.720783	5.11	.279217	16
45	.668027	4.00	.946937	1.11	.721089	5.11	.278911	15
46	.668267	4.00	.946871	1.11	.721396	5.11	.278604	14
47	.668506	3.99	.946804	1.11	.721702	5.10	.278298	13
48	.668746	3.99	.946738	1.11	.722009	5.10	.277991	12
49	.668986	3.99	.946671	1.11	.722315	5.10	.277685	11
50	9.669225	3.99	9.946604	1.11	9.722621	5.10	10.277379	10
51	.669464	3.98	.946538	1.11	.722927	5.10	.277073	9
52	.669703	3.98	.946471	1.11	.723232	5.09	.276768	8
53	.669942	3.98	.946404	1.11	.723538	5.09	.276462	7
54	.670181	3.98	.946337	1.11	.723844	5.09	.276156	6
55	.670419	3.97	.946270	1.12	.724149	5.09	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.09	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.96	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.96	.946002	1.12	.725370	5.08	.274630	1
60	.671609		.945935		.725674		.274326	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

117° **62°**

28° **151°**

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	M.
0	9.671609	3.96	9.945935	1.12	9.725674	5.08	10.274326	60
1	.671847	3.96	.945868	1.12	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.12	.726892	5.07	.273108	56
5	.672795	3.94	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.94	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.94	.945464	1.13	.727805	5.06	.272195	53
8	.673505	3.94	.945396	1.13	.728109	5.06	.271891	52
9	.673741	3.93	.945328	1.13	.728412	5.06	.271588	51
10	9.673977	3.93	9.945261	1.13	9.728716	5.06	10.271284	50
11	.674213	3.93	.945193	1.13	.729020	5.06	.270980	49
12	.674448	3.93	.945125	1.13	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.92	.944990	1.13	.729929	5.05	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.05	.269767	45
16	.675390	3.91	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.91	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.91	.944718	1.13	.731141	5.04	.268859	42
19	.676094	3.91	.944650	1.13	.731444	5.04	.268556	41
20	9.676328	3.90	9.944582	1.14	9.731746	5.04	10.268254	40
21	.676562	3.90	.944514	1.14	.732048	5.04	.267952	39
22	.676796	3.90	.944446	1.14	.732351	5.04	.267649	38
23	.677030	3.90	.944377	1.14	.732653	5.03	.267347	37
24	.677264	3.89	.944309	1.14	.732955	5.03	.267045	36
25	.677498	3.89	.944241	1.14	.733257	5.03	.266743	35
26	.677731	3.89	.944172	1.14	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.14	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.14	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.14	.734463	5.02	.265537	31
30	9.678663	3.88	9.943899	1.14	9.734764	5.02	10.265236	30
31	.678895	3.87	.943830	1.14	.735066	5.02	.264934	29
32	.679128	3.87	.943761	1.15	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.01	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.01	.264031	26
35	.679824	3.86	.943555	1.15	.736269	5.01	.263731	25
36	.680056	3.86	.943486	1.15	.736570	5.01	.263430	24
37	.680288	3.86	.943417	1.15	.736870	5.01	.263130	23
38	.680519	3.86	.943348	1.15	.737171	5.01	.262829	22
39	.680750	3.85	.943279	1.15	.737471	5.00	.262529	21
40	9.680982	3.85	9.943210	1.15	9.737771	5.00	10.262229	20
41	.681213	3.85	.943141	1.15	.738071	5.00	.261929	19
42	.681443	3.84	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.84	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.84	.942934	1.15	.738971	4.99	.261029	16
45	.682135	3.84	.942864	1.16	.739271	4.99	.260729	15
46	.682365	3.83	.942795	1.16	.739570	4.99	.260430	14
47	.682595	3.83	.942726	1.16	.739870	4.99	.260130	13
48	.682825	3.83	.942656	1.16	.740169	4.99	.259831	12
49	.683055	3.83	.942587	1.16	.740468	4.98	.259532	11
50	9.683284	3.82	9.942517	1.16	9.740767	4.98	10.259233	10
51	.683514	3.82	.942448	1.16	.741066	4.98	.258934	9
52	.683743	3.82	.942378	1.16	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.16	.741664	4.98	.258336	7
54	.684201	3.81	.942239	1.16	.741962	4.98	.258038	6
55	.684430	3.81	.942169	1.16	.742261	4.97	.257739	5
56	.684658	3.81	.942099	1.16	.742559	4.97	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	.685571		.941819		.743752		.256248	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	M.

118° **61°**

29° **150°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.685571	3.80	9.941819	1.17	9.743752	4.96	10.256248	60
1	.685799	3.79	.941749	1.17	.744050	4.96	.255950	59
2	.686027	3.79	.941679	1.17	.744348	4.96	.255652	58
3	.686254	3.79	.941609	1.17	.744645	4.96	.255355	57
4	.686482	3.79	.941539	1.17	.744943	4.96	.255057	56
5	.686709	3.78	.941469	1.17	.745240	4.96	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.78	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.17	.746132	4.95	.253868	52
9	.687616	3.77	.941187	1.17	.746429	4.95	.253571	51
10	9.687843	3.77	9.941117	1.18	9.746726	4.95	10.253274	50
11	.688069	3.77	.941046	1.18	.747023	4.95	.252977	49
12	.688295	3.77	.940975	1.18	.747319	4.94	.252681	48
13	.688521	3.76	.940905	1.18	.747616	4.94	.252384	47
14	.688747	3.76	.940834	1.18	.747913	4.94	.252087	46
15	.688972	3.76	.940763	1.18	.748209	4.94	.251791	45
16	.689198	3.76	.940693	1.18	.748505	4.94	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	9.690098	3.75	9.940409	1.18	9.749689	4.93	10.250311	40
21	.690323	3.74	.940338	1.18	.749985	4.93	.250015	39
22	.690548	3.74	.940267	1.19	.750281	4.93	.249719	38
23	.690772	3.74	.940196	1.19	.750576	4.92	.249424	37
24	.690996	3.74	.940125	1.19	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.19	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.19	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.19	.751757	4.92	.248243	33
28	.691892	3.73	.939840	1.19	.752052	4.92	.247948	32
29	.692115	3.72	.939768	1.19	.752347	4.91	.247653	31
30	9.692339	3.72	9.939697	1.19	9.752642	4.91	10.247358	30
31	.692562	3.72	.939625	1.19	.752937	4.91	.247063	29
32	.692785	3.72	.939554	1.19	.753231	4.91	.246769	28
33	.693008	3.71	.939482	1.19	.753526	4.91	.246474	27
34	.693231	3.71	.939410	1.19	.753820	4.91	.246180	26
35	.693453	3.71	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.71	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.20	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	9.694564	3.70	9.938980	1.20	9.755585	4.89	10.244415	20
41	.694786	3.69	.938908	1.20	.755878	4.89	.244122	19
42	.695007	3.69	.938836	1.20	.756172	4.89	.243828	18
43	.695229	3.69	.938763	1.20	.756465	4.89	.243535	17
44	.695450	3.69	.938691	1.20	.756759	4.89	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.89	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.21	.757638	4.88	.242362	13
48	.696334	3.68	.938402	1.21	.757931	4.88	.242069	12
49	.696554	3.67	.938330	1.21	.758224	4.88	.241776	11
50	9.696775	3.67	9.938258	1.21	9.758517	4.88	10.241483	10
51	.696995	3.67	.938185	1.21	.758810	4.88	.241190	9
52	.697215	3.67	.938113	1.21	.759102	4.87	.240898	8
53	.697435	3.66	.938040	1.21	.759395	4.87	.240605	7
54	.697654	3.66	.937967	1.21	.759687	4.87	.240313	6
55	.697874	3.66	.937895	1.21	.759979	4.87	.240021	5
56	.698094	3.66	.937822	1.21	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.21	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.21	.760856	4.86	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.86	.238852	1
60	.698970		.937531		.761439		.238561	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

119° **60°**

30° 149°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.698970	3.65	9.937531	1.22	9.761439	4.86	10.238561	60
1	.699189	3.64	.937458	1.22	.761731	4.86	.238269	59
2	.699407	3.64	.937385	1.22	.762023	4.86	.237977	58
3	.699626	3.64	.937312	1.22	.762314	4.86	.237686	57
4	.699844	3.64	.937238	1.22	.762606	4.86	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.63	.936946	1.22	.763770	4.85	.236230	52
9	.700933	3.62	.936872	1.22	.764061	4.85	.235939	51
10	9.701151	3.62	9.936799	1.22	9.764352	4.85	10.235648	50
11	.701368	3.62	.936725	1.23	.764643	4.84	.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.84	.235067	48
13	.701802	3.61	.936578	1.23	.765224	4.84	.234776	47
14	.702019	3.61	.936505	1.23	.765514	4.84	.234486	46
15	.702236	3.61	.936431	1.23	.765805	4.84	.234195	45
16	.702452	3.61	.936357	1.23	.766095	4.84	.233905	44
17	.702669	3.60	.936284	1.23	.766385	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	9.703317	3.60	9.936062	1.23	9.767255	4.83	10.232745	40
21	.703533	3.59	.935988	1.23	.767545	4.83	.232455	39
22	.703749	3.59	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.59	.935840	1.23	.768124	4.82	.231876	37
24	.704179	3.59	.935766	1.24	.768413	4.82	.231587	36
25	.704395	3.59	.935692	1.24	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.24	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.24	.769281	4.82	.230719	33
28	.705040	3.58	.935469	1.24	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.24	.769860	4.82	.230140	31
30	9.705469	3.57	9.935320	1.24	9.770148	4.81	10.229852	30
31	.705683	3.57	.935246	1.24	.770437	4.81	.229563	29
32	.705898	3.57	.935171	1.24	.770726	4.81	.229274	28
33	.706112	3.57	.935097	1.24	.771015	4.81	.228985	27
34	.706326	3.56	.935022	1.24	.771303	4.81	.228697	26
35	.706539	3.56	.934948	1.24	.771592	4.81	.228408	25
36	.706753	3.56	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.56	.934798	1.25	.772168	4.80	.227832	23
38	.707180	3.55	.934723	1.25	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	9.707606	3.55	9.934574	1.25	9.773033	4.80	10.226967	20
41	.707819	3.55	.934499	1.25	.773321	4.80	.226679	19
42	.708032	3.54	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.54	.934349	1.25	.773896	4.79	.226104	17
44	.708458	3.54	.934274	1.25	.774184	4.79	.225816	16
45	.708670	3.54	.934199	1.25	.774471	4.79	.225529	15
46	.708882	3.54	.934123	1.25	.774759	4.79	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.79	.224954	13
48	.709306	3.53	.933973	1.26	.775333	4.79	.224667	12
49	.709518	3.53	.933898	1.26	.775621	4.78	.224379	11
50	9.709730	3.53	9.933822	1.26	9.775908	4.78	10.224092	10
51	.709941	3.52	.933747	1.26	.776195	4.78	.223805	9
52	.710153	3.52	.933671	1.26	.776482	4.78	.223518	8
53	.710364	3.52	.933596	1.26	.776769	4.78	.223231	7
54	.710575	3.52	.933520	1.26	.777055	4.78	.222945	6
55	.710786	3.51	.933445	1.26	.777342	4.78	.222658	5
56	.710997	3.51	.933369	1.26	.777628	4.77	.222372	4
57	.711208	3.51	.933293	1.26	.777915	4.77	.222085	3
58	.711419	3.51	.933217	1.26	.778201	4.77	.221799	2
59	.711629	3.51	.933141	1.26	.778488	4.77	.221512	1
60	.711839		.933066		.778774		.221226	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

120° 59°

31° **148°**

M.	Sine.	D.1''.	Cosine.	D.1''.	Tang.	D.1''.	Cotang.	M.
0	9.711839	3.50	9.933066	1.27	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.50	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.76	.220368	57
4	.712679	3.49	.932762	1.27	.779918	4.76	.220082	56
5	.712889	3.49	.932685	1.27	.780203	4.76	.219797	55
6	.713098	3.49	932609	1.27	.780489	4.76	.219511	54
7	.713308	3.49	.932533	1.27	.780775	4.76	.219225	53
8	.713517	3.48	.932457	1.27	.781060	4.76	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.76	.218654	51
10	9.713935	3.48	9.932304	1.27	9.781631	4.75	10.218369	50
11	.714144	3.48	.932228	1.27	.781916	4.75	.218084	49
12	.714352	3.48	.932151	1.28	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.47	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.28	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.46	.931768	1.28	.783626	4.74	.216374	43
18	.715602	3.46	.931691	1.28	.783910	4.74	.216090	42
19	.715809	3.46	.931614	1.28	.784195	4.74	.215805	41
20	9.716017	3.46	9.931537	1.28	9.784479	4.74	10.215521	40
21	.716224	3.46	.931460	1.28	.784764	4.74	.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.74	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.74	.214668	37
24	.716846	3.45	.931229	1.29	.785616	4.73	.214384	36
25	.717053	3.45	.931152	1.29	.785900	4.73	.214100	35
26	.717259	3.44	.931075	1.29	.786184	4.73	.213816	34
27	.717466	3.44	.930998	1.29	.786468	4.73	.213532	33
28	.717673	3.44	.930921	1.29	.786752	4.73	.213248	32
29	.717879	3.44	.930843	1.29	.787036	4.73	.212964	31
30	9.718085	3.43	9.930766	1.29	9.787319	4.73	10.212681	30
31	.718291	3.43	.930688	1.29	.787603	4.72	.212397	29
32	.718497	3.43	.930611	1.29	.787886	4.72	.212114	28
33	.718703	3.43	.930533	1.29	.788170	4.72	.211830	27
34	.718909	3.43	.930456	1.29	.788453	4.72	.211547	26
35	.719114	3.42	.930378	1.29	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.30	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.71	.210415	22
39	.719935	3.41	.930067	1.30	.789868	4.71	.210132	21
40	9.720140	3.41	9.929989	1.30	9.790151	4.71	10.209849	20
41	.720345	3.41	.929911	1.30	.790433	4.71	.209567	19
42	.720549	3.41	.929833	1.30	.790716	4.71	.209284	18
43	.720754	3.41	.929755	1.30	.790999	4.71	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.71	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.70	.208437	15
46	.721366	3.40	.929521	1.30	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.31	.792128	4.70	.207872	13
48	.721774	3.39	.929364	1.31	792410	4.70	.207590	12
49	.721978	3.39	.929286	1.31	.792692	4.70	.207308	11
50	9.722181	3.39	9.929207	1.31	9.792974	4.70	10.207026	10
51	.722385	3.39	.929129	1.31	.793256	4.70	.206744	9
52	.722588	3.39	.929050	1.31	.793538	4.70	.206462	8
53	.722791	3.38	.928972	1.31	.793819	4.69	.206181	7
54	.722994	3.38	.928893	1.31	.794101	4.69	.205899	6
55	.723197	3.38	.928815	1.31	.794383	4.69	.205617	5
56	.723400	3.38	.928736	1.31	.794664	4.69	.205336	4
57	.723603	3.37	.928657	1.31	.794945	4.69	.205055	3
58	.723805	3.37	.928578	1.31	795227	4.69	.204773	2
59	.724007	3.37	.928499	1.32	.795508	4.69	.204492	1
60	.724210		.928420		.795789		.204211	0
M.	Cosine.	D.1''.	Sine.	D.1''.	Cotang.	D.1''.	Tang.	M.

121° **58°**

32° 1

M.	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	
0	9.724210	3.37	9.928420	1.32	9.795789	4.68	10.204211	6
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	5
2	.724614	3.36	.928263	1.32	.796351	4.68	.203649	5
3	.724816	3.36	.928183	1.32	.796632	4.68	.203368	6
4	.725017	3.36	.928104	1.32	.796913	4.68	.203087	5
5	.725219	3.36	.928025	1.32	.797194	4.68	.202806	5
6	.725420	3.36	.927946	1.32	.797475	4.68	.202525	5
7	.725622	3.35	.927867	1.32	.797755	4.68	.202245	5
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	9.726225	3.35	9.927629	1.32	9.798596	4.67	10.201404	50
11	.726426	3.34	.927549	1.33	.798877	4.67	.201123	49
12	.726626	3.34	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.34	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.34	.927310	1.33	.799717	4.67	.200283	46
15	.727228	3.34	.927231	1.33	.799997	4.66	.200003	45
16	.727428	3.33	.927151	1.33	.800277	4.66	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.66	.199443	43
18	.727828	3.33	.926991	1.33	.800836	4.66	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.66	.198884	41
20	9.728227	3.33	9.926831	1.33	9.801396	4.66	10.198604	40
21	.728427	3.32	.926751	1.33	.801675	4.66	.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.66	.198045	38
23	.728825	3.32	.926591	1.34	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.34	.802513	4.65	.197487	36
25	.729223	3.31	.926431	1.34	.802792	4.65	.197208	35
26	.729422	3.31	.926351	1.34	.803072	4.65	.196928	34
27	.729621	3.31	.926270	1.34	.803351	4.65	.196649	33
28	.729820	3.31	.926190	1.34	.803630	4.65	.196370	32
29	.730018	3.31	.926110	1.34	.803908	4.65	.196092	31
30	9.730216	3.30	9.926029	1.34	9.804187	4.65	10.195813	30
31	.730415	3.30	.925949	1.34	.804466	4.64	.195534	29
32	.730613	3.30	.925868	1.34	.804745	4.64	.195255	28
33	.730811	3.30	.925788	1.34	.805023	4.64	.194977	27
34	.731009	3.30	.925707	1.35	.805302	4.64	.194698	26
35	.731206	3.29	.925626	1.35	.805580	4.64	.194420	25
36	.731404	3.29	.925545	1.35	.805859	4.64	.194141	24
37	.731602	3.29	.925465	1.35	.806137	4.64	.193863	23
38	.731799	3.29	.925384	1.35	.806415	4.64	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	9.732193	3.28	9.925222	1.35	9.806971	4.63	10.193029	20
41	.732390	3.28	.925141	1.35	.807249	4.63	.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.28	.924979	1.35	.807805	4.63	.192195	17
44	.732980	3.27	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.63	.191639	15
46	.733373	3.27	.924735	1.36	.808638	4.63	.191362	14
47	.733569	3.27	.924654	1.36	.808916	4.62	.191084	13
48	.733765	3.27	.924572	1.36	.809193	4.62	.190807	12
49	.733961	3.26	.924491	1.36	.809471	4.62	.190529	11
50	9.734157	3 26	9.924409	1.36	9.809748	4.62	10.190252	10
51	.734353	3.26	.924328	1.36	.810025	4.62	.189975	9
52	.734549	3.26	.924246	1.36	.810302	4.62	.189698	8
53	.734744	3.26	.924164	1.36	.810580	4.62	.189420	7
54	.734939	3.25	.924083	1.36	.810857	4.62	.189143	6
55	.735135	3.25	.924001	1.36	.811134	4.61	.188866	5
56	.735330	3.25	.923919	1.36	.811410	4.61	.188590	4
57	.735525	3.25	.923837	1.37	.811687	4.61	.188313	3
58	.735719	3.25	.923755	1.37	.811964	4.61	.188036	2
59	.735914	3.24	.923673	1.37	.812241	4.61	.187759	1
60	.736109		.923591		.812517		.187483	0
M.	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	.

122° 57°

33° **146°**

M.	Sine.	D.1".	Cosine.	D.1"	Tang.	D.1".	Cotang.	M.
0	9.736109	3.24	9.923591	1.37	9.812517	4.61	10.187483	60
1	.736303	3.24	.923509	1.37	.812794	4.61	.187206	59
2	.736498	3.24	.923427	1.37	.813070	4.61	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.61	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.37	.813899	4.60	.186101	55
6	.737274	3.23	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.37	.814452	4.60	.185548	53
8	.737661	3.22	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	9.738048	3.22	9.922768	1.38	9.815280	4.60	10.184720	50
11	.738241	3.22	.922686	1.38	.815555	4.60	.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.59	.184169	48
13	.738627	3.21	.922520	1.38	.816107	4.59	.183893	47
14	.738820	3.21	.922438	1.38	.816382	4.59	.183618	46
15	.739013	3.21	.922355	1.38	.816658	4.59	.183342	45
16	.739206	3.21	.922272	1.38	.816933	4.59	.183067	44
17	.739398	3.21	.922189	1.38	.817209	4.59	.182791	43
18	.739590	3.20	.922106	1.38	.817484	4.59	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.59	.182241	41
20	9.739975	3.20	9.921940	1.39	9.818035	4.59	10.181965	40
21	.740167	3.20	.921857	1.39	.818310	4.58	.181690	39
22	.740359	3.20	.921774	1.39	.818585	4.58	.181415	38
23	.740550	3.19	.921691	1.39	.818860	4.58	.181140	37
24	.740742	3.19	.921607	1.39	.819135	4.58	.180865	36
25	.740934	3.19	.921524	1.39	.819410	4.58	.180590	35
26	.741125	3.19	.921441	1.39	.819684	4.58	.180316	34
27	.741316	3.19	.921357	1.39	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.39	.820234	4.58	.179766	32
29	.741699	3.18	.921190	1.39	.820508	4.58	.179492	31
30	9.741889	3.18	9.921107	1.39	9.820783	4.57	10.179217	30
31	.742080	3.18	.921023	1.39	.821057	4.57	.178943	29
32	.742271	3.18	.920939	1.40	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.17	.920688	1.40	.822154	4.57	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.16	.920436	1.40	.822977	4.57	.177023	32
39	.743602	3.16	.920352	1.40	.823251	4.56	.176749	21
40	9.743792	3.16	9.920268	1.40	9.823524	4.56	10.176476	20
41	.743982	3.16	.920184	1.40	.823798	4.56	.176202	19
42	.744171	3.16	.920099	1.40	.824072	4.56	.175928	18
43	.744361	3.15	.920015	1.41	.824345	4.56	.175655	17
44	.744550	3.15	.919931	1.41	.824619	4.56	.175381	16
45	.744739	3.15	.919846	1.41	.824893	4.56	.175107	15
46	.744928	3.15	.919762	1.41	.825166	4.56	.174834	14
47	.745117	3.15	.919677	1.41	.825439	4.56	.174561	13
48	.745306	3.14	.919593	1.41	.825713	4.55	.174287	12
49	.745494	3.14	.919508	1.41	.825986	4.55	.174014	11
50	9.745683	3.14	9.919424	1.41	9.826259	4.55	10.173741	10
51	.745871	3.14	.919339	1.41	.826532	4.55	.173468	9
52	.746060	3.14	.919254	1.41	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.41	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.41	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.13	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.54	.171830	3
58	.747187	3.12	.918745	1.42	.828442	4.54	.171558	2
59	.747374	3.12	.918659	1.42	.828715	4.54	.171285	1
60	.747562		.918574		.828987		.171013	0
M.	Cosine.	D.1".	Sine.	D.1".	Cotang.	D.1".	Tang.	M.

123° **56°**

34° **145°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.747562	3.12	9.918574	1.42	9.828987	4.54	10.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.54	.170740	59
2	.747936	3.12	.918404	1.42	.829532	4.54	.170468	58
3	.748123	3.11	.918318	1.42	.829805	4.54	.170195	57
4	.748310	3.11	.918233	1.42	.830077	4.54	.169923	56
5	.748497	3.11	.918147	1.43	.830349	4.54	.169651	55
6	.748683	3.11	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.11	.917976	1.43	.830893	4.53	.169107	53
8	.749056	3.10	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	9.749429	3.10	9.917719	1.43	9.831709	4.53	10.168291	50
11	.749615	3.10	.917634	1.43	.831981	4.53	.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.10	.917462	1.43	.832525	4.53	.167475	47
14	.750172	3.09	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.09	.917290	1.43	.833068	4.53	.166932	45
16	.750543	3.09	.917204	1.43	.833339	4.52	.166661	44
17	.750729	3.09	.917118	1.44	.833611	4.52	.166389	43
18	.750914	3.09	.917032	1.44	.833882	4.52	.166118	42
19	.751099	3.08	.916946	4.44	.834154	4.52	.165846	41
20	9.751284	3.08	9.916859	1.44	9.834425	4.52	10.165575	40
21	.751469	3.08	.916773	1.44	.834696	4.52	.165304	39
22	.751654	3.08	.916687	1.44	.834967	4.52	.165033	38
23	.751839	3.08	.916600	1.44	.835238	4.52	.164762	37
24	.752023	3.07	.916514	1.44	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.44	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.44	.836051	4.51	.163949	34
27	.752576	3.07	.916254	1.44	.836322	4.51	.163678	33
28	.752760	3.07	.916167	1.45	.836593	4.51	.163407	32
29	.752944	3.06	.916081	1.45	.836864	4.51	.163136	31
30	9.753128	3.06	9.915994	1.45	9.837134	4.51	10.162866	30
31	.753312	3.06	.915907	1.45	.837405	4.51	.162595	29
32	.753495	3.06	.915820	1.45	.837675	4.51	.162325	28
33	.753679	3.06	.915733	1.45	.837946	4.51	.162054	27
34	.753862	3.05	.915646	1.45	.838216	4.51	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.51	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.45	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.50	.160703	22
39	.754778	3.05	.915210	1.46	.839568	4.50	.160432	21
40	9.754960	3.04	9.915123	1.46	9.839838	4.50	10.160162	20
41	.755143	3.04	.915035	1.46	.840108	4.50	.159892	19
42	.755326	3.04	.914948	1.46	.840378	4.50	.159622	18
43	.755508	3.04	.914860	1.46	.840648	4.50	.159352	17
44	.755690	3.04	.914773	1.46	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.46	.841187	4.49	.158813	15
46	.756054	3.03	.914598	1.46	.841457	4.49	.158543	14
47	.756236	3.03	.914510	1.46	.841727	4.49	.158273	13
48	.756418	3.03	.914422	1.46	.841996	4.49	.158004	12
49	.756600	3.03	.914334	1.46	.842266	4.49	.157734	11
50	9.756782	3.02	9.914246	1.47	9.842535	4.49	10.157465	10
51	.756963	3.02	.914158	1.47	.842805	4.49	.157195	9
52	.757144	3.02	.914070	1.47	.843074	4.49	.156926	8
53	.757326	3.02	.913982	1.47	.843343	4.49	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.49	.156388	6
55	.757688	3.02	.913806	1 47	.843882	4.49	.156118	5
56	.757869	3.01	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.01	.913630	1.47	.844420	4.48	.155580	3
58	.758230	3.01	.913541	1.47	.844689	4.48	.155311	2
59	.758411	3.01	.913453	1.47	.844958	4.48	.155652	1
60	.758591		.913365		.845227		.154773	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

35° **144°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.758591	3.01	9.913365	1.47	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.48	.154504	59
2	.758952	3.00	.913187	1.48	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.48	.846302	4.48	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	2.99	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.99	.912744	1.48	.847108	4.47	.152892	53
8	.760031	2.99	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.99	.912566	1.48	.847644	4.47	.152356	51
10	9.760390	2.99	9.912477	1.48	9.847913	4.47	10.152087	50
11	.760569	2.99	.912388	1.48	.848181	4.47	.151819	49
12	.760748	2.98	.912299	1.49	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.49	.848717	4.47	.151283	47
14	.761106	2.98	.912121	1.49	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.49	.849254	4.47	.150746	45
16	.761464	2.98	.911942	1.49	.849522	4.47	.150478	44
17	.761642	2.97	.911853	1.49	.849790	4.46	.150210	43
18	.761821	2.97	.911763	1.49	.850057	4.46	.149943	42
19	.761999	2.97	.911674	1.49	.850325	4.46	.149675	41
20	9.762177	2.97	9.911584	1.49	9.850593	4.46	10.149407	40
21	.762356	2.97	.911495	1.49	.850861	4.46	.149139	39
22	.762534	2.97	.911405	1.49	.851129	4.46	.148871	38
23	.762712	2.96	.911315	1.50	.851396	4.46	.148604	37
24	.762889	2.96	.911226	1.50	.851664	4.46	.148336	36
25	.763067	2.96	.911136	1.50	.851931	4.46	.148069	35
26	.763245	2.96	.911046	1.50	.852199	4.46	.147801	34
27	.763422	2.96	.910956	1.50	.852466	4.46	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.46	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	9.763954	2.95	9.910686	1.50	9.853268	4.45	10.146732	30
31	.764131	2.95	.910596	1.50	.853535	4.45	.146465	29
32	.764308	2.95	.910506	1.50	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.51	.854069	4.45	.145931	27
34	.764662	2.94	.910325	1.51	.854336	4.45	.145664	26
35	.764838	2.94	.910235	1.51	.854603	4.45	.145397	25
36	.765015	2.94	.910144	1.51	.854870	4.45	.145130	24
37	.765191	2.94	.910054	1.51	.855137	4.45	.144863	23
38	.765367	2.94	.909963	1.51	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.51	.855671	4.44	.144329	21
40	9.765720	2.93	9.909782	1.51	9.855938	4.44	10.144062	20
41	.765896	2.93	.909691	1.51	.856204	4.44	.143796	19
42	.766072	2.93	.909601	1.51	.856471	4.44	.143529	18
43	.766247	2.93	.909510	1.51	.856737	4.44	.143263	17
44	.766423	2.93	.909419	1.52	.857004	4.44	.142996	16
45	.766598	2.92	.909328	1.52	.857270	4.44	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.44	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.44	.142197	13
48	.767124	2.92	.909055	1.52	.858069	4.44	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.44	.141664	11
50	9.767475	2.91	9.908873	1.52	9.858602	4.44	10.141398	10
51	.767649	2.91	.908781	1.52	.858868	4.43	.141132	9
52	.767824	2.91	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.91	.908599	1.52	.859400	4.43	.140600	7
54	.768173	2.91	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.91	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.90	.908324	1.53	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.43	.139270	2
59	.769045	2.90	.908049	1.53	.860995	4.43	.139005	1
60	.769219		.907958		.861261		.138739	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

125° **54°**

40° **139°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.808067	2.51	9.884254	1.77	9.923813	4.28	10.076187	60
1	.808218	2.51	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.51	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.77	.924583	4.27	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.77	.925352	4.27	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.27	.074135	52
9	.809419	2.50	.883297	1.78	.926122	4.27	.073878	51
10	9.809569	2.49	9.883191	1.78	9.926378	4.27	10.073622	50
11	.809718	2.49	.883084	1.78	.926634	4.27	.073366	49
12	.809868	2.49	.882977	1.78	.926890	4.27	.073110	48
13	.810017	2.49	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.49	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.49	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.79	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.79	.928427	4.27	.071573	42
19	.810912	2.48	.882229	1.79	.928683	4.27	.071317	41
20	9.811061	2.48	9.882121	1.79	9.928940	4.27	10.071060	40
21	.811210	2.48	.882014	1.79	.929196	4.27	.070804	39
22	.811358	2.48	.881907	1.79	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.79	.929708	4.27	.070292	37
24	.811655	2.47	.881692	1.79	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.79	.930220	4.27	.069780	35
26	.811952	2.47	.881477	1.79	.930475	4.26	.069525	34
27	.812100	2.47	.881369	1.79	.930731	4.26	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.26	.069013	32
29	.812396	2.47	.881153	1.80	.931243	4.26	.068757	31
30	9.812544	2.46	9.881046	1.80	9.931499	4.26	10.068501	30
31	.812692	2.46	.880938	1.80	.931755	4.26	.068245	29
32	.812840	2.46	.880830	1.80	.932010	4.26	.067990	28
33	.812988	2.46	.880722	1.80	.932266	4.26	.067734	27
34	.813135	2.46	.880613	1.80	.932522	4.26	.067478	26
35	.813283	2.46	.880505	1.80	.932778	4.26	.067222	25
36	.813430	2.46	.880397	1.81	.933033	4.26	.066967	24
37	.813578	2.45	.880289	1.81	.933289	4.26	.066711	23
38	.813725	2.45	.880180	1.81	.933545	4.26	.066455	22
39	.813872	2.45	.880072	1.81	.933800	4.26	.066200	21
40	9.814019	2.45	9.879963	1.81	9.934056	4.26	10.065944	20
41	.814166	2.45	.879855	1.81	.934311	4.26	.065689	19
42	.814313	2.45	.879746	1.81	.934567	4.26	.065433	18
43	.814460	2.45	.879637	1.81	.934823	4.26	.065177	17
44	.814607	2.44	.879529	1.81	.935078	4.26	.064922	16
45	.814753	2.44	.879420	1.81	.935333	4.26	.064667	15
46	.814900	2.44	.879311	1.81	.935589	4.26	.064411	14
47	.815046	2.44	.879202	1.82	.935844	4.26	.064156	13
48	.815193	2.44	.879093	1.82	.936100	4.26	.063900	12
49	.815339	2.44	.878984	1.82	.936355	4.26	.063645	11
50	9.815485	2.44	9.878875	1.82	9.936611	4.26	10.063389	10
51	.815631	2.43	.878766	1.82	.936866	4.26	.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.26	.062879	8
53	.815924	2.43	.878547	1.82	.937376	4.25	.062624	7
54	.816069	2.43	.878438	1.82	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.83	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.25	.061858	4
57	.816507	2.43	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.42	.877999	1.83	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	.816943		.877780		.939163		.060837	0
M	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

130° **49°**

41° **138°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.42	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.42	.877340	1.84	.940183	4.25	.059817	56
5	.817668	2.41	.877230	1.84	.940438	4.25	.059562	55
6	.817813	2.41	.877120	1.84	.940694	4.25	.059306	54
7	.817958	2.41	.877010	1.84	.940949	4.25	.059051	53
8	.818103	2.41	.876899	1.84	.941204	4.25	.058796	52
9	.818247	2.41	.876789	1.84	.941458	4.25	.058542	51
10	9.818392	2.41	9.876678	1.84	9.941713	4.25	10.058287	50
11	.818536	2.41	.876568	1.84	.941968	4.25	.058032	49
12	.818681	2.40	.876457	1.84	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.84	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.85	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.25	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.39	.875682	1.85	.944007	4.25	.055993	41
20	9.819832	2.39	9.875571	1.85	9.944262	4.25	10.055738	40
21	.819976	2.39	.875459	1.85	.944517	4.25	.055483	39
22	.820120	2.39	.875348	1.85	.944771	4.24	.055229	38
23	.820263	2.39	.875237	1.86	.945026	4.24	.054974	37
24	.820406	2.39	.875126	1.86	.945281	4.24	.054719	36
25	.820550	2.39	.875014	1.86	.945535	4.24	.054465	35
26	.820693	2.38	.874903	1.86	.945790	4.24	.054210	34
27	820836	2.38	.874791	1.86	.946045	4.24	.053955	33
28	.820979	2.38	.874680	1.86	.946299	4.24	.053701	32
29	.821122	2.38	.874568	1.86	.946554	4.24	.053446	31
30	9.821265	2.38	9.874456	1.86	9.946808	4.24	10.053192	30
31	.821407	2.38	.874344	1.86	.947063	4.24	.052937	29
32	.821550	2.38	.874232	1.87	.947318	4.24	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.24	.052428	27
34	.821835	2.37	.874009	1.87	.947826	4.24	.052174	26
35	.821977	2.37	.873896	1.87	.948081	4.24	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.24	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.24	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.24	.051156	22
39	.822546	2.37	.873448	1.87	.949099	4.24	.050901	21
40	9.822688	2.37	9.873335	1.87	9.949353	4.24	10.050647	20
41	.822830	2.36	.873223	1.88	.949608	4.24	.050392	19
42	.822972	2.36	.873110	1.88	.949862	4.24	.050138	18
43	.823114	2.36	.872998	1.88	.950116	4.24	.049884	17
44	.823255	2.36	.872885	1.88	.950371	4.24	.049629	16
45	.823397	2.36	.872772	1.88	.950625	4.24	.049375	15
46	.823539	2.36	.872659	1.88	.950879	4.24	.049121	14
47	.823680	2.36	.872547	1.88	.951133	4.24	.048867	13
48	.823821	2.35	.872434	1.88	.951388	4.24	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.24	.048358	11
50	9.824104	2.35	9.872208	1.89	9.951896	4.24	10.048104	10
51	.824245	2.35	.872095	1.89	.952150	4.24	.047850	9
52	.824386	2.35	.871981	1.89	.952405	4.24	.047595	8
53	.824527	2.35	.871868	1.89	.952659	4.24	.047341	7
54	.824668	2.35	.871755	1.89	.952913	4.24	.047087	6
55	.824808	2.34	.871641	1.89	.953167	4.24	.046833	5
56	.824949	2.34	.871528	1.89	.953421	4.24	.046579	4
57	.825090	2.34	.871414	1.89	.953675	4.23	.046325	3
58	.825230	2.34	.871301	1.89	.953929	4.23	.046071	2
59	.825371	2.34	.871187	1.90	.954183	4.23	.045817	1
60	.825511		.871073		.954437		.045563	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

131° **48°**

42° **137°**

M.	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	M.
0	9.825511	2.34	9.871073	1.90	9.954437	4.23	10.045563	60
1	.825651	2.34	.870960	1.90	.954691	4.23	.045309	59
2	.825791	2.33	.870846	1.90	.954945	4.23	.045055	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955707	4.23	.044293	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.90	.956215	4.23	.043785	53
8	.826631	2.33	.870161	1.91	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.91	.956723	4.23	043277	51
10	9.826910	2.32	9.869933	1.91	9.956977	4.23	10.043023	50
11	.827049	2.32	.869818	1.91	.957231	4.23	.042769	49
12	.827189	2.32	.869704	1.91	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.91	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.91	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.91	.958247	4.23	.041753	45
16	.827745	2.32	.869245	1.91	.958500	4.23	.041500	44
17	.827884	2.31	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.31	.869015	1.92	.959008	4 23	.040992	42
19	.828162	2.31	.868900	1.92	.959262	4.23	.040738	41
20	9.828301	2.31	9.868785	1.92	9.959516	4.23	10.040484	40
21	.828439	2.31	.868670	1.92	.959769	4.23	.040231	39
22	.828578	2.31	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.31	.868440	1.92	.960277	4.23	.039723	37
24	.828855	2.31	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.92	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.93	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.23	.038708	33
28	.829407	2.30	.867862	1.93	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1 93	.961799	4.23	.038201	31
30	9.829683	2.30	9.867631	1.93	9.962052	4.23	10.037948	30
31	.829821	2.30	.867515	1.93	.962306	4.23	.037694	29
32	.829959	2.29	.867399	1.93	.962560	4.23	.037440	28
33	.830097	2.29	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.29	.867167	1.93	.963067	4.23	.036933	26
35	.830372	2.29	.867051	1.94	.963320	4.23	.036680	25
36	.830509	2.29	.866935	1.94	.963574	4.23	.036426	24
37	.830646	2.29	.866819	1.94	.963828	4.23	.036172	23
38	.830784	2.29	.866703	1.94	.964081	4.23	.035919	22
39	.830921	2.29	.866586	1.94	.964335	4.23	.035665	21
40	9.831058	2.28	9.866470	1.94	9.964588	4.22	10.035412	20
41	.831195	2.28	.866353	1.94	.964842	4.22	.035158	19
42	.831332	2.28	.866237	1.94	.965095	4.22	.034905	18
43	.831469	2.28	.866120	1.94	.965349	4.22	.034651	17
44	.831606	2.28	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.22	.034145	15
46	.831879	2.28	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.27	.865653	1.95	.966362	4.22	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.27	.865419	1.95	.966869	4.22	.033131	11
50	9.832425	2.27	9.865302	1.95	9.967123	4.22	10.032877	10
51	.832561	2.27	.865185	1.95	.967376	4.22	.032624	9
52	.832697	2.27	.865068	1.95	.967629	4.22	.032371	8
53	.832833	2.27	.864950	1.96	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.96	.968136	4.22	.031864	6
55	.833105	2.26	.864716	1.96	.968389	4.22	.031611	5
56	.833241	2.26	.864598	1.96	.968643	4.22	.031357	4
57	.833377	2.26	.864481	1.96	.968896	4.22	.031104	3
58	.833512	2.26	.864363	1.96	.969149	4.22	.030851	2
59	.833648	2.26	.864245	1.96	.969403	4.22	.030597	1
60	.833783		.864127		.969656		.030344	0
M.	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	M.

132° **47°**

43° **136°**

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.833783	2.26	9.864127	1.96	9.969656	4.22	10.030344	60
1	.833919	2.26	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.22	.029838	58
3	.834189	2.25	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.97	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.22	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.25	.863183	1.97	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	9.835134	2.24	9.862946	1.98	9.972188	4.22	10.027812	50
11	.835269	2.24	.862827	1.98	.972441	4.22	.027559	49
12	.835403	2.24	.862709	1.98	.972694	4.22	.027306	48
13	.835538	2.24	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.24	.862471	1.98	.973201	4.22	.026799	46
15	.835807	2.24	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.24	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.99	.974466	4.22	.025534	41
20	9.836477	2.23	9.861758	1.99	9.974720	4.22	10.025280	40
21	.836611	2.23	.861638	1.99	.974973	4.22	.025027	39
22	.836745	2.23	.861519	1.99	.975226	4.22	.024774	38
23	.836878	2.23	.861400	1.99	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.99	.975732	4.22	.024268	36
25	.837146	2.22	.861161	1.99	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.99	.976238	4.22	.023762	34
27	.837412	2.22	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	9.837812	2.22	9.860562	2.00	9.977250	4.22	10.022750	30
31	.837945	2.22	.860442	2.00	.977503	4.22	.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.21	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.21	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.21	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.21	.859842	2.01	.978768	4.22	.021232	24
37	.838742	2.21	859721	2.01	.979021	4.22	.020979	23
38	.838875	2.21	.859601	2.01	.979274	4.22	.020726	22
39	.839007	2.21	.859480	2.01	.979527	4.22	.020473	21
40	9.839140	2.21	9.859360	2.01	9.979780	4.22	10.020220	20
41	.839272	2.20	.859239	2.01	.980033	4.22	.019967	19
42	839404	2.20	.859119	2.01	.980286	4.22	.019714	18
43	.839536	2.20	.858998	2.01	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2 02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.21	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.21	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.21	.018450	13
48	.840196	2.19	.858393	2.02	.981803	4 21	.018197	12
49	.840328	2.19	.858272	2.02	.982056	4.21	.017944	11
50	9.840459	2.19	9.858151	2.02	9.982309	4.21	10.017691	10
51	.840591	2.19	.858029	2.02	.982562	4.21	.017438	9
52	.840722	2.19	.857908	2.02	.982814	4.21	.017186	8
53	.840854	2.19	.857786	2.03	.983067	4.21	.016933	7
54	.840985	2.19	.857665	2.03	.983320	4.21	.016680	6
55	.841116	2.19	.857543	2.03	.983573	4.21	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.21	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.21	.015921	3
58	.841509	2.18	.857178	2.03	.984331	4.21	.015669	2
59	.841640	2.18	.857056	2.03	.984584	4.21	.015416	1
60	.841771		.856934		.984837		.015163	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

133° **46°**

44° 135°

M.	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	M.
0	9.841771	2.18	9.856934	2.03	9.984837	4.21	10.015163	60
1	.841902	2.18	.856812	2.04	.985090	4.21	.014910	59
2	.842033	2.18	.856690	2.04	.985343	4.21	.014657	58
3	.842163	2.18	.856568	2.04	.985596	4.21	.014404	57
4	.842294	2.17	.856446	2.04	.985848	4.21	.014152	56
5	.842424	2.17	.856323	2.04	.986101	4.21	.013899	55
6	.842555	2.17	.856201	2.04	.986354	4.21	.013646	54
7	.842685	2.17	.856078	2.04	.986607	4.21	.013393	53
8	.842815	2.17	.855956	2.04	.986860	4.21	.013140	52
9	.842946	2.17	.855833	2.04	.987112	4.21	.012888	51
10	9.843076	2.17	9.855711	2.05	9.987365	4.21	10.012635	50
11	.843206	2.17	.855588	2.05	.987618	4.21	.012382	49
12	.843336	2.16	.855465	2.05	.987871	4.21	.012129	48
13	.843466	2.16	.855342	2.05	.988123	4.21	.011877	47
14	.843595	2.16	.855219	2.05	.988376	4.21	.011624	46
15	.843725	2.16	.855096	2.05	.988629	4.21	.011371	45
16	.843855	2.16	.854973	2.05	.988882	4.21	.011118	44
17	.843984	2.16	.854850	2.05	.989134	4.21	.010866	43
18	.844114	2.16	.854727	2.06	.989387	4.21	.010613	42
19	.844243	2.16	.854603	2.06	.989640	4.21	.010360	41
20	9.844372	2.15	9.854480	2.06	9.989893	4.21	10.010107	40
21	.844502	2.15	.854356	2.06	.990145	4.21	.009855	39
22	.844631	2.15	.854233	2.06	.990398	4.21	.009602	38
23	.844760	2.15	.854109	2.06	.990651	4.21	.009349	37
24	.844889	2.15	.853986	2.06	.990903	4.21	.009097	36
25	.845018	2.15	.853862	2.06	.991156	4.21	.008844	35
26	.845147	2.15	.853738	2.06	.991409	4.21	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.21	.008338	33
28	.845405	2.14	.853490	2.07	.991914	4.21	.008086	32
29	.845533	2.14	.853366	2.07	.992167	4.21	.007833	31
30	9.845662	2.14	9.853242	2.07	9.992420	4.21	10.007580	30
31	.845790	2.14	.853118	2.07	.992672	4.21	.007328	29
32	.845919	2.14	.852994	2.07	.992925	4.21	.007075	28
33	.846047	2.14	.852869	2.07	.993178	4.21	.006822	27
34	.846175	2.14	.852745	2.07	.993431	4.21	.006569	26
35	.846304	2.14	.852620	2.08	.993683	4.21	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.21	.006064	24
37	.846560	2.13	.852371	2.08	.994189	4.21	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.21	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.21	.005306	21
40	9.846944	2.13	9.851997	2.08	9.994947	4.21	10.005053	20
41	.847071	2.13	.851872	2.08	.995199	4.21	.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.21	.004548	18
43	.847327	2.13	.851622	2.09	.995705	4.21	.004295	17
44	.847454	2.12	.851497	2.09	.995957	4.21	.004043	16
45	.847582	2.12	.851372	2.09	.996210	4.21	.003790	15
46	.847709	2.12	.851246	2.09	.996463	4.21	.003537	14
47	.847836	2.12	.851121	2.09	.996715	4.21	.003285	13
48	.847964	2.12	.850996	2.09	.996968	4.21	.003032	12
49	.848091	2.12	.850870	2.09	.997221	4.21	.002779	11
50	9.848218	2.12	9.850745	2.09	9.997473	4.21	10.002527	10
51	.848345	2.12	.850619	2.10	.997726	4.21	.002274	9
52	.848472	2.11	.850493	2.10	.997979	4.21	.002021	8
53	.848599	2.11	.850368	2.10	.998231	4.21	.001769	7
54	.848726	2.11	.850242	2.10	.998484	4.21	.001516	6
55	.848852	2.11	.850116	2.10	.998737	4.21	.001263	5
56	.848979	2.11	.849990	2.10	.998989	4.21	.001011	4
57	.849106	2.11	.849864	2.10	.999242	4.21	.000758	3
58	.849232	2.11	.849738	2.10	.999495	4.21	.000505	2
59	.849359	2.11	.849611	2.11	.999748	4.21	.000252	1
60	.849485		.849485		0.000000		.000000	0
M.	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	M.

134° 45°

TABLE V.

LATITUDES AND DEPARTURES,

OR

TRAVERSE TABLE.

B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5		B'ng
° '	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	° '
0 15	1.0000	0.0044	2.0000	0.0087	3.0000	0.0131	4.0000	0.0175	5.0000	0.0218	89 45
30	0000	0087	1.9999	0175	2.9999	0262	3.9998	0349	4.9998	0436	30
45	0.9999	0131	9998	0262	9997	0393	9997	0524	9996	0654	15
1 0	9998	0175	9997	0349	9995	0524	9994	0698	9992	0873	89 0
15	9998	0218	9995	0436	9993	0654	9990	0873	9988	1091	45
30	9997	0262	9993	0524	9990	0785	9986	1047	9983	1309	30
45	9995	0305	9991	0611	9986	0916	9981	1222	9977	1527	15
2 0	9994	0349	9988	0698	9982	1047	9976	1396	9970	1745	88 0
15	9992	0393	9985	0785	9977	1178	9969	1570	9961	1963	45
30	9990	0436	9981	0872	9971	1309	9962	1745	9952	2181	30
45	0.9988	0.0480	1.9977	0.0960	2.9965	0.1439	3.9954	0.1919	4.9942	0.2399	15
3 0	9986	0523	9973	1047	9959	1570	9945	2093	9931	2617	87 0
15	9984	0567	9968	1134	9952	1701	9936	2268	9920	2835	45
30	9981	0610	9963	1221	9944	1831	9925	2442	9907	3052	30
45	9979	0654	9957	1308	9936	1962	9914	2616	9893	3270	15
4 0	9976	0698	9951	1395	9927	2093	9903	2790	9878	3488	86 0
15	9973	0741	9945	1482	9918	2223	9890	2964	9863	3705	45
30	9969	0785	9938	1569	9908	2354	9877	3138	9846	3923	30
45	9966	0828	9931	1656	9897	2484	9863	3312	9828	4140	15
5 0	9962	0872	9924	1743	9886	2615	9848	3486	9810	4358	85 0
15	0.9958	0.0915	1.9916	0.1830	2.9874	0.2745	3.9832	0.3660	4.9790	0.4575	45
30	9954	0958	9908	1917	9862	2875	9816	3834	9770	4792	30
45	9950	1002	9899	2004	9849	3006	9799	4008	9748	5009	15
6 0	9945	1045	9890	2091	9836	3136	9781	4181	9726	5226	84 0
15	9941	1089	9881	2177	9822	3266	9762	4355	9703	5443	45
30	9936	1132	9871	2264	9807	3396	9743	4528	9679	5660	30
45	9931	1175	9861	2351	9792	3526	9723	4701	9653	5877	15
7 0	9925	1219	9851	2437	9776	3656	9702	4875	9627	6093	83 0
15	9920	1262	9840	2524	9760	3786	9680	5048	9600	6310	45
30	9914	1305	9829	2611	9743	3916	9658	5221	9572	6526	30
45	0.9909	0.1349	1.9817	0.2697	2.9726	0.4046	3.9635	0.5394	4.9543	0.6743	15
8 0	9903	1392	9805	2783	9708	4175	9611	5567	9513	6959	82 0
15	9897	1435	9793	2870	9690	4305	9586	5740	9483	7175	45
30	9890	1478	9780	2956	9670	4434	9561	5912	9451	7390	30
45	9884	1521	9767	3042	9651	4564	9534	6085	9418	7606	15
9 0	9877	1564	9754	3129	9631	4693	9508	6257	9384	7822	81 0
15	9870	1607	9740	3215	9610	4822	9480	6430	9350	8037	45
30	9863	1650	9726	3301	9589	4951	9451	6602	9314	8252	30
45	9856	1693	9711	3387	9567	5080	9422	6774	9278	8467	15
10 0	9848	1736	9696	3473	9544	5209	9392	6946	9240	8682	80 0
15	0.9840	0.1779	1.9681	0.3559	2.9521	0.5338	3.9362	0.7118	4.9202	0.8897	45
30	9833	1822	9665	3645	9498	5467	9330	7289	9163	9112	30
45	9825	1865	9649	3730	9474	5596	9298	7461	9123	9326	15
11 0	9816	1908	9633	3816	9449	5724	9265	7632	9081	9540	79 0
15	9808	1951	9616	3902	9424	5853	9231	7804	9039	9755	45
30	9799	1994	9598	3987	9398	5981	9197	7975	8996	9968	30
45	9790	2036	9581	4073	9371	6109	9162	8146	8952	1.0182	15
12 0	9781	2079	9563	4158	9344	6237	9126	8316	8907	0396	78 0
15	9772	2122	9545	4244	9317	6365	9089	8487	8862	0609	45
30	9763	2164	9526	4329	9289	6493	9052	8658	8815	0822	30
45	0.9753	0.2207	1.9507	0.4414	2.9260	0.6621	3.9014	0.8828	4.8767	1.1035	15
13 0	9744	2250	9487	4499	9231	6749	8975	8998	8719	1248	77 0
15	9734	2292	9468	4584	9201	6876	8935	9168	8669	1460	45
30	9724	2334	9447	4669	9171	7003	8895	9338	8618	1672	30
45	9713	2377	9427	4754	9140	7131	8854	9507	8567	1884	15
14 0	9703	2419	9406	4838	9109	7258	8812	9677	8515	2096	76 0
15	9692	2462	9385	4923	9077	7385	8769	9846	8462	2308	45
30	9681	2504	9363	5008	9044	7511	8726	1.0015	8407	2519	30
45	9670	2546	9341	5092	9011	7638	8682	0184	8352	2730	15
15 0	9659	2588	9319	5176	8978	7765	8637	0353	8296	2941	75 0
° '	Dep.	Lat	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° '
B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng

B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
0 15	5.9999	0.0262	6.9999	0.0305	7.9999	0.0349	8.9999	0.0393	9.9999	0.0436	89 45
30	9998	0524	9997	0611	9997	0698	9997	0785	9996	0873	30
45	9995	0785	9994	0916	9993	1047	9992	1178	9991	1309	15
1 0	9991	1047	9989	1222	9988	1396	9986	1571	9985	1745	89 0
15	9986	1309	9983	1527	9981	1745	9979	1963	9976	2181	45
30	9979	1571	9976	1832	9973	2094	9969	2356	9966	2618	30
45	9972	1832	9967	2138	9963	2443	9958	2748	9953	3054	15
2 0	9963	2094	9957	2443	9951	2792	9945	3141	9939	3490	88 0
15	9954	2356	9946	2748	9938	3141	9931	3533	9923	3926	45
30	9943	2617	9933	3053	9924	3490	9914	3926	9905	4362	30
45	5.9931	0.2879	6.9919	0.3358	7.9908	0.3838	8.9896	0.4318	9.9885	0.4798	15
3 0	9918	3140	9904	3664	9890	4187	9877	4710	9863	5234	87 0
15	9904	3402	9887	3968	9871	4535	9855	5102	9839	5669	45
30	9888	3663	9869	4273	9851	4884	9832	5494	9813	6105	30
45	9872	3924	9850	4578	9829	5232	9807	5886	9786	6540	15
4 0	9854	4185	9829	4883	9805	5581	9781	6278	9756	6976	86 0
15	9835	4447	9808	5188	9780	5929	9753	6670	9725	7411	45
30	9815	4708	9784	5492	9753	6277	9723	7061	9692	7846	30
45	9794	4968	9760	5797	9725	6625	9691	7453	9657	8281	15
5 0	9772	5229	9734	6101	9696	6972	9658	7844	9619	8716	85 0
15	5.9748	0.5490	6.9706	0.6405	7.9664	0.7320	8.9622	0 8235	9.9580	0.9150	45
30	9724	5751	9678	6709	9632	7668	9586	8626	9540	9585	30
45	9698	6011	9648	7013	9597	8015	9547	9017	9497	1.0019	15
6 0	9671	6272	9617	7317	9562	8362	9507	9408	9452	0453	84 0
15	9643	6532	9584	7621	9525	8709	9465	9798	9406	0887	45
30	9614	6792	9550	7924	9486	9056	9421	1.0188	9357	1320	30
45	9584	7052	9515	8228	9445	9403	9376	0578	9307	1754	15
7 0	9553	7312	9478	8531	9404	9750	9329	0968	9255	2187	83 0
15	9520	7572	9440	8834	9360	1.0096	9280	1358	9200	2620	45
30	9487	7832	9401	9137	9316	0442	9230	1747	9144	3053	30
45	5.9452	0.8091	6.9361	0.9440	7.9269	1.0788	8.9178	1.2137	9.9087	1.3485	15
8 0	9416	8350	9319	9742	9221	1134	9124	2526	9027	3917	82 0
15	9379	.8610	9276	1.0044	9172	1479	9069	2914	8965	4349	45
30	9341	8869	9231	0347	9121	1825	9011	3303	8902	4781	30
45	9302	9127	9185	0649	9069	2170	8953	3691	8836	5212	15
9 0	9261	9386	9138	0950	9015	2515	8892	4079	8769	5643	81 0
15	9220	9645	9090	1252	8960	2859	8830	4467	8700	6074	45
30	9177	9903	9040	1553	8903	3204	8766	4854	8629	6505	30
45	9133	1.0161	8989	1854	8844	3548	8700	5241	8556	6935	15
10 0	9088	0419	8937	2155	8785	3892	8633	5628	8481	7365	80 0
15	5.9042	1.0677	6.8883	1.2456	7.8723	1.4235	8.8564	1.6015	9.8404	1.7794	45
30	8995	0934	8828	2756	8660	4579	8493	6401	8325	8224	30
45	8947	1191	8772	3057	8596	4922	8421	6787	8245	8652	15
11 0	8898	1449	8714	3357	8530	5265	8346	7173	8163	9081	79 0
15	8847	1705	8655	3656	8463	5607	8271	7558	8079	9509	45
30	8795	1962	8595	3956	8394	5949	8193	7943	7992	9937	30
45	8743	2219	8533	4255	8324	6291	8114	8328	7905	2.0364	15
12 0	8689	2475	8470	4554	8252	6633	8033	8712	7815	0791	78 0
15	8634	2731	8406	4852	8178	6974	7951	9096	7723	1218	45
30	8578	2986	8341	5151	8104	7315	7867	9480	7630	1644	30
45	5.8521	1.3242	6.8274	1.5449	7.8027	1.7656	8.7781	1.9863	9.7534	2.2070	15
13 0	8462	3497	8206	5747	7950	7996	7693	2.0246	7437	2495	77 0
15	8403	3752	8137	6044	7870	8336	7604	0628	7338	2920	45
30	8342	4007	8066	6341	7790	8676	7513	1010	7237	3345	30
45	8281	4261	7994	6638	7707	9015	7421	1392	7134	3769	15
14 0	8218	4515	7921	6935	7624	9354	7327	1773	7030	4192	76 0
15	8154	4769	7846	7231	7538	9692	7231	2154	6923	4615	45
30	8089	5023	7770	7527	7452	2.0030	7133	2534	6815	5038	30
45	8023	5276	7693	7822	7364	0368	7034	2914	6705	5460	15
15 0	7956	5529	7615	8117	7274	0706	6933	3294	6593	5882	75 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° ′
B'ng	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng

B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng
° ′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° ′
15 15	0.9648	0.2630	1.9296	0.5261	2.8944	0.7891	3.8591	1.0521	4.8239	1.3152	74 45
30	9636	2672	9273	5345	8909	8017	8545	0690	8182	3362	30
45	9625	2714	9249	5429	8874	8143	8498	0858	8123	3572	15
16 0	9613	2756	9225	5513	8838	8269	8450	1025	8063	3782	74 0
15	9600	2798	9201	5597	8801	8395	8402	1193	8002	3991	45
30	9588	2840	9176	5680	8765	8520	8353	1361	7941	4201	30
45	9576	2882	9151	5764	8727	8646	8303	1528	7879	4410	15
17 0	9563	2924	9126	5847	8689	8771	8252	1695	7815	4619	73 0
15	9550	2965	9100	5931	8651	8896	8201	1862	7751	4827	45
30	9537	3007	9074	6014	8612	9021	8149	2028	7686	5035	30
45	0.9524	0.3049	1.9048	0.6097	2.8572	0.9146	3.8096	1.2195	4.7620	1.5243	15
18 0	9511	3090	9021	6180	8532	9271	8042	2361	7553	5451	72 0
15	9497	3132	8994	6263	8491	9395	7988	2527	7485	5658	45
30	9483	3173	8966	6346	8450	9519	7933	2692	7416	5865	30
45	9469	3214	8939	6429	8408	9643	7877	2858	7347	6072	15
19 0	9455	3256	8910	6511	8366	9767	7821	3023	7276	6278	71 0
15	9441	3297	8882	6594	8323	9891	7764	3188	7204	6485	45
30	9426	3338	8853	6676	8279	1.0014	7706	3352	7132	6690	30
45	9412	3379	8824	6758	8235	0138	7647	3517	7059	6896	15
20 0	9397	3420	8794	6840	8191	0261	7588	3681	6985	7101	70 0
15	0.9382	0.3461	1.8764	0.6922	2.8146	1.0384	3.7528	1.3845	4.6910	1.7306	45
30	9367	3502	8733	7004	8100	0506	7467	4008	6834	7510	30
45	9351	3543	8703	7086	8054	0629	7405	4172	6757	7715	15
21 0	9336	3584	8672	7167	8007	0751	7343	4335	6679	7918	69 0
15	9320	3624	8640	7249	7960	0873	7280	4498	6600	8122	45
30	9304	3665	8608	7330	7913	0995	7217	4660	6521	8325	30
45	9288	3706	8576	7411	7864	1117	7152	4822	6440	8528	15
22 0	9272	3746	8544	7492	7816	1238	7087	4984	6359	8730	68 0
15	9255	3786	8511	7573	7766	1359	7022	5146	6277	8932	45
30	9239	3827	8478	7654	7716	1481	6955	5307	6194	9134	30
45	0.9222	0.3867	1.8444	0.7734	2.7666	1.1601	3.6888	1.5468	4.6110	1.9336	15
23 0	9205	3907	8410	7815	7615	1722	6820	5629	6025	9537	67 0
15	9188	3947	8376	7895	7564	1842	6752	5790	5940	9737	45
30	9171	3987	8341	7975	7512	1962	6682	5950	5853	9937	30
45	9153	4027	8306	8055	7459	2082	6612	6110	5766	2.0137	15
24 0	9135	4067	8271	8135	7406	2202	6542	6269	5677	0337	66 0
15	9118	4107	8235	8214	7353	2322	6470	6429	5588	0536	45
30	9100	4147	8199	8294	7299	2441	6398	6588	5498	0735	30
45	9081	4187	8163	8373	7244	2560	6326	6746	5407	0933	15
25 0	9063	4226	8126	8452	7189	2679	6252	6905	5315	1131	65 0
15	0.9045	0.4266	1.8089	0.8531	2.7134	1.2797	3.6178	1.7063	4.5223	2.1328	45
30	9026	4305	8052	8610	7078	2915	6103	7220	5129	1526	30
45	9007	4344	8014	8689	7021	3033	6028	7378	5035	1722	15
26 0	8988	4384	7976	8767	6964	3151	5952	7535	4940	1919	64 0
15	8969	4423	7937	8846	6906	3269	5875	7692	4844	2114	45
30	8949	4462	7899	8924	6848	3386	5797	7848	4747	2310	30
45	8930	4501	7860	9002	6789	3503	5719	8004	4649	2505	15
27 0	8910	4540	7820	9080	6730	3620	5640	8160	4550	2700	63 0
15	8890	4579	7780	9157	6671	3736	5561	8315	4451	2894	45
30	8870	4617	7740	9235	6610	3852	5480	8470	4351	3087	30
45	0.8850	0.4656	1.7700	0.9312	2.6550	1.3968	3.5400	1.8625	4.4249	2.3281	15
28 0	8829	4695	7659	9389	6488	4084	5318	8779	4147	3474	62 0
15	8809	4733	7618	9466	6427	4200	5236	8933	4045	3666	45
30	8788	4772	7576	9543	6365	4315	5153	9086	3941	3858	30
45	8767	4810	7535	9620	6302	4430	5069	9240	3836	4049	15
29 0	8746	4848	7492	9696	6239	4544	4985	9392	3731	4240	61 0
15	8725	4886	7450	9772	6175	4659	4900	9545	3625	4431	45
30	8704	4924	7407	9848	6111	4773	4814	9697	3518	4621	30
45	8682	4962	7364	9924	6046	4886	4728	9849	3410	4811	15
30 0	8660	5000	7321	1.0000	5981	5000	4641	2.0000	3301	5000	60 0
° ′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	′ °
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15	15	5.7887	1.5782	6.7535	1.8412	7.7183	2.1042	8.6831	2.3673	9.6479	2.6303	74	45
	30	7818	6034	7454	8707	7090	1379	6727	4051	6363	6724		30
	45	7747	6286	7372	9001	6996	1715	6621	4430	6246	7144		15
16	0	7676	6538	7288	9295	6901	2051	6514	4807	6126	7564	74	0
	15	7603	6790	7203	9588	6804	2386	6404	5185	6005	7983		45
	30	7529	7041	7117	9881	6706	2721	6294	5561	5882	8402		30
	45	7454	7292	7030	2.0174	6606	3056	6181	5938	5757	8820		15
17	0	7378	7542	6941	0466	6504	3390	6067	6313	5630	9237	73	0
	15	7301	7792	6851	0758	6402	3723	5952	6689	5502	9654		45
	30	7223	8042	6760	1049	6297	4056	5835	7064	5372	3.0071		30
	45	5.7144	1.8292	6.6668	2.1341	7.6192	2.4389	8.5716	2.7438	9.5240	3.0486		15
18	0	7063	8541	6574	1631	6085	4721	5595	7812	5106	0902	72	0
	15	6982	8790	6479	1921	5976	5053	5473	8185	4970	1316		45
	30	6899	9038	6383	2211	5866	5384	5349	8557	4832	1730		30
	45	6816	9286	6285	2501	5754	5715	5224	8930	4693	2144		15
19	0	6731	9534	6186	2790	5641	6045	5097	9301	4552	2557	71	0
	15	6645	9781	6086	3078	5527	6375	4968	9672	4409	2969		45
	30	6558	2.0028	5985	3366	5411	6705	4838	3.0043	4264	3381		30
	45	6471	0275	5882	3654	5294	7033	4706	0413	4118	3792		15
20	0	6382	0521	5778	3941	5175	7362	4572	0782	3969	4202	70	0
	15	5.6291	2.0767	6.5673	2.4228	7.5055	2.7689	8.4437	3.1151	9.3819	3.4612		45
	30	6200	1012	5567	4515	4934	8017	4300	1519	3667	5021		30
	45	6108	1257	5459	4800	4811	8343	4162	1886	3514	5429		15
21	0	6015	1502	5351	5086	4686	8669	4022	2253	3358	5837	69	0
	15	5920	1746	5241	5371	4561	8995	3881	2619	3201	6244		45
	30	5825	1990	5129	5655	4433	9320	3738	2985	3042	6650		30
	45	5729	2233	5017	5939	4305	9645	3593	3350	2881	7056		15
22	0	5631	2476	4903	6222	4175	9969	3447	3715	2718	7461	68	0
	15	5532	2719	4788	6505	4043	3.0292	3299	4078	2554	7865		45
	30	5433	2961	4672	6788	3910	0615	3149	4442	2388	8268		30
	45	5.5332	2.3203	6.4554	2.7070	7.3776	3.0937	8.2998	3.4804	9.2220	3.8671		15
23	0	5230	3444	4435	7351	3640	1258	2845	5166	2050	9073	67	0
	15	5127	3685	4315	7632	3503	1580	2691	5527	1879	9474		45
	30	5024	3925	4194	7912	3365	1900	2535	5887	1706	9875		30
	45	4919	4165	4072	8192	3225	2220	2378	6247	1531	4.0275		15
24	0	4813	4404	3948	8472	3084	2539	2219	6606	1355	0674	66	0
	15	4706	4643	3823	8750	2941	2858	2059	6965	1176	1072		45
	30	4598	4882	3697	9029	2797	3175	1897	7322	0996	1469		30
	45	4489	5120	3570	9306	2651	3493	1733	7679	0814	1866		15
25	0	4378	5357	3442	9583	2505	3809	1568	8036	0631	2262	65	0
	15	5.4267	2.5594	6.3312	2.9860	7.2356	3.4125	8.1401	3.8391	9.0446	4.2657		45
	30	4155	5831	3181	3.0136	2207	4441	1233	8746	0259	3051		30
	45	4042	6067	3049	0411	2056	4756	1063	9100	0070	3445		15
26	0	3928	6302	2916	0686	1904	5070	0891	9453	8.9879	3837	64	0
	15	3812	6537	2781	0960	1750	5383	0719	9806	9687	4229		45
	30	3696	6772	2645	1234	1595	5696	0544	4.0158	9493	4620		30
	45	3579	7006	2509	1507	1438	6008	0368	0509	9298	5010		15
27	0	3460	7239	2370	1779	1281	6319	0191	0859	9101	5399	63	0
	15	3341	7472	2231	2051	1121	6630	0012	1209	8902	5787		45
	30	3221	7705	2091	2322	0961	6940	7.9831	1557	8701	6175		30
	45	5.3099	2.7937	6.1949	3.2593	7.0799	3.7249	7.9649	4.1905	8.8499	4.6561		15
28	0	2977	8168	1806	2863	0636	7558	9465	2252	8295	6947	62	0
	15	2853	8399	1662	3132	0471	7866	9280	2599	8089	7332		45
	30	2729	8630	1517	3401	0305	8173	9094	2944	7882	7716		30
	45	2604	8859	1371	3669	0138	8479	8905	3289	7673	8099		15
29	0	2477	9089	1223	3937	6.9970	8785	8716	3633	7462	8481	61	0
	15	2350	9317	1075	4203	9800	9090	8525	3976	7250	8862		45
	30	2221	9545	0925	4470	9628	9394	8332	4318	7036	9242		30
	45	2092	9773	0774	4735	9456	9697	8138	4659	6820	9622		15
30	0	1962	3.0000	0622	5000	9282	4.0000	7942	5000	6603	5.0000	60	0
°	′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	°	′
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B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng
° '	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	° '
30 15	0.8638	0.5038	1.7277	1.0075	2.5915	1.5113	3.4553	2.0151	4.3192	2.5189	59 45
30	8616	5075	7233	0151	5849	5226	4465	0302	3081	5377	30
45	8594	5113	7188	0226	5782	5339	4376	0452	2970	5565	15
31 0	8572	5150	7142	0301	5715	5451	4287	0602	2858	5752	59 0
15	8549	5188	7098	0375	5647	5563	4196	0751	2746	5939	45
30	8526	5225	7053	0450	5579	5675	4106	0900	2632	6125	30
45	8504	5262	7007	0524	5511	5786	4014	1049	2518	6311	15
32 0	8480	5299	6961	0598	5441	5898	3922	1197	2402	6496	58 0
15	8457	5336	6915	0672	5372	6008	3829	1345	2286	6681	45
30	8434	5373	6868	0746	5302	6119	3736	1492	2170	6865	30
45	0.8410	0.5410	1.6821	1.0819	2.5231	1.6229	3.3642	2.1639	4.2052	2.7049	15
33 0	8387	5446	6773	0893	5160	6339	3547	1786	1934	7232	57 0
15	8363	5483	6726	0966	5089	6449	3451	1932	1814	7415	45
30	8339	5519	6678	1039	5017	6558	3355	2077	1694	7597	30
45	8315	5556	6629	1111	4944	6667	3259	2223	1573	7779	15
34 0	8290	5592	6581	1184	4871	6776	3162	2368	1452	7960	56 0
15	8266	5628	6532	1256	4798	6884	3064	2512	1329	8140	45
30	8241	5664	6483	1328	4724	6992	2965	2656	1206	8320	30
45	8216	5700	6433	1400	4649	7100	2866	2800	1082	8500	15
35 0	8192	5736	6383	1472	4575	7207	2766	2943	0958	8679	55 0
15	0.8166	0.5771	1.6333	1.1543	2.4499	1.7314	3.2666	2.3086	4.0832	2.8857	45
30	8141	5807	6282	1614	4423	7421	2565	3228	0706	9035	30
45	8116	5842	6231	1685	4347	7527	2463	3370	0579	9212	15
36 0	8090	5878	6180	1756	4271	7634	2361	3511	0451	9389	54 0
15	8064	5913	6129	1826	4193	7739	2258	3652	0322	9565	45
30	8039	5948	6077	1896	4116	7845	2154	3793	0193	9741	30
45	8013	5983	6025	1966	4038	7950	2050	3933	0063	9916	15
37 0	7986	6018	5973	2036	3959	8054	1945	4073	3.9932	3.0091	53 0
15	7960	6053	5920	2106	3880	8159	1840	4212	9800	0265	45
30	7934	6088	5867	2175	3801	8263	1734	4350	9668	0438	30
45	0.7907	0.6122	1.5814	1.2244	2.3721	1.8367	3.1628	2.4489	3.9534	3.0611	15
38 0	7880	6157	5760	2313	3640	8470	1520	4626	9400	0783	52 0
15	7853	6191	5706	2382	3560	8573	1413	4764	9266	0955	45
30	7826	6225	5652	2450	3478	8675	1304	4901	9130	1126	30
45	7799	6259	5598	2518	3397	8778	1195	5037	8994	1296	15
39 0	7771	6293	5543	2586	3314	8880	1086	5173	8857	1466	51 0
15	7744	6327	5488	2654	3232	8981	0976	5308	8720	1635	45
30	7716	6361	5432	2722	3149	9082	0865	5443	8581	1804	30
45	7688	6394	5377	2789	3065	9183	0754	5578	8442	1972	15
40 0	7660	6428	5321	2856	2981	9284	0642	5712	8302	2139	50 0
15	0.7632	0.6461	1.5265	1.2922	2.2897	1.9384	3.0529	2.5845	3.8162	3.2306	45
30	7604	6494	5208	2989	2812	9483	0416	5978	8020	2472	30
45	7576	6528	5151	3055	2727	9583	0303	6110	7878	2638	15
41 0	7547	6561	5094	3121	2641	9682	0188	6242	7735	2803	49 0
15	7518	6593	5037	3187	2555	9780	0074	6374	7592	2967	45
30	7490	6626	4979	3252	2469	9879	2.9958	6505	7448	3131	30
45	7461	6659	4921	3318	2382	9976	9842	6635	7303	3294	15
42 0	7431	6691	4863	3383	2294	2.0074	9726	6765	7157	3457	48 0
15	7402	6724	4804	3447	2207	0171	9609	6895	7011	3618	45
30	7373	6756	4746	3512	2118	0268	9491	7024	6864	3780	30
45	0.7343	0.6788	1.4686	1.3576	2.2030	2.0364	2.9373	2.7152	3.6716	3.3940	15
43 0	7314	6820	4627	3640	1941	0460	9254	7280	6568	4100	47 0
15	7284	6852	4567	3704	1851	0555	9135	7407	6419	4259	45
30	7254	6884	4507	3767	1761	0651	9015	7534	6269	4418	30
45	7224	6915	4447	3830	1671	0745	8895	7661	6118	4576	15
44 0	7193	6947	4387	3893	1580	0840	8774	7786	5967	4733	46 0
15	7163	6978	4326	3956	1489	0934	8652	7912	5815	4890	45
30	7133	7009	4265	4018	1398	1027	8530	8036	5663	5045	30
45	7102	7040	4204	4080	1306	1120	8407	8161	5509	5201	15
45 0	7071	7071	4142	4142	1213	1213	8284	8284	5355	5355	45 0
° '	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	° '
B'ng	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		B'ng

B'ng		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng	
°	′	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	°	′
30	15	5.1830	3.0226	6.0468	3.5264	6.9107	4.0302	7.7745	4.5340	8.6384	5.0377	59	45
	30	1698	0452	0314	5528	8930	0603	7547	5678	6163	0754		30
	45	1564	0678	0158	5791	8753	0903	7347	6016	5941	1129		15
31	0	1430	0902	0002	6053	8573	1203	7145	6353	5717	1504	59	0
	15	1295	1126	5.9844	6314	8393	1502	6942	6690	5491	1877		45
	30	1158	1350	9685	6575	8211	1800	6738	7025	5264	2250		30
	45	1021	1573	9525	6835	8028	2097	6532	7359	5035	2621		15
32	0	0883	1795	9363	7094	7844	2394	6324	7693	4805	2992	58	0
	15	0744	2017	9201	7353	7658	2689	6116	8025	4573	3361		45
	30	0603	2238	9037	7611	7471	2984	5905	8357	4339	3730		30
	45	5.0462	3.2458	5.8873	3.7868	6.7283	4.3278	7.5694	4.8688	8.4104	5.4097		15
33	0	0320	2678	8707	8125	7094	3571	5480	9018	3867	4464	57	0
	15	0177	2898	8540	8381	6903	3863	5266	9346	3629	4829		45
	30	0033	3116	8372	8636	6711	4155	5050	9674	3389	5194		30
	45	4.9888	3334	8203	8890	6518	4446	4832	5.0001	3147	5557		15
34	0	9742	3552	8033	9144	6323	4735	4613	0327	2904	5919	56	0
	15	9595	3768	7861	9396	6127	5024	4393	0652	2659	6280		45
	30	9448	3984	7689	9648	5930	5312	4171	0977	2413	6641		30
	45	9299	4200	7515	9900	5732	5600	3948	1300	2165	7000		15
35	0	9149	4415	7341	4.0150	5532	5886	3724	1622	1915	7358	55	0
	15	4.8998	3.4629	5.7165	4.0400	6.5331	4.6172	7.3498	5.1943	8.1664	5.7715		45
	30	8847	4842	6988	0649	5129	6456	3270	2263	1412	8070		30
	45	8694	5055	6810	0897	4926	6740	3042	2582	1157	8425		15
36	0	8541	5267	6631	1145	4721	7023	2812	2901	0902	8779	54	0
	15	8387	5479	6451	1392	4516	7305	2580	3218	0644	9131		45
	30	8231	5689	6270	1638	4309	7586	2347	3534	0386	9482		30
	45	8075	5899	6088	1883	4100	7866	2113	3849	0125	9832		15
37	0	7918	6109	5904	2127	3891	8145	1877	4163	7.9864	6.0182	53	0
	15	7760	6318	5720	2371	3680	8424	1640	4476	9600	0529		45
	30	7601	6526	5535	2613	3468	8701	1402	4789	9335	0876		30
	45	4.7441	3.6733	5.5348	4.2855	6.3255	4.8977	7.1162	5.5100	7.9069	6.1222		15
38	0	7281	6940	5161	3096	3041	9253	0921	5410	8801	1566	52	0
	15	7119	7146	4972	3337	2825	9528	0679	5718	8532	1909		45
	30	6956	7351	4783	3576	2609	9801	0435	6026	8261	2251		30
	45	6793	7555	4592	3815	2391	5.0074	0190	6333	7988	2592		15
39	0	6629	7759	4400	4052	2172	0346	6.9943	6639	7715	2932	51	0
	15	6464	7962	4207	4289	1951	0616	9695	6943	7439	3271		45
	30	6297	8165	4014	4525	1730	0886	9446	7247	7162	3608		30
	45	6131	8366	3819	4761	1507	1155	9196	7550	6884	3944		15
40	0	5963	8567	3623	4995	1284	1423	8944	7851	6604	4279	50	0
	15	4.5794	3.8767	5.3426	4.5229	6.1059	5.1690	6.8691	5.8151	7.6323	6.4612		45
	30	5624	8967	3228	5461	0832	1956	8437	8450	6041	4945		30
	45	5454	9166	3030	5693	0605	2221	8181	8748	5756	5276		15
41	0	5283	9364	2830	5924	0377	2485	7924	9045	5471	5606	49	0
	15	5110	9561	2629	6154	0147	2748	7666	9341	5184	5935		45
	30	4937	9757	2427	6383	5.9916	3010	7406	9636	4896	6262		30
	45	4763	9953	2224	6612	9685	3271	7145	9929	4606	6588		15
42	0	4589	4.0148	2020	6839	9452	3530	6883	6.0222	4314	6913	48	0
	15	4413	0342	1815	7066	9217	3789	6620	0513	4022	7237		45
	30	4237	0535	1609	7291	8982	4047	6355	0803	3728	7559		30
	45	4.4059	4.0728	5.1403	4.7516	5.8746	5.4304	6.6089	6.1092	7.3432	6.7880		15
43	0	3881	0920	1195	7740	8508	4560	5822	1380	3135	8200	47	0
	15	3702	1111	0986	7963	8270	4815	5553	1666	2837	8518		45
	30	3522	1301	0776	8185	8030	5068	5284	1952	2537	8835		30
	45	3342	1491	0565	8406	7789	5321	5013	2236	2236	9151		15
44	0	3160	1680	0354	8626	7547	5573	4741	2519	1934	9466	46	0
	15	2978	1867	0141	8845	7304	5823	4467	2801	1630	9779		45
	30	2795	2055	4.9928	9064	7060	6073	4193	3082	1325	7.0091		30
	45	2611	2241	9713	9281	6815	6321	3917	3361	1019	0401		15
45	0	2426	2426	9497	9497	6569	6569	3640	3640	0711	0711	45	0
°	′	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	°	′
B'ng		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		B'ng	

For Correction of Courses on Random Lines.

Minutes.	10 Chains.	20 Chains.	40 Chains.	80 Chains.	Minutes.
1	.003	.006	.012	.023	1
2	006	012	023	046	2
3	009	017	035	070	3
4	012	023	046	093	4
5	014	029	058	116	5
6	017	035	070	140	6
7	020	041	081	163	7
8	023	046	093	186	8
9	026	052	105	209	9
10	029	058	116	233	10
11	032	064	128	256	11
12	035	070	140	279	12
13	038	076	151	302	13
14	041	081	163	326	14
15	044	087	174	349	15
16	046	093	186	372	16
17	049	099	198	396	17
18	052	105	209	419	18
19	055	110	221	442	19
20	058	116	233	466	20
21	061	122	244	488	21
22	064	128	256	512	22
23	067	134	268	535	23
24	070	140	279	558	24
25	073	145	291	581	25
26	076	151	302	605	26
27	078	157	314	628	27
28	081	163	326	651	28
29	084	169	337	674	29
30	087	174	349	698	30
31	090	180	361	722	31
32	093	186	372	744	32
33	096	192	384	767	33
34	099	198	395	790	34
35	102	204	407	814	35
36	105	209	419	837	36
37	108	215	430	860	37
38	110	221	442	883	38
39	113	227	454	906	39
40	116	233	465	929	40
41	119	238	477	953	41
42	122	244	488	976	42
43	125	250	500	999	43
44	128	256	512	1.022	44
45	131	262	523	1.045	45
46	134	268	535	1.068	46
47	137	273	546	1.092	47
48	140	279	558	1.115	48
49	142	285	570	1.138	49
50	145	291	581	1.161	50
51	148	297	593	1.184	51
52	151	302	605	1.207	52
53	154	308	616	1.230	53
54	157	314	628	1.253	54
55	160	320	639	1.276	55
56	163	326	651	1.299	56
57	166	331	663	1.323	57
58	169	337	674	1.346	58
59	172	343	686	1.369	59
60	174	349	698	1.392	60

1° - - - - - - 11°			11° - - - - - - 21°			21° - - - - - - 31°			31° - - - - - - 41°		
Angle.		Secant.	Angle.		Secant.	Angle.		Secant.	Angle.		Secant.
°	'		°	'		°	'		°	'	
1		1.00015	11		1.01872	21		1.07115	31		1.16663
	10	1.00021		10	1.01930		10	1.07235		10	1.16868
	20	1.00027		20	1.01989		20	1.07356		20	1.17075
	30	1.00034		30	1.02049		30	1.07479		30	1.17283
	40	1.00042		40	1.02110		40	1.07602		40	1.17493
	50	1.00051		50	1.02171		50	1.07727		50	1.17704
2		1.00061	12		1.02234	22		1.07853	32		1.17918
	10	1.00072		10	1.02298		10	1.07981		10	1.18133
	20	1.00083		20	1.02362		20	1.08109		20	1.18350
	30	1.00095		30	1.02428		30	1.08239		30	1.18569
	40	1.00108		40	1.02494		40	1.08370		40	1.18790
	50	1.00122		50	1.02562		50	1.08503		50	1.19012
3		1.00137	13		1.02630	23		1.08636	33		1.19236
	10	1.00153		10	1.02700		10	1.08771		10	1.19463
	20	1.00169		20	1.02770		20	1.08907		20	1.19691
	30	1.00187		30	1.02842		30	1.09044		30	1.19920
	40	1.00205		40	1.02914		40	1.09183		40	1.20152
	50	1.00224		50	1.02987		50	1.09323		50	1.20386
4		1.00244	14		1.03061	24		1.09464	34		1.20622
	10	1.00265		10	1.03137		10	1.09606		10	1.20859
	20	1.00287		20	1.03213		20	1.09750		20	1.21099
	30	1.00309		30	1.03290		30	1.09895		30	1.21341
	40	1.00333		40	1.03368		40	1.10041		40	1.21584
	50	1.00357		50	1.03447		50	1.10189		50	1.21830
5		1.00382	15		1.03528	25		1.10338	35		1.22070
	10	1.00408		10	1.03609		10	1.10488		10	1.22327
	20	1.00435		20	1.03691		20	1.10640		20	1.22579
	30	1.00463		30	1.03774		30	1.10793		30	1.22833
	40	1.00491		40	1.03858		40	1.10947		40	1.23089
	50	1 00521		50	1.03944		50	1.11103		50	1.23347
6		1.00551	16		1.04030	26		1.11260	36		1.23607
	10	1.00582		10	1.04117		10	1.11419		10	1.23869
	20	1.00614		20	1.04206		20	1.11579		20	1.24134
	30	1.00647		30	1.04295		30	1.11740		30	1.24400
	40	1.00681		40	1.04385		40	1.11903		40	1.24669
	50	1.00715		50	1.04477		50	1.12067		50	1.24940
7		1.00751	17		1.04569	27		1.12233	37		1.25214
	10	1.00787		10	1.04663		10	1.12400		10	1.25489
	20	1.00825		20	1.04757		20	1.12568		20	1.25767
	30	1.00863		30	1.04853		30	1.12738		30	1.26047
	40	1.00902		40	1.04950		40	1.12910		40	1 26330
	50	1.00942		50	1.05047		50	1.13083		50	1.26615
8		1.00983	18		1.05146	28		1.13257	38		1.26902
	10	1.01024		10	1.05246		10	1.13433		10	1.27191
	20	1.01067		20	1.05347		20	1.13610		20	1.27483
	30	1.01111		30	1.05449		30	1.13789		30	1.27778
	40	1.01155		40	1.05552		40	1.13970		40	1.28075
	50	1.01200		50	1.05657		50	1.14152		50	1.28374
9		1.01247	19		1.05762	29		1.14335	39		1.28676
	10	1.01294		10	1.05869		10	1.14521		10	1.28980
	20	1.01342		20	1.05976		20	1.14707		20	1.29287
	30	1.01391		30	1.06085		30	1.14896		30	1.29597
	40	1.01440		40	1.06195		40	1.15085		40	1.29909
	50	1.01491		50	1.06306		50	1.15277		50	1.30223
10		1.01543	20		1.06418	30		1.15470	40		1.30541
	10	1.01595		10	1.06531		10	1.15665		10	1.30831
	20	1.01649		20	1.06645		20	1.15861		20	1.31183
	30	1.01703		30	1.06761		30	1.16059		30	1.31509
	40	1.01758		40	1.06878		40	1.16259		40	1.31837
	50	1.01815		50	1.06995		50	1.16460		50	1.32168

41° - - - - - - 46°			46° - - - - - - 51°			51° - - - - - - 56°			56° - - - - - - 61°		
Angle.		Secant.	Angle.		Secant.	Angle.		Secant.	Angle.		Secant.
°	'		°	'		°	'		°	'	
41		1.32501	46		1.43956	51		1.58902	56		1.78829
	10	1.32838		10	1.44391		10	1.59475		10	1.79604
	20	1.33177		20	1.44831		20	1.60054		20	1.80388
	30	1.33519		30	1.45274		30	1.60639		30	1.81180
	40	1.33864		40	1.45721		40	1.61229		40	1.81981
	50	1.34212		50	1.46173		50	1.61825		50	1.82790
42		1.34563	47		1.46628	52		1.62427	57		1.83608
	10	1.34917		10	1.47087		10	1.63035		10	1.84435
	20	1.35274		20	1.47551		20	1.63648		20	1.85271
	30	1.35634		30	1.48019		30	1.64268		30	1.86116
	40	1.35997		40	1.48491		40	1.64894		40	1.86990
	50	1.36363		50	1.48967		50	1.65526		50	1.87834
43		1.36733	48		1.49448	53		1.66164	58		1.88708
	10	1.37105		10	1.49933		10	1.66809		10	1.89591
	20	1.37481		20	1.50422		20	1.67460		20	1.90485
	30	1.37860		30	1.50916		30	1.68117		30	1.91388
	40	1.38242		40	1.51415		40	1.68782		40	1.92302
	50	1.38628		50	1.51918		50	1.69452		50	1.93226
44		1.39016	49		1.52425	54		1.70130	59		1.94160
	10	1.39409		10	1.52938		10	1.70815		10	1.95106
	20	1.39804		20	1.53455		20	1.71506		20	1.96062
	30	1.40203		30	1.53977		30	1.72205		30	1.97029
	40	1.40606		40	1.54504		40	1.72911		40	1.98008
	50	1.41012		50	1.55036		50	1.73624		50	1.98998
45		1.41421	50		1.55572	55		1.74345	60		2.00000
	10	1.41835		10	1.56114		10	1.75073		10	2.01014
	20	1.42251		20	1.56661		20	1.75808		20	2.02039
	30	1.42670		30	1.57213		30	1.76552		30	2.03077
	40	1.43096		40	1.57771		40	1.77303		40	2.04128
	50	1.43524		50	1.58333		50	1.78062		50	2.05191

TABLE VIII. AZIMUTH OF POLARIS AT ELONGATION.

Lat.	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895
°	° '	° '	° '	° '	° '	° '	° '	° '	° '	° '
+ 25	1 26.0	1 25.7	1 25.3	1 25.0	1 24.6	1 24.3	1 23.9	1 23.6	1 23 2	1 22.9
26	26.7	26.4	26.0	25.7	25.3	25.0	24.6	24.3	23.9	23.6
27	27.5	27.1	26.8	26.4	26.0	25.7	25.4	25.1	24.7	24.3
28	28.3	27.9	27.6	27.2	26.8	26.5	26.2	25.8	25.4	25.1
29	29.1	28.8	28.4	28.0	27.6	27.3	27.0	26.6	26.3	25.9
30	30.0	29.6	29.3	28.9	28 5	28.2	27.8	27.5	27.1	26.8
31	30.9	30.5	30.2	29.8	29.4	29.1	28.8	28.4	28.0	27.6
32	31.9	31.5	31.2	30.8	30.4	30.1	29.7	29.3	29.0	28.6
33	33.0	32.6	32.2	31.8	31.4	31.1	30.7	30.3	30.0	29.6
34	34.0	33.6	33.3	32.9	32.5	32.1	31.8	31.4	31.0	30.6
35	35 2	34.8	34.4	34.0	33.6	33.2	32.9	32.5	32.1	31.7
36	36.4	36.0	35.6	35.2	34.8	34.4	34.0	33.6	33.2	32.9
37	37.6	37.2	36.8	36.4	36.0	35.6	35.2	34.8	34.5	34.1
38	38.9	38.5	38.1	37.7	37.3	36.9	36.5	36.1	35.7	35.3
39	40.3	39.9	39.5	39.1	38.7	38.3	37.9	37.5	37.1	36.7
40	41.8	41.4	41.0	40.5	40.1	39.7	39.3	38.9	38.5	38.1
41	43.3	42.9	42.5	42.0	41.6	41.2	40.8	40.4	40.0	39.6
42	44.9	44.5	44.1	43.6	43.2	42.8	42.4	42.0	41.5	41.1
43	46.6	46.1	45.7	45.3	44.9	44.4	44.0	43.6	43.2	42.7
44	48.4	47.9	47.5	47.1	46.6	46.2	45.8	45.3	44.9	44.4
45	50.3	49.8	49.4	48.9	48.5	48.1	47.6	47.1	46.7	46.2
46	52.2	51.8	51.3	50.9	50.4	50.0	49.5	49.0	48.6	48.2
47	54.3	53.8	53.4	52.9	52.5	52.0	51.5	51.0	50.6	50.2
48	56.5	56.0	55.6	55.1	54.6	54.2	53.7	53.2	52.8	52.3
49	1 58.8	1 58.3	1 57.9	57.4	56.9	56.5	56.0	55.5	55.0	54 5
+ 50	2 01.3	2 00.8	2 00.3	1 59.8	1 59.3	1 58.8	1 58.4	1 57.9	1 57.4	1 56.9

For one revolution of Gradienter Screw, used in finding d' *and* d, *Art. 142.*

Elevation.	Multipliers of r.		Elevation.	Multipliers of r.		Elevat'n.	Multipliers of r.	
$e.$	Inc. Dist.	Hor. Dist.	$e.$	Inc. Dist.	Hor. Dist.	$e.$	Inc. Dist.	Hor. Dist.
° '			° '			° '		
1 00	99.97	99.95	14	96.79	93.91	22 30	92.01	85.01
2	99.90	99.84	14 30	96.56	93.49	23	91.66	84.37
3	99.81	99.67	15	96.33	93.05	23 30	91.31	83.73
4	99.69	99.44	15 30	96.09	92.59	24	90.95	83.08
5	99.53	99.15	16	95.85	92.13	24 30	90.58	82.42
6	99.35	98.80	16 30	95.60	91.66	25	90.21	81.75
7	99.13	98.39	17	95.34	91.17	25 30	89.83	81.08
8	98.89	97.92	17 30	95.07	90.67	26	89.44	80.39
9	98.61	97.39	18	94.80	90.15	26 30	89.05	79.69
10	98.31	96.81	18 30	94.52	89.63	27	88.65	78.99
10 30	98.14	96.50	19	94.23	89.09	27 30	88.24	78.27
11	97.97	96.17	19 30	93.93	88.54	28	87.83	77.55
11 30	97.79	95.83	20	93.63	87.97	28 30	87.40	76.81
12	97.61	95.47	20 30	93.32	87.41	29	86.98	76.07
12 30	97.41	95.10	21	93.00	86.82	29 30	86.54	75.32
13	97.21	94.72	21 30	92.68	86.23	30	86.10	74.57
13 30	97.00	94.22	22	92.34	85.61	30 30	85.66	73.81

TABLE X. ANGLES OF ELEVATION,

Corresponding to numbers of Revolution of the Gradienter Screw.

Screw.	Angle.	Screw.	Angle.	Screw.	Angle.
Rev. Div.	° ' "	Rev. Div.	° ' "	Rev. Div.	° ' "
0 1	0 00 21	0 10	0 3 26	1 00	0 34 23
2	0 41	20	6 53	2	1 08 45
3	1 02	30	10 19	3	1 43 08
4	1 23	40	13 45	4	2 17 30
5	1 43	50	17 11	5	2 51 53
6	2 04	60	20 38	6	3 26 15
7	2 24	70	24 04	7	4 00 38
8	2 44	80	27 30	8	4 35 00
9	3 06	90	30 56	9	5 09 23
0 10	0 3 26	1 00	0 34 23	10 00	5 43 45

In Declination, for use with Solar Compass.

Hour Angle.	Declinations.								
	For Latitude 30°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06
	For Latitude 32° 30′.								
0h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′03	1′15	1′31	1 53	2 20	3 05	4 25	7 36	
	For Latitude 35°.								
0h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 00
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	
	For Latitude 37° 30′.								
0h.	18″	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″
2	22	28	35	42	50	1′00	1′12	1 26	1 45
3	29	36	43	52	1′02	1 14	1 29	1 49	2 16
4	43	51	1′01	1′13	1 27	1 49	2 14	2 54	4 05
5	1′11	1′26	1 54	2 10	2 49	3 55	6 15	14 58	
	For Latitude 40°.								
0h.	21″	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″
2	25	32	39	46	52	1 06	1 19	1 35	1 57
3	33	40	48	57	1′08	1 21	1 38	2 02	2 36
4	47	55	1′06	1′19	1 36	1 58	2 30	3 21	4 59
5	1′15	1′31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 42° 30′.								
0h.	24″	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″
2	28	35	39	50	1′00	1 12	1 26	1 45	2 11
3	36	43	52	1′02	1 13	1 29	1 49	2 17	2 59
4	50	1′00	1′11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′16	1 36	1 58	2 30	3 22	5 00	9 24		
	For Latitude 45°.								
0h.	27″	33″	40″	48″	57″	1′08″	1′21″	1′39″	2′02″
2	32	39	46	52	1′06	1 19	1 35	1 57	2 29
3	40	47	56	1′07	1 21	1 38	2 00	2 34	3 29
4	54	1′04	1′16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′23	1 41	2 05	2 41	3 40	5 40	12 02		
	For Latitude 47° 30″								
0h.	30″	36″	44″	52″	1′02″	1′14″	1′29″	1′49″	2′18″
2	35	42	50	1′00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′27	1 46	2 12	2 52	4 01	6 30	16 19		

Showing Number of Acres served by drains having bottom widths from 1 ft. to 10 ft., with side slopes of 1 to 1, on the supposition of 1 inch rainfall in 24 hours, one-half of which reaches the drain.

Computed by B. F. WELLES, C. E., *Marshall, Mich.*

Fall in feet per			Bottom Widths.					
			1 ft.		2 ft.		3 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.	2 ft, deep.	3 ft. deep.
1.6	.030	.04	407	981	594	1311	780	1649
2.0	.038	.05	462	1105	665	1473	879	1861
2.4	.045	.06	508	1218	732	1622	968	2047
2.8	.053	.07	553	1319	797	1762	1053	2217
3.2	.060	.08	592	1416	853	1889	1128	2377
3 6	.070	.09	631	1505	939	2009	1198	2529
4.0	.076	.10	666	1590	959	2115	1264	2665
4.8	.091	.12	733	1748	1057	2333	1391	2935
5.6	.110	.14	794	1895	1143	2523	1499	3172
6.4	.120	.16	852	2030	1225	2700	1612	3401
7 2	.136	.18	905	2154	1300	2869	1715	3612
8.0	.150	.20	956	2273	1373	3031	1809	3815

Fall in Feet per			Bottom Widths.					
			4 ft.		5 ft.		6 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.
1.6	.030	.04	976	2003	1171	2357	1368	2716
2.0	.038	.05	1094	2249	1316	2650	1541	3046
2.4	.045	.06	1206	2477	1448	2910	1699	3362
2.8	.053	.07	1308	2684	1572	3158	1835	3642
3.2	.060	.08	1404	2872	1684	3384	1970	3908
3.6	.070	.09	1494	3049	1790	3598	2097	4150
4.0	.076	.10	1579	3227	1894	3800	2211	4322
4.8	.091	.12	1731	3553	2089	4173	2436	4810
5.6	.110	.14	1878	3849	2257	4512	2632	5203
6.4	.120	.16	2013	4115	2415	4838	2820	5571
7 2	.136	.18	2137	4372	2566	5141	3001	5927
8.0	.150	.20	2256	4609	2705	5412	3165	6257

Fall in Feet per			Bottom Widths.					
			7 ft.		8 ft.		10 ft.	
1 mi.	100 ft.	8 rd.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.	2 ft. deep.	3 ft. deep.
1.6	.030	.04	1574	3074	1767	3458	2177	4179
2.0	.038	.05	1768	3469	1983	3877	2448	4710
2.4	.045	.06	1946	3807	2181	4265	2695	5169
2.8	.053	.07	2115	4131	2369	4622	2921	5609
3.2	.060	.08	2259	4427	2538	4948	3136	6014
3.6	.070	.09	2403	4695	2697	5258	3327	6378
4.0	.076	.10	2538	4963	2848	5552	3508	6745
4.8	.091	.12	2792	5443	3130	6094	3857	7405
5.6	.110	.14	3029	5894	3393	6591	4184	8010
6.4	.120	.16	3240	6317	3628	7057	4489	8578
7.2	.136	.18	3443	6715	3854	7507	4760	9110
8.0	.150	.20	3629	7078	4070	7910	5038	9623

FORMULAS: $v = \left\{ \frac{af \times 9000}{p} \right\}^{1/2} - 0.11.$ $Q = av.$ $A = Q \times 47.6 =$ Acreage.

Showing Number of Acres drained by different sizes of tile, the rainfall being considered as equal to one-half inch in depth each 24 hours. Computed by R. C. CARPENTER, *Lansing, Mich.*

Rate of Inclination.	Acres Drained.						
Feet to one of rise.	2-in Tile.	3-in. Tile.	4-in. Tile.	6-in. Tile.	8-in. Tile.	10-in. Tile.	12-in. Tile.
1 foot in 10 feet	6.6	18.9	....	....	....	....	....
1 " 20 "	4.7	13.0	26.8	....	....	....	....
1 " 25 "	4.2	11.4	24.0	66.2	....	....	....
1 " 30 "	3·9	10.9	21.9	61.5	126.4	....	....
1 " 40 "	3.4	9.4	19.0	53.3	109.6	190.5	....
1 " 50 "	3.0	8.4	17.0	47.7	98.0	170.4	269.0
1 " 60 "	2.7	7.6	15.6	43.4	90.0	156.0	246.0
1 " 70 "	2 5	6.9	14.5	39.9	83.0	144.4	228.1
1 " 80 "	2.3	6.5	13.4	37.2	77.0	135.0	213.0
1 " 90 "	2.2	6.1	12.6	35.0	72.5	127.0	200.5
1 " 100 "	2.0	5.7	11.9	33.1	69.2	120.6	190.5
1 " 150 "	1.6	4.5	9.5	26.6	56.0	97.3	154.4
1 " 200 "	....	3.9	8.2	22.8	48.0	83.9	132.5
1 " 250 "	....	3.5	7.5	20.4	43.4	74.4	117.0
1 " 300 "	....	....	6.9	18.4	38.2	65.5	107.0
1 " 400 "	....	....	5.9	16.5	34.6	60.3	90.7
1 " 500 "	....	....	....	14.8	30.1	54.0	81.6
1 " 600 "	....	....	....	13.3	28.0	48.6	74.0
1 " 800 "	....	....	....	....	24.0	41.9	65.0
1 " 1,000 "	....	....	....	....	21.2	37.2	56.0
1 " 1,500 "	....	....	....	....	....	30.8	47.0
1 " 2,000 "	....	....	....	....	....	....	40.8

NOTE.—Tile should not be laid to grades where numbers are replaced by dashes.

TABLE XIV. CAPACITY OF TILE.

Showing carrying capacity of different sizes of tile, in gallons. From Catalogue of the Bennett Sewer Pipe Co., Jackson, Mich.

Size of pipes.	Carrying Capacity—Gallons per Minute.							
	1½ in. fall per 100 ft.	3 in. fall per 100 ft.	6 in. fall per 100 ft.	9 in. fall per 100 ft.	1 ft. fall per 100 ft.	1½ ft. fall per 100 ft.	2 ft. fall per 100 ft.	3 ft. fall per 100 ft.
2¼ in.	14	20	28	34	40	49	55	68
3 "	21	30	42	52	60	74	85	104
4 "	36	52	76	92	108	132	148	184
5 "	54	78	111	134	159	192	219	269
6 "	84	120	169	206	240	294	338	414
8 "	144	208	304	368	432	528	592	736
9 "	232	330	470	570	660	810	930	1140
10 "	267	378	463	655	803	926	1340	1613
12 "	470	680	960	1160	1360	1670	1920	2350
15 "	830	1180	1680	2040	2370	2920	3340	4100
18 "	1300	1850	2630	3200	3740	4600	5270	6470
20 "	1760	2450	3450	4180	4860	5980	6850	8410
24 "	3000	4152	5871	7202	8303	10021	11743	14466

Latitude.	1 mile.			2 miles.			3 miles.			4 miles.		
°	°	′	″	°	′	″	°	′	″	°	′	″
30	89	59	30	89	58	59.9	89	58	29.9	89	57	59.9
31		59	28.8		58	57.5		58	26.3		57	55.0
32		59	27.5		58	55.0		58	22.5		57	50.0
33		59	26.2		58	52.5		58	18.7		57	44.9
34		59	24.9		58	49.9		58	14.8		57	39.7
35		59	23.6		58	47.2		58	10.8		57	34.4
36		59	22.2		58	44.4		58	06.8		57	28.9
37		59	20.8		58	41.6		58	02.5		57	23.3
38		59	19.4		58	38.8		57	58.2		57	17.5
39		59	17.9		58	35.8		57	53.7		57	11.6
40		59	16.4		58	32.8		57	49.2		57	05.5
41		59	14.8		58	29.6		57	44.4		56	59.3
42		59	13.2		58	26.4		57	39.6		56	52.8
43		59	11.5		58	23.1		57	34.6		56	46.2
44		59	09.8		58	19.6		57	29.5		56	39.3
45		59	08.0		58	16.1		57	24.1		56	32.1
46		59	06.2		58	12.4		57	18.6		56	24.8
47	89	59	04.3	89	58	08.6	89	57	12.9	89	56	17.1

Latitude.	5 miles.			6 miles.			7 miles.			8 miles.		
°	°	′	″	°	′	″	°	′	″	°	′	″
30	89	57	29.9	89	56	59.8	89	56	29.8	89	55	59.8
31		57	23.8		56	52.5		56	21.3		55	50.0
32		57	17.5		56	45.0		56	12.5		55	40.0
33		57	11.2		56	37.4		56	03.6		55	29.9
34		57	04.6		56	29.6		55	54.5		55	19.4
35		56	58.0		56	21.6		55	45.2		55	08.8
36		56	51.1		56	13.4		55	35.6		54	57.8
37		56	44.1		56	05.0		55	25.8		54	46.6
38		56	36.9		55	56.3		55	15.7		54	35.1
39		56	29.6		55	47.5		55	05.4		54	23.3
40		56	21.9		55	38.3		54	54.7		54	11.1
41		56	14.1		55	28.9		54	43.7		53	58.5
42		56	06.0		55	19.2		54	32.4		53	45.6
43		55	57.7		55	09.2		54	20.8		53	32.3
44		55	49.1		54	58.9		54	08.7		53	18.5
45		55	40.2		54	48.2		53	56.3		53	04.3
46		55	31.0		54	37.2		53	43.4		52	49.5
47	89	55	21.4	89	54	25.7	89	53	30.0	89	52	34.3

Latitude.	9 miles.			10 miles.			11 miles.			12 miles.		
°	°	′	″	°	′	″	°	′	″	°	′	″
30	89	55	29.8	89	54	59.7	89	54	29.7	89	53	59.7
31		55	18.8		54	47.6		54	16.3		53	45.1
32		55	07.6		54	35.1		54	02.6		53	30.1
33		54	56.1		54	22.3		53	48.5		53	14.8
34		54	44.4		54	09.3		53	34.2		52	59.1
35		54	32.3		53	55.9		53	19.5		52	43.1
36		54	20.0		53	42.3		53	04.5		52	26.7
37		54	07.4		53	28.2		52	49.1		52	09.9
38		53	54.5		53	13.9		52	33.2		51	52.6
39		53	41.2		52	59.1		52	17.0		51	34.9
40		53	27.5		52	43.8		52	00.2		51	16.6
41		53	13.4		52	28.2		51	43.0		50	57.8
42		52	58.8		52	12.0		51	25.2		50	38.4
43		52	43.8		51	55.4		51	06.9		50	18.5
44		52	28.4		51	38.2		50	48.0		49	57.8
45		52	12.3		51	20.4		50	28.4		49	36.4
46		51	55.7		51	01.9		50	08.1		49	14.3
47	89	51	38.6	89	50	42.9	89	49	47.2	89	48	51.4

Latitude.	1 mile.	2 miles.	3 miles.	4 miles.
°	*Feet.*	*Feet.*	*Feet.*	*Feet.*
30	0.39	1.54	3.47	6.17
31	0.40	1.60	3.61	6.42
32	0.42	1.67	3.76	6.67
33	0.43	1.73	3.90	6.93
34	0.45	1.80	4.05	7.20
35	0.47	1.87	4.20	7.47
36	0.48	1.94	4.36	7.75
37	0.50	2.01	4.52	8.04
38	0.52	2.08	4.69	8.33
39	0.54	2.16	4.86	8.63
40	0.56	2.24	5.03	8.95
41	0.58	2.32	5.21	9.27
42	0.60	2.40	5.40	9.59
43	0.62	2.48	5.59	9.93
44	0.64	2.57	5.79	10.29
45	0.67	2.66	5.99	10.65
46	0.69	2.76	6.20	11.02
47	0.71	2.85	6.42	11.41

Latitude.	5 miles.	6 miles.	7 miles.	8 miles.
°	*Feet.*	*Feet.*	*Feet.*	*Feet.*
30	9.64	13.88	18.89	24.67
31	10.03	14.44	19.66	25.68
32	10.42	15.02	20.44	26.69
33	10.82	15.60	21.23	27.74
34	11.25	16.20	22.05	28.80
35	11.68	16.81	22.89	29.89
36	12.11	17.41	23.74	31.01
37	12.57	18.09	24.62	32.16
38	13.02	18.75	25.52	33.33
39	13.49	19.43	26.44	34.54
40	13.98	20.11	27.40	35.78
41	14.48	20.85	28.37	37.06
42	14.99	21.59	29.38	38.38
43	15.52	22.35	30.42	39.74
44	16.07	23.14	31.50	41.14
45	16.64	23.96	32.61	42.59
46	17.21	24.80	33.76	44.10
47	17.83	25.68	34.95	45.65

Latitude.	9 miles.	10 miles.	11 miles.	12 miles.
°	*Feet.*	*Feet.*	*Feet.*	*Feet.*
30	31.23	38.55	46.65	55.52
31	32.49	40.12	48.54	57.77
32	33.78	41.71	50.47	60.06
33	35.10	43.34	52.44	62.41
34	36.45	45.00	54.45	64.80
35	37.83	46.71	56.62	67.26
36	39.25	48.45	58.63	69.77
37	40.70	50.24	60.79	72.35
38	42.19	52.08	63.02	75.00
39	43.71	53.97	65.30	77.71
40	45.29	55.91	67.65	80.51
41	46.90	57.91	70.07	83.39
42	48.57	59.97	72.56	86.35
43	50.29	62.09	75.13	89.41
44	52.07	64.28	77.78	92.57
45	53.91	66.55	80.53	95.84
46	55.81	68.90	83.37	99.22
47	57.78	71.34	86.32	102.72